§3.3 #58 To be difb'l you need

① Cnt's $\lim_{x\to -1} f(x) = f(-1)$

② difb'l $\lim_{h\to 0} \frac{f(-1+h) - f(-1)}{h}$

This boils down to

Then Do this

$$\lim_{x\to -1^-} f'(x) = \lim_{x\to -1^+} f'(x)$$

$$\frac{d}{dx}\left[bx^2-3\right]\Big|_{x=-1} = \frac{d}{dx}\left[ax+b\right]\Big|_{x=-1}$$

$$2bx = a$$

$$-2b = a = 3$$

$$\boxed{b = -\frac{3}{2}}$$

1st Do THIS

$$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^+} f(x)$$

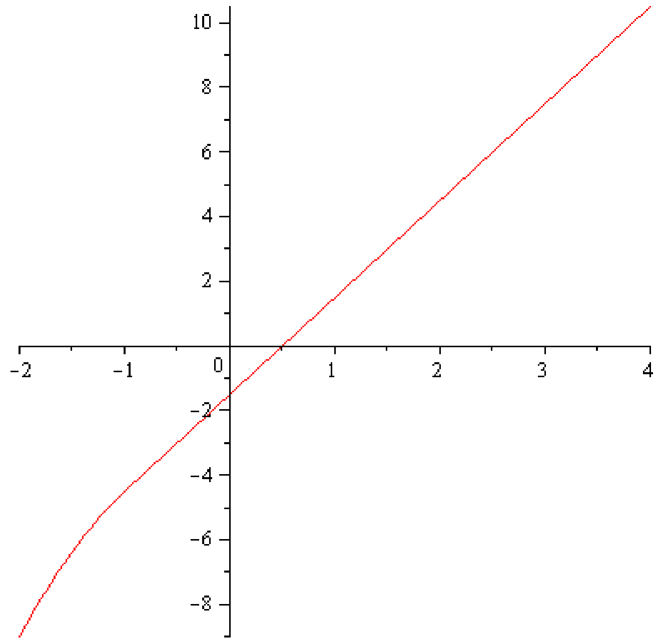
$$\lim_{x\to -1^-} (bx^2-3) = \lim_{x\to -1^+} (ax+b)$$

$$b(-1)^2-3 = a(-1)+b$$

$$b-3 = b-a$$

$$-3 = -a$$

$$3 = a$$




Sep 19-12:48 PM

The thing about #58:

Practical:

$$f(x)=\begin{cases} bx^2-3 & \text{if } x \le -1 \\ ax+b & \text{if } x > -1 \end{cases}$$

cnt$\Sigma$: Need $b(-1)^2-3 = a(-1)+b$

difbl: $\therefore$ $2b(-1) = a$

From:

$$f'(x)=\begin{cases} 2bx & \text{if } x < -1 \\ a & \text{if } x > -1 \end{cases}$$

and we ain't sure about $x=-1$.

$b-3=-a+b$

$3=a$

AND NOW $-2b=a=3 \Longrightarrow$

$b=-\frac{3}{2}$

---

Thought process:

Continuity: want

$$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^+} f(x) = f(-1)$$

Differentiability: want

$\lim_{h\to 0} f(x)$ to exist.

That means

$$\lim_{h\to 0^-} \frac{f(-1+h)-f(-1)}{h} = \lim_{h\to 0^+} \frac{f(-1+h)-f(-1)}{h}$$

on the left, it's $bx^2-3$ & $\lim_{h\to 0}$ etc is

on the right, it's $ax+b$ & $\lim_{h\to 0}$ (etc) is

$f'_-(-1) = 2bx\Big|_{x=-1}$

$f'_+(-1) = a\Big|_{x=-1}$

Derivative from the left @ $x=-1$

Practice Test instead of BIG TEST

1$^{st}$ go-round 100% for participating

After this, some sort of

100% if you score above 70%

75% otherwise.

We'll discuss how much they'll weigh at a later date.

Sep 20-12:21 PM

Pattern recognition

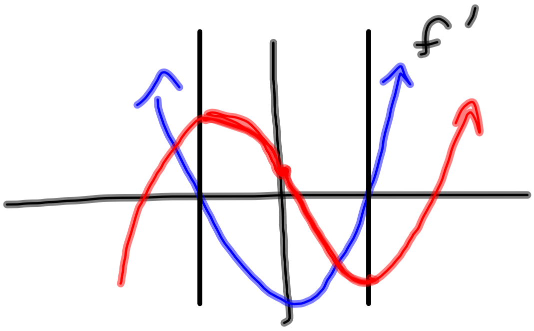
$f'(7) =$

43 3.3 b Minimize $f'(x) = 3x^2 - 4$

It occurs @ $x = 0$.

$f(0) = 1 \rightsquigarrow (0,1)$

from $f(x) = x^3 - 4x + 1$




Sep 20-12:25 PM

Pattern recognition

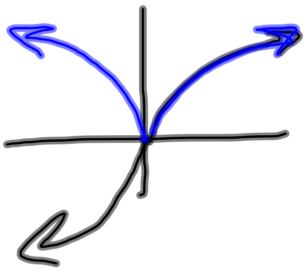
$$f'(7) = \lim_{h\to 0}\frac{f(7+h)-f(7)}{h} = \lim_{x\to 7}\frac{f(x)-f(7)}{x-7}$$

See #56 §3.3

$$\lim_{x\to -1}\frac{x^{2/9}-1}{x+1} = \lim_{x\to -1}\frac{