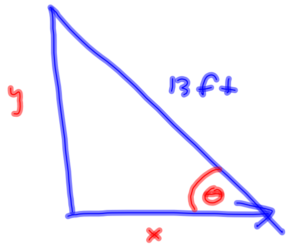
Questions?

$2^{nd}$ "Test" wed.




$$\left.\frac{dx}{dt}\right|_{x=12} = \frac{5\,ft}{s}$$

(a) $\left.\frac{dy}{dt}\right|_{x=12} = ?$

$$12^2 + y^2 = 13^2$$
$$y^2 = 169 - 144 = 25$$
$$y = \pm 5$$
Take $y > 0$
$$y = 5$$

$$\frac{d}{dt}\left[x^2 + y^2 = 13^2\right]$$
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$2(12)(5) + 2(5)\frac{dy}{dt} = 0$$

(b) $\left.\frac{dA}{dt}\right|_{x=12} = ?$

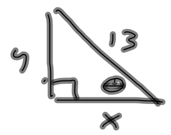
$$(fg)' = f'g + fg'$$
$$(xy)' = x'y + xy'$$

$$A = \frac{1}{2}xy$$

$$\left.\frac{dA}{dt}\right|_{x=12} = \left.\left(\frac{1}{2}\frac{dx}{dt}y + \frac{1}{2}x\frac{dy}{dt}\right)\right|_{x=12}$$

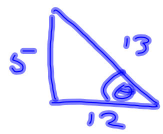
$$= \frac{1}{2}(5)(5) + \frac{1}{2}(12)(\text{Part a})$$

(c) $\left.\frac{d\theta}{dt}\right|_{x=12} = ?$




$$\frac{y}{13} = \sin\theta$$

$$y = 13\sin\theta$$




$$\left.\frac{dy}{dt}\right|_{x=12} = \left.13\cos\theta\,\frac{d\theta}{dt}\right|_{x=12}$$

$$(\text{Part a}) = \left.13\cdot\frac{12}{13}\cdot\frac{d\theta}{dt}\right|_{x=12}$$

$$\frac{y}{x} = \tan\theta$$

$$\left.\frac{y'x - yx'}{x^2}\right|_{x=12} = \left.\sec^2\theta\,\frac{d\theta}{dt}\right|_{x=12}$$

$$\frac{(\text{part a})(12) - (5)(5)}{12^2} = \left.\frac{13^2}{12^2}\cdot\frac{d\theta}{dt}\right|_{x=12}$$

$$A = \frac{1}{2}r^2\theta \qquad \theta \text{ in Radians.}$$

$$A = \frac{1}{2}r^2\cdot 2\pi = \pi r^2$$

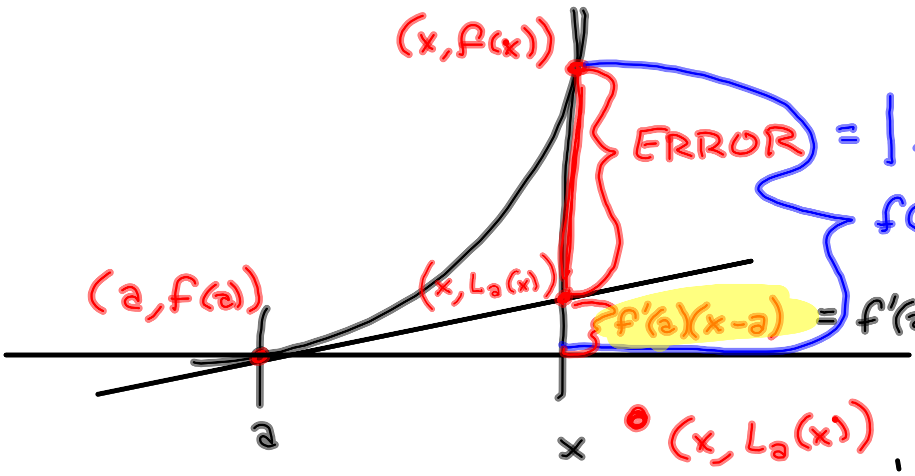
Sep 28-12:09 PM

$L_a(x)$ = Linearization of $f$ @ $x=a$

= Tangent line approximation (near $x=a$)

