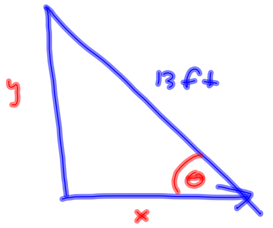
Questions?

$2^{nd}$ "Test" Wed.




$$\left.\frac{dx}{dt}\right|_{x=12} = \frac{5 \text{ ft}}{s}$$

(a) $\left.\frac{dy}{dt}\right|_{x=12} = ?$

$$12^2 + y^2 = 13^2$$
$$y^2 = 169 - 144 = 25$$
$$y = \pm 5$$
Take $y > 0$
$$y = 5$$

$$\frac{d}{dt}\left[x^2 + y^2 = 13^2\right]$$
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$2(12)(5) + 2(5)\frac{dy}{dt} = 0$$

(b) $\left.\frac{dA}{dt}\right|_{x=12} = ?$

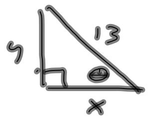
$$(fg)' = f'g + fg'$$
$$(xy)' = x'y + xy'$$

$$A = \frac{1}{2}xy$$

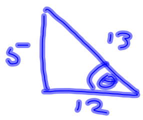
$$\left.\frac{dA}{dt}\right|_{x=12} = \left.\left(\frac{1}{2}\frac{dx}{dt}y + \frac{1}{2}x\frac{dy}{dt}\right)\right|_{x=12}$$

$$= \frac{1}{2}(5)(5) + \frac{1}{2}(12)(\text{Part a})$$

(c) $\left.\frac{d\theta}{dt}\right|_{x=12} = ?$




$$\frac{y}{13} = \sin\theta$$




$$y = 13\sin\theta$$

$$\left.\frac{dy}{dt}\right|_{x=12} = 13\cos\theta \left.\frac{d\theta}{dt}\right|_{x=12}$$

$$(\text{Part a}) = 13 \cdot \frac{12}{13} \cdot \left.\frac{d\theta}{dt}\right|_{x=12}$$

$$\frac{y}{x} = \tan\theta$$

$$\left.\frac{y'x - yx'}{x^2}\right|_{x=12} = \sec^2\theta \left.\frac{d\theta}{dt}\right|_{x=12}$$

$$\frac{(\text{part a})(12) - (5)(5)}{12^2} = \frac{13^2}{12^2} \cdot \left.\frac{d\theta}{dt}\right|_{x=12}$$

$A = \frac{1}{2}r^2\theta$    $\theta$ in Radians.

