3.5 #33

$\frac{d}{dx}[5f(x)] = 5\frac{d}{dx}[f(x)]$

$y = \csc x \Longrightarrow$

$y' = -\csc x \cot x \Longrightarrow$

$y'' = -(-\csc x \cot x)\cot x + (-\csc x)(-\csc^2 x)$

$\quad\quad f' \quad g \quad + \quad f \quad g'$

$= \boxed{\csc x \cot^2 x + \csc^3 x}$

$\cot^2 x + 1 = \csc^2 x$

$= \csc x[\csc^2 x - 1] + \csc^3 x$

$= 2\csc^3 x - \csc x$

$= 2\csc^3 x - \csc x$

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Find y' if $y = \sec x = \frac{1}{\cos x} = (\cos x)^{-1}$

$$\boxed{y' = \frac{0 \cdot \cos x - 1 \cdot (-\sin x)}{\cos^2 x}}$$

$$= \frac{\sin x}{\cos^2 x} = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} = \sec x \tan x$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

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$$\frac{d^{999}}{dx^{999}}\left[\sin x\right] = ?$$

$y^{(0)} = \sin x$
$y^{(1)} = \cos x$
$y^{(2)} = -\sin x$
$y^{(3)} = -\cos x$
$y^{(4)} = \sin x = y^{(0)}$
$y^{(5)} = \cos x = y^{(1)}$
$y^{(6)} = -\sin x = y^{(2)}$
$y^{(7)} = -\cos x = y^{(3)}$

~~$\mathbb{Z} \bmod 3\mathbb{Z}$~~

$\mathbb{Z} \bmod 4\mathbb{Z}$

$\bar{0} = \bar{4} = \bar{8} = \overline{12}$

$\bar{1} = \bar{5} = \bar{9} = \overline{13}$

How do I know the class of

$z = 25 = \bar{1}$

$\frac{25}{4} = 6 + \frac{1}{4}$

$(i^{500}) = (i^2)^{250}$

$(-1)^{250} = 1$

$$\frac{999}{4} = 249 + \frac{3}{4}$$

So $999 = 3 \bmod 4$

So $\frac{d^{999}}{dx^{999}}(\sin x) = -\cos x$

0 $\cos x$
1 $-\sin x$
2 $-\cos x$
3 $\sin x$
4 $\cos x$

$$\frac{d^{999}}{dx^{999}}\left[\cos x\right] = \sin x$$

§3.6 ?

$x^n \longrightarrow n x^{n-1}$

Find $\frac{dg}{dt}$ for

$$g(t) = \left(\frac{1+\sin(3t)}{3-2t}\right)^{-1} = f(h(t)), \text{ where } f(t) = t^{-1} \text{ and } h(t) =$$

$$g'(t) = -1\left(\frac{1+\sin(3t)}{3-2t}\right)^{-2}\left(\frac{(\cos(3t)\cdot 3)(3-2t) - (1+\sin(3t))(-2)}{(3-2t)^2}\right)$$

$$\frac{dg}{d\left(\frac{1+\sin(3t)}{3-2t}\right)}$$

$$\frac{d}{dt}\left[1+\sin(3t)\right] = \cos(3t)\cdot 3$$

$$g(t) = \left(\frac{1+\sin(3t)}{3-2t}\right)^{-1} = f(h(t)), \text{ where } f(t) = t^{-1} \text{ and } \boxed{h(t) = \frac{1+\sin(3t)}{3-2t}}$$

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$$h(x) = x^2 \sec\left(\frac{1}{x}\right)$$

$$h'(x) = 2x\sec\left(\frac{1}{x}\right) + x^2\left(\sec\left(\frac{1}{x}\right)\tan\left(\frac{1}{x}\right)\right)\left(-\frac{1}{x^2}\right)$$

$$\frac{d}{dx}\left[\sec\left(\frac{1}{x}\right)\right] = \left(\sec\left(\frac{1}{x}\right)\tan\left(\frac{1}{x}\right)\right)\frac{d}{dx}\left[\frac{1}{x}\right]$$

$$\frac{d}{dx}\left[\left(x^2+5x\right)^{37}\right] = \underline{\underline{37\left(x^2+5x\right)^{36}}}\,\underline{\underline{(2x+5)}}$$