$$L_a(x) = f'(a)(x-a) + f(a)$$

$$= f(a) + f'(a)(x-a)$$




$= |\Delta y - dy|$

$f(x) - f(a) = \Delta y$

$= f'(a)\Delta x \equiv f'(a)dx$

$dx \equiv \Delta x$

$$\frac{dy}{dx} = f'(x)$$

$$dy = f'(x)\,dx \approx \Delta y$$

$$L_a(x) = f(a) + f'(x)(x-a)$$

Sep 28-12:29 PM

$$L_a(x) = f'(a)\,(x-a) + f(a)$$

$$\left(\text{steepness}\right)\left(\begin{matrix}\text{change}\\ \text{horiz.}\end{matrix}\right) + \text{old ht}$$

$$\begin{aligned} dy &= f'(x)\,(x-a) \\ &= f'(x)\,dx \\ &= f'(x)\,\Delta x \end{aligned}$$

Typically you'll be given $\Delta x$ as a number.
Differentials

Typically you'll be given initial x-value & new x-value
Linearization

$dx = \Delta x = x - a$

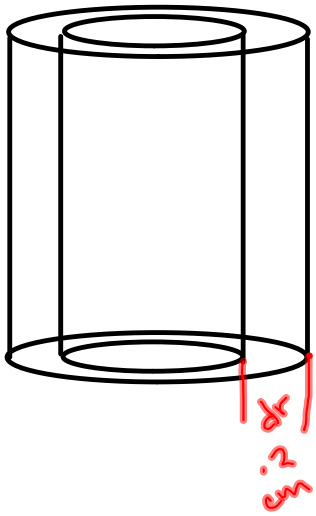
Recall paint sides of can.

How much paint.

$h = 30\text{cm}$

$r = 20\text{cm}$

$dr = \Delta r = .2\text{ cm}$ = thickness of coat of paint.




$$V = \pi r^2 h$$

$$\frac{dV}{dr} = 2\pi r h + \pi r^2 \frac{dh}{dr}$$

$\frac{dh}{dr} \to 0$

$$\frac{dV}{dr} = 2\pi r h$$

$$dV = 2\pi r h\, dr$$

$$\Delta V \approx dV = 2\pi(20)(30)(.2)$$

$$\approx 753.9822369\text{ cm}^3$$

Actual $\Delta V$: $V(20.2) - V(20)$

$$= \pi(20.2)^2(30) - \pi(20)^2(30)$$

$$\approx 38456.86399 - 37699.1184$$

$$\approx 757.752511\text{ cm}^3$$

Error $= |\Delta V - dV| \approx |757.75\ldots - 753.98\ldots|$

See #s 29-34 $\approx 3.7699142$

what's the % error?

$$\left(\frac{3.7699142}{757.752511}\right)(100\%) \approx .4975\%$$

$$L_a(x) = f'(a)(x-a) + f(a) \approx f(x)$$

$$dy = f'(x)\,dx \approx \Delta y$$

differential of y

differential of x.

$$f(x) \approx f'(a)(x-a) + \underline{f(a)}$$

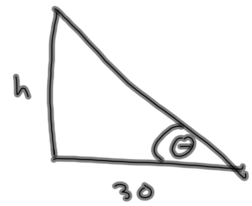
$$\boxed{\underbrace{f(x) - f(a)}_{\Delta y}} \approx f'(a)(x-a)$$

$$\Delta y \approx f'(a)\,\Delta x = f'(a)\,dx$$

$$\boxed{dy \approx \Delta y \approx f'(a)\,dx}$$

3.9 #s 2, 8, 13, 15, 18, 22, 29, 30, 41, 44, 48, 54

44



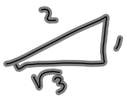

Want error in $h < 4\%$ of actual.