$$A = \frac{1}{2}r^2 \cdot 2\pi = \pi r^2$$

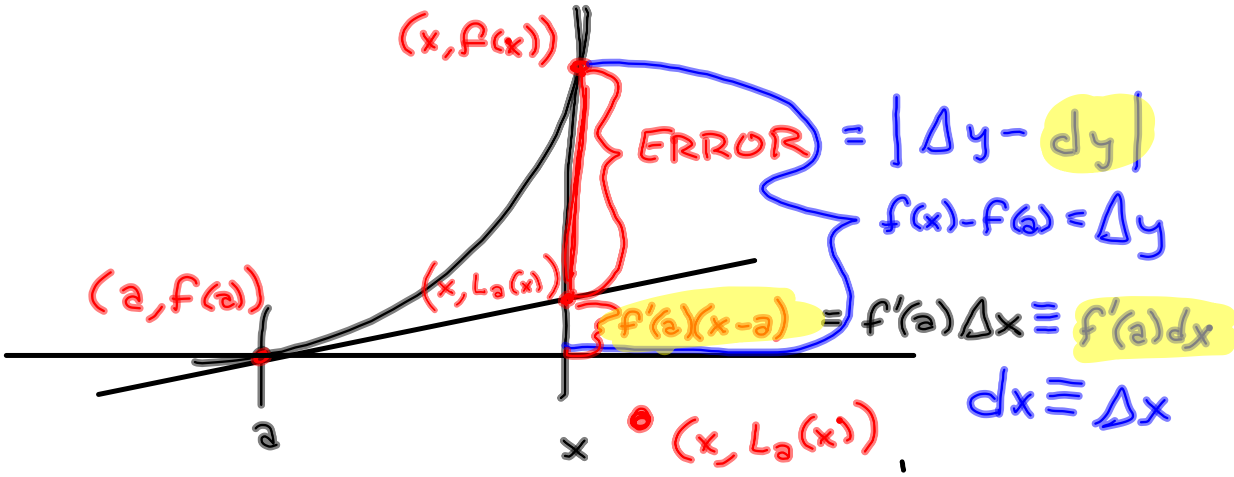
$$\text{Arclength} = s = r\theta$$

$L_a(x)$ = Linearization of $f$ @ $x=a$

= Tangent line approximation (near $x=a$)

$$L_a(x) = f'(a)(x-a) + f(a)$$

$$= f(a) + f'(a)(x-a)$$




$$\frac{dy}{dx} = f'(x)$$

$$dy = f'(x)\,dx \approx \Delta y$$

$$L_a(x) = f(a) + f'(x)(x-a)$$

$$L_a(x) = f'(a)(x-a) + f(a)$$

$$\left(\text{steepness}\right)\left(\begin{array}{c}\text{change}\\ \text{horiz.}\end{array}\right) + \text{old ht}$$

$$dy = f'(x)(x-a)$$
$$= f'(x)\,dx$$
$$= f'(x)\,\Delta x$$

Typically you'll be given $\Delta x$ as a number.
Differentials

$dx = \Delta x = x - a$

Typically you'll be given initial x-value & new x-value
Linearization

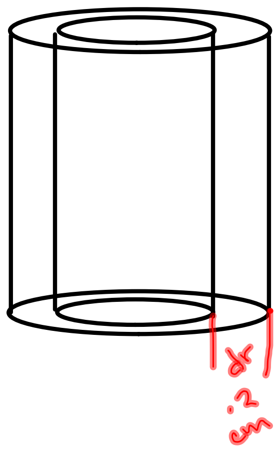
Recall paint sides of can.

How much paint.

$h = 30\text{cm}$

$r = 20\text{cm}$

$dr = \Delta r = .2\text{ cm}$ = thickness of coat of paint.




$V = \pi r^2 h$

$\frac{dV}{dr} = 2\pi r h + \pi r^2 \frac{dh}{dr} \to 0$

$\frac{dV}{dr} = 2\pi r h$

$dV = 2\pi r h\, dr$

$\Delta V \approx dV = 2\pi(20)(30)(.2)$

$\approx 753.9822369\text{ cm}^3$

Actual $\Delta V$: $V(20.2) - V(20)$

$= \pi(20.2)^2(30) - \pi(20)^2(30)$

$\approx 38456.86399 - 37699.11184$

$\approx 757.752511\text{ cm}^3$

Error $= |\Delta V - dV| \approx |757.75\ldots - 753.98\ldots|$

See #s 29-34 $\approx 3.7699142$

what's the % error?

$\left(\frac{3.7699142}{757.752511}\right)(100\%) \approx .4975\%$

$$L_a(x) = f'(a)(x-a) + f(a) \approx f(x)$$

$$dy = f'(x)\,dx \approx \Delta y$$

differential of y

differential of x.

$$f(x) \approx f'(a)(x-a) + f(a)$$

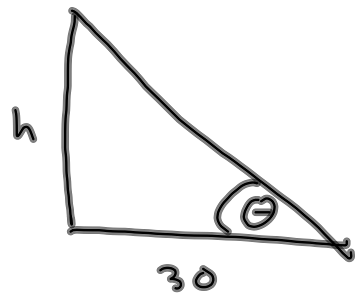
$$f(x) - f(a) \approx f'(a)(x-a)$$

$$\Delta y \approx f'(a)\Delta x = f'(a)\,dx$$

$$dy \approx \Delta y \approx f'(a)\,dx$$

3.9 #s 2, 8, 13, 15, 18, 22, 29, 30, 41, 44, 48, 54

44



$h = 30 \tan \theta$

want $|dh| = \left| \dfrac{30 \sec^2 \theta \, d\theta}{30 \tan \theta} \right| <$