3.3 #20

$$f(t) = \frac{t^2-1}{t^2+t-2}$$

$$f'(t) = \frac{2t(t^2+t-2) - (t^2-1)(2t+1)}{(t^2+t-2)^2}$$

$$= \frac{2t^3+2t^2-4t-(2t^3+t^2-2t-1)}{((t+2)(t-1))^2}$$

$$= \frac{2t^3+2t^2-4t-2t^3-t^2+2t+1}{(\quad)^2}$$

$$= \frac{t^2-2t+1}{(\quad)^2} = \frac{(t-1)^2}{(t+2)^2(t-1)^2} = \frac{1}{(t+2)^2}$$

Dummy $f(t) = \frac{t^2-1}{t^2+t-2} = \frac{\cancel{(t-1)}(t+1)}{(t+2)\cancel{(t-1)}} = \boxed{\frac{t+1}{t+2}}$

$$= \frac{t+1}{t+2} \quad (t \neq 1)$$

$$\Rightarrow f'(t) = \frac{1(t+2)-(t+1)(1)}{(t+2)^2} = \frac{t+2-t-1}{(t+2)^2}$$

$$= \frac{1}{(t+2)^2}$$

3.7 #36

$$x\sin(2y) = y\cos(2x) \quad @ \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$$

$$\frac{\pi}{4}\sin(\pi) = \frac{\pi}{2}\cos\left(\frac{\pi}{2}\right)$$

$$0 = 0$$

$$1\cdot\sin(2y) + x\cdot 2y'\cos(2y) = y'\cos(2x) + y(-\sin(2x))(2)$$

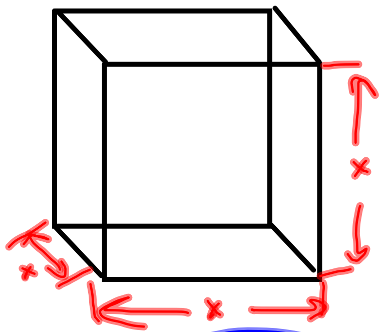
$$\sin(2y) + 2xy'\cos(2y) = y'\cos(2x) - 2y\sin(2x)$$

And then a miracle occurs. ⋮

$$y' = \frac{-2y\sin(2x) - \sin(2y)}{2x\cos(2y) - \cos(2x)}$$

$$\frac{d}{dx}\left[\cos\left(\frac{\pi}{6}x\right)\right] = -\sin\left(\frac{\pi}{6}x\right)\cdot\frac{\pi}{6}$$

$$= -\frac{\pi}{6}\sin\left(\frac{\pi}{6}x\right)$$



Surface area of cube is

$$\boxed{A(x) = 6x^2}$$

$$\boxed{\frac{dA}{dt} = 72}\ \frac{in^2}{s} \text{ is}$$

Given

find $\left.\frac{dV}{dt}\right|_{x=3\text{ in}}$

$$\boxed{\frac{dA}{dt} = 72 = 12x\frac{dx}{dt}}$$

This'll give you $\left.\frac{dx}{dt}\right|_{x=3}$ which you'll need for the final act.

$V = x^3$

$$\left.\frac{dV}{dt}\right|_{x=3} = \left.3x^2\frac{dx}{dt}\right|_{x=3} = 3(3)^2 \cdot \left.\frac{dx}{dt}\right|_{x=3}$$

To date, we've used slope of a straight line and limits to talk about the slope of a curve at one pt.

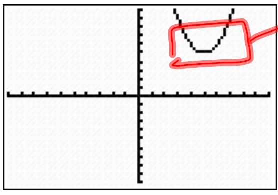
$$\frac{y_2-y_1}{x_2-x_1} \rightsquigarrow \frac{f(x_2)-f(x_1)}{x_2-x_1} \text{ "="} \frac{f(x+h)-f(x)}{h}$$

where $x_2 = x_1 + h$
$= x + h$

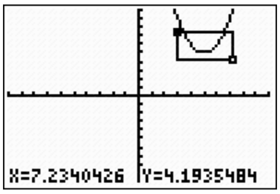
$\xrightarrow{h \to 0} f'(x)$.