$h = 30 \tan(75°)$

$h = 30 \dfrac{\sin(30° + 45°)}{\cos(30° + 45°)}$

$h = 30 \tan\theta$

$$= 30 \frac{\sin\left(\frac{\pi}{6} + \frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{6} + \frac{\pi}{4}\right)} = \frac{30 \sin\frac{\pi}{6}\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos\frac{\pi}{6}}{\cos\frac{\pi}{6}\cos\frac{\pi}{4} - \sin\frac{\pi}{6}\sin\frac{\pi}{4}}$$




$$30\left(\frac{\frac{1}{2}\cdot\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}} - \frac{1}{2}\cdot\frac{1}{\sqrt{2}}}\right) = \frac{30\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)}{\frac{\sqrt{3}-1}{2\sqrt{2}}}$$


$= h = 30\left(\dfrac{\sqrt{3}+1}{\sqrt{3}-1}\right)$ our guess of actual ht.

$h = 30 \tan\theta$

$\dfrac{dh}{d\theta} = 30\sec^2\theta \implies$ Differential of h is

$$\underbrace{\Delta h \approx dh = (30\sec^2\theta)\, d\theta}_{\text{I'm using this as an estimate of error of measurement.}} \overset{\text{want}}{<} \underbrace{\frac{4}{100}\left(30\,\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)}_{4\%\text{ of actual}}$$

Want $|\text{Actual} - \text{Projected}| <$

$|h_{act} - h_{guess}| <$

error in measurement using differentials.

Use $\Delta h \approx dh$ as our measure of the error.

Relate $\Delta h$ to $\Delta\theta$

$$\left(\frac{2}{5\cancel{0}}\right)(3\cancel{0}) = \frac{6}{5}$$

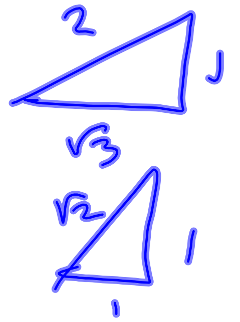
$$\underbrace{dh = (30\sec^2\theta)\,d\theta}_{\text{Relates change in height to change in angle}} \quad \overset{\text{want}}{<} \frac{4}{100}\left(30\,\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$$

$$d\theta \leq \frac{\left(\frac{6}{5}\right)\frac{\sqrt{3}+1}{\sqrt{3}-1}}{30\sec^2\left(\frac{\pi}{6}+\frac{\pi}{4}\right)} = \frac{\frac{\cancel{6}^{1}}{(5)(\cancel{30}_{5})}\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)}{\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^2}$$

$$= \frac{1}{25}\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)\left(\frac{8}{(\sqrt{3}-1)^2}\right) = \frac{8}{25}\left(\frac{\sqrt{3}+1}{(\sqrt{3}-1)^3}\right)$$

$$\approx 2.228512513 \text{ rads}$$

Book Says: 0.01 rads $\approx .57^\circ$




$$\frac{1}{\cos^2\left(\frac{\pi}{6}+\frac{\pi}{4}\right)} = \frac{1}{\left(\cos\frac{\pi}{6}\cos\frac{\pi}{4} - \sin\frac{\pi}{6}\sin\frac{\pi}{4}\right)^2}$$

$$= \left(\frac{\sqrt{3}}{2}\frac{1}{\sqrt{2}} - \frac{1}{2}\frac{1}{\sqrt{2}}\right)^2 = \left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^2$$

Oct 1-12:34 PM

$\frac{\sin}{\cos} \cdot \cos^2 = \sin \cos$


$\frac{\pi}{6} + \frac{\pi}{4} = \frac{4\pi + 3\pi}{12}$

$= \frac{7\pi}{12}$

$30 \sec^2 \theta \, d\theta < (.04)(30 \tan(75°))$

$d\theta < (.04)\cancel{\left(\frac{30}{30}\right)}\left(\frac{\tan 75°}{\sec^2 75°}\right)$

$= .04 \sin \frac{7\pi}{12} \cos \frac{7\pi}{12}$

Maple says

$$.04 \cdot \sin\left(\frac{7 \cdot \mathrm{Pi}}{12}\right) \cdot \cos\left(\frac{7 \cdot \mathrm{Pi}}{12}\right)$$

$$-0.04 \sin\left(\frac{5}{12}\pi\right) \cos\left(\frac{5}{12}\pi\right)$$

$evalf(\%)$

$$-0.01000000000$$ radians

$$\frac{\% \cdot 180}{\mathrm{Pi}}$$

$$-\frac{1.800000000}{\pi}$$

$evalf(\%)$

$$-0.5729577950$$ degrees

Estimate $\sin(32^\circ)$ w/o calc.

$f(a) = f(\frac{\pi}{6}) = \sin(\frac{\pi}{6}) = \frac{1}{2}$

$f(x) = \sin x$

$x_0 = a = 30^\circ = \frac{\pi}{6}$ rads.

$f'(x) = \cos x$

$f'(a) = \cos(\frac{\pi}{6})$

$= \frac{\sqrt{3}}{2}$

$\Delta x = 2^\circ$

$a + \Delta x = 30^\circ + 2^\circ$

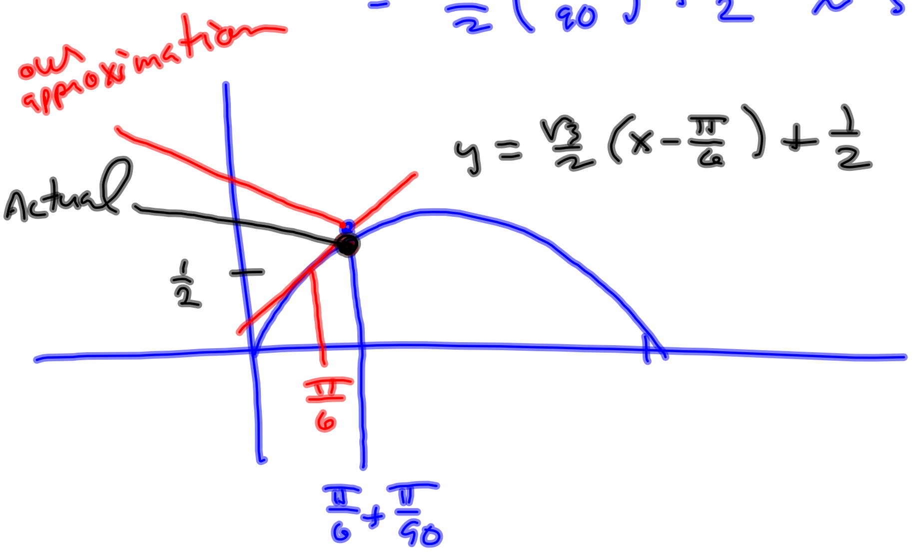
$= \frac{\pi}{6} + \frac{2\pi}{180} = \underline{\frac{\pi}{6} + \frac{\pi}{90}}$

$f(a + \Delta x) \approx \underline{f(a)} + \underline{f'(a)}(\underline{x - a}) = L_a(x)$

$= L_{\frac{\pi}{6}}(x) = f'(a)(x-a) + f(a)$

$= \frac{\sqrt{3}}{2}\left(\frac{\pi}{6} + \frac{\pi}{90} - \frac{\pi}{6}\right) + \frac{1}{2}$

$= \frac{\sqrt{3}}{2}\left(\frac{\pi}{90}\right) + \frac{1}{2} \approx \sin(32^\circ)$



Paint a 1" cube with a $\frac{1}{10}$th inch coat of paint. Use differentials to estimate paint used

$V = x^3$

$$\boxed{\begin{aligned} &\underline{dV} = 3x^2\,\underline{dx} \approx \Delta V \\ &dV\Big|_{\substack{x=1 \\ dx=.1}} = 3(1)^2(.1) = .3 \text{ cubic inches of paint.}\end{aligned}}$$

$\frac{dV}{dx} = 3x^2 \rightsquigarrow dV = 3x^2 dx$

$(1.1)^3 = 1.331$

$\Delta V = 1.331 - 1 = .331$ cubic inches