$\left(x^2+5x\right)^{37} = f(g(x))$, where

$f(x) = x^{37}$ & $g(x) = x^2+5x$

$$\frac{d}{dx}\left[f(g(x))\right] = \frac{df}{dg}\cdot\frac{dg}{dx} = 37\left(x^2+5x\right)^{36}\cdot(2x+5)$$

$$x^2 + y^2 = 25$$

Explicit

Find eq'n(s) of tangent line(s) to the circle @ $x = 4$

$y^2 = 25 - x^2$

$y = \pm\sqrt{25 - x^2}$

$y_1 = (25 - x^2)^{1/2}$

$\frac{dy_1}{dx} = \frac{1}{2}(25 - x^2)^{-\frac{1}{2}}(-2x)$

$= \frac{-x}{\sqrt{25 - x^2}}$

$y_2 = -(25 - x^2)^{1/2}$

$\frac{dy_2}{dx} = -\frac{1}{2}(25 - x^2)^{-1/2}(-2x)$

$= \frac{x}{\sqrt{25 - x^2}}$

Implicit, viewing y as $f(x)$, and applying the Chain Rule.

$x^2 + y^2 = 25$

$2x + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{2x}{2y} = -\frac{x}{y}$

$= \frac{-x}{\pm\sqrt{25 - x^2}}$

@ $x = 4$, $y = \pm\sqrt{25 - 16} = \pm 3$

Gives $(4, 3)$, $(4, -3)$

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$$x^2+y^2=25$$

$$y=\pm\sqrt{25-x^2}$$

$$=\pm(25-x^2)^{1/2}$$

y_1 = top 1/2

y_2 = bottom 1/2

$x=4 \Rightarrow y=\pm 3$

$(4,3)$ & $(4,-3)$

m_{tan} @ $x=4$

$$y_1'(x) = -\frac{x}{\sqrt{25-x^2}}$$

$$y_1'(4) = -\frac{4}{\sqrt{9}} = -\frac{4}{3} = m$$

$$y=-\frac{4}{3}(x-4)+3$$

$$y_2'(4) = \frac{4}{3} = m$$

$$y=\frac{4}{3}(x-4)-3$$

$$\frac{d}{dx}\left[x^2+y^2=25\right] \qquad y' = \frac{dy}{dx}$$

$$2x+2yy'=0$$

$$2yy'=-2x$$

$$y'=\frac{-2x}{2y}=-\frac{x}{y}$$

Now @ $x=4$:

$$4^2+y^2=25$$

$$y^2=25-16=9$$

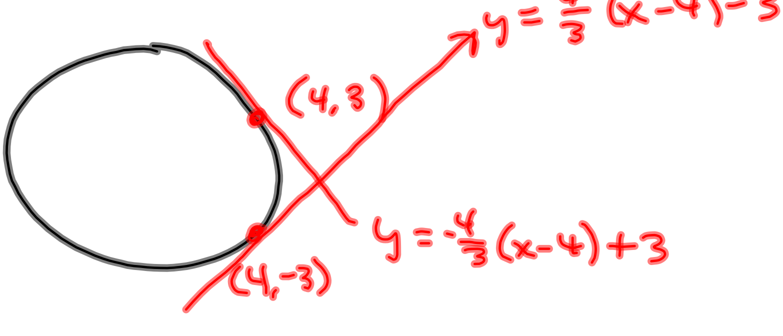
$$y=\pm 3$$

$x=4$, $y=3 \Rightarrow y' = -\frac{4}{3}$ OR $-\frac{4}{-3}=\frac{4}{3}$

$x=4$, $y=-3$

$$y=\frac{4}{3}(x-4)-3$$

$$y=-\frac{4}{3}(x-4)+3$$



$$\frac{d}{dx}\left[(f(x))^{37}\right] = 37\, f(x)^{36}\, \frac{df}{dx}$$

$$x^3 + y^3 - \underline{3xy} = 7$$

$$3x^2 + 3y^2y' - \underbrace{3y - 3xy'}_{\text{Product Rule \& Chain Rule}} = 0$$

$$\frac{d}{dx}[fg] = f'g + fg'$$

$$\frac{d}{dx}[xy] = 1 \cdot y + xy'$$

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