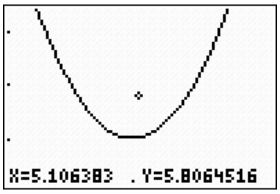
This works for "smooth" functions.

Smooth functions are "locally linear"

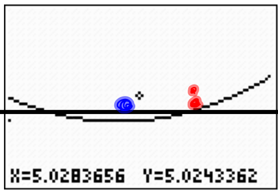


zoom in





$(x-5)^2+5$
in $[-10,10]\times[-10,10]$



Zoomed-in 3 times
small window
curve flattens out.
$\therefore$ straightens out,
very close to a straight line

Line stays "close" over short distances.

$y = f'(x_0)(x-x_0) + f(x_0)$ is tangent line to $f(x)$ @ $(x_0, f(x_0))$

Very good approximation to $f(x)$ when $x \approx x_0$

Use the tangent line to approximate

$\sqrt{98}$ Let $x_0 = 100$

$$f(x) = \sqrt{x} = x^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$f(x_0) = \sqrt{100} = 10$$

$$f'(x_0) = \frac{1}{2\sqrt{100}} = \frac{1}{2(10)} = \frac{1}{20} = .05$$

Then $f(98) \approx$ tangent line

$$y = f'(x_0)(x - x_0) + f(x_0)$$

$$= .05(x - 100) + 10$$

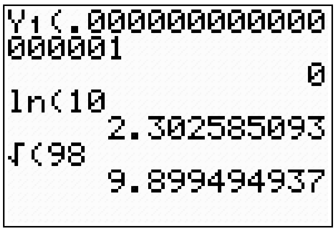
@ $x = 98$, we have

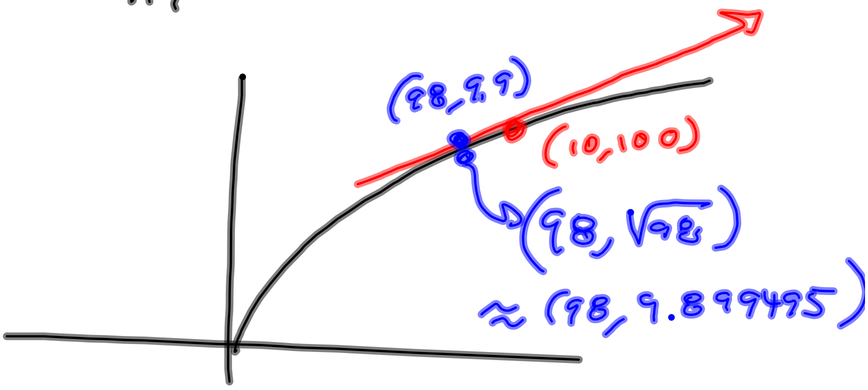
$$L(x) = .05(98 - 100) + 10$$

$$= .05(-2) + 10$$

$$= -.1 + 10$$

$$= 9.9$$





$L(x)$ is tangent line approximation

"Linearization"

$$L(x) = f'(x_0)(x - x_0) + f(x_0)$$

$$f(x) \approx L(x) = f'(x_0)(x - x_0) + f(x_0)$$

for x close to x_0

$$\underbrace{f(x) - f(x_0)}_{\Delta y} \approx f'(x)\,\underbrace{(x - x_0)}_{\Delta x}$$

$$\Delta y \approx f'(x)\,\Delta x$$

$$\frac{\Delta y}{\Delta x} \approx f'(x)$$

$$\Delta y \approx f'(x)\,\Delta x \approx f'(x)\,dx$$

How much paint to make a .2 cm coat on a cylinder with radius 20 cm and a height of 30 cm? (Paint the sides)

Can be viewed as volume of a .2 cm "shell" around the cylinder.

$\to dr = \Delta r$