

§ 3.6 #s 1-4, 9-12, → optional, if it helps. 19-24, 37-44, 58-60,

65, 67, 73, 79  
→ optional

$$\frac{d}{dx} \left[ (x^5 + \sin x)^{37} \right] = 37 (x^5 + \sin x)^{36} (5x^4 + \cos x)$$

$$\frac{d}{dt} \left[ \cos(27t^2 - 15t) \right] = (-\sin(27t^2 - 15t))(54t - 15)$$

Derivative of the outside w.r.t. the inside

TIMES

Derivative of the inside with respect to the  
variable

My profit function is

$$P(x) = 2x^2 - 3x \quad (\text{in } \$), \text{ where}$$

$x = \text{the \# of shirts I'm making.}$

Suppose  $x = 5t$ , where  $t = \text{time}$   
in hours

How fast am I making money?

What's the rate of money change as  
a function of time?

Want  $\frac{dP}{dt}$

$$\frac{dP}{dx} = 4x - 3$$

But I want  $\frac{dP}{dt}$ .

We know  $x = 5t$

$$\begin{aligned} \text{So } P(x) &= P(x(t)) = 2x^2 - 3x = 2(5t)^2 - 3(5t) \\ &= 50t^2 - 15t = P(t). \end{aligned}$$

$$\begin{aligned} \frac{dP}{dt} &= 100t - 15 \\ &= 5(20t - 3) \\ &= 5(4(5t) - 3) \\ &= 5(4x - 3) \\ &= (4x - 3)(5) \\ &= \frac{dP}{dx} \cdot \frac{dx}{dt} \end{aligned}$$

$$\begin{aligned} &\frac{d}{dt} [P(x(t))] \\ &= \frac{dP}{dx} \cdot \frac{dx}{dt} \\ &= \underbrace{P'(x)}_{\substack{\text{prime} \\ \text{w.r.t } x}} \cdot \underbrace{x'(t)}_{\substack{\text{w.r.t. } t}} \\ &= (f(g(x)))' \\ &= f'(g) g'(x) \end{aligned}$$

$$f(x) = \cos^2(5x)$$

$$= (\cos(5x))^2$$

$$\Rightarrow f'(x) = 2(\cos(5x))' (-\sin(5x))(5)$$

$$\frac{d((\cos(5x))^2)}{d(\cos(5x))} \cdot \frac{d(\cos(5x))}{d(5x)} \cdot \frac{d(5x)}{dx}$$

we have  $f(g(h(x)))$ , where

$$f(g) = g^2$$

$$g(h) = \cos(h)$$

$$h(x) = 5x = \frac{df}{dg} \cdot \frac{dg}{dh} \cdot \frac{dh}{dx}$$

$$\frac{d}{dx} [f(g(h(x)))]$$

$$\frac{d}{dx} \left[ \tan(\sin(x^2 - \sqrt{x})) \right]$$

$$= \sec^2(\sin(x^2 - \sqrt{x})) (\cos(x^2 - \sqrt{x})) \left( 2x - \frac{1}{2\sqrt{x}} \right)$$

$$\frac{d}{dx} [\sqrt{x}] = \frac{d}{dx} [x^{\frac{1}{2}}] = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2} \cdot \frac{1}{\sqrt{x}} =$$

8.4

$$\text{Speed} = |\text{Velocity}|$$

$$s(t) = \text{position}$$

$$s'(t) = \text{velocity} = v(t)$$

$$s''(t) = \text{acceleration} = v'(t) = a(t)$$

$$s'''(t) = \text{jerk} = j(t)$$

$$\frac{t^4}{4} = \left(\frac{1}{4}t\right)^4$$

$$\frac{d}{dt} \left[ \frac{t^4}{4} \right] = \frac{4t^3 - t^4 \cdot 0}{4^2} = \frac{4t^3}{4} = t^3 \text{ owie!}$$

$$4 \cdot \frac{1}{4} t^3 = t^3$$

$$(3) \quad s(t) = -t^3 + 3t^2 - 3t \quad t \in [0, 3]$$

$$(a) \quad s(3) - s(0) = \Delta s$$

$$= -3^3 + 3(3)^2 - 3(3) = -27 + 27 - 9 = -9 \text{ m}$$

$$v_{\text{AVG}} = \frac{s(3) - s(0)}{3 - 0} = \frac{-9}{3} = -3 \text{ m/s}$$

$$(b) \quad \text{speed} = |v(t)|$$

$$v(t) = -3t^2 + 6t - 3$$

$$|v(0)| = |-3| = 3 \text{ m/s}$$

$$|v(3)| = |-3(3)^2 + 6(3) - 3| = |-27 + 18 - 3| = |-12|$$

$$= 12 \text{ m/s}$$

$$a(t) = v'(t) = s''(t)$$

$$= -6t + 6 \rightarrow$$

$$a(0) = 6 \text{ m/s}^2$$

$$a(3) = -6(3) + 6 = -12 \text{ m/s}^2$$

$$(c) \quad v(t) = -3t^2 + 6t - 3 \stackrel{!}{=} 0$$

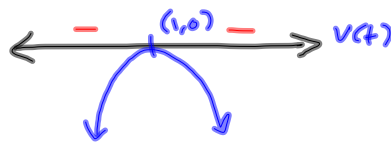
$$= -3(t^2 - 2t + 1) = 0 \Rightarrow$$

$$t^2 - 2t + 1 = 0$$

$$(t-1)^2 = 0$$

So this is really

$$v(t) = -3(t-1)^2$$



So  $s(t)$  is doing this:



$$\textcircled{1} \quad \frac{d}{dx} [\sin x] =$$

$$\textcircled{2} \quad \frac{d}{dx} [\cos x] =$$

$$\textcircled{3} \quad \frac{d}{dx} [\tan x] =$$

$$\textcircled{4} \quad \frac{d}{dx} [\sec x] =$$

$$\textcircled{5} \quad \frac{d}{dx} [\csc x] =$$

$$\textcircled{6} \quad \frac{d}{dx} [\cot x] =$$

$$\textcircled{1} \quad \cos x$$

$$\textcircled{2} \quad -\sin x =$$

$$\textcircled{3} \quad \sec^2 x$$

$$\textcircled{4} \quad \sec x \tan x$$

$$\textcircled{5} \quad -\csc x \cot x$$

$$\textcircled{6} \quad -\csc^2 x$$

$$\frac{d}{dx} [\sec x] = \frac{d}{dx} \left[ \frac{1}{\cos x} \right] = \frac{d}{dx} [(\cos x)^{-1}]$$

$$= -(\cos x)^{-2} (-\sin x)$$

$$= \frac{\sin x}{\cos^2 x} = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} = \underline{\sec x \tan x}$$

$$\textcircled{1} \quad \cos x$$

$$\textcircled{2} \quad -\sin x$$

$$\textcircled{3} \quad \sec^2 x$$

$$\textcircled{4} \quad \sec x \tan x$$

$$\textcircled{5} \quad -\csc x \cot x$$

$$\textcircled{6} \quad -\csc^2 x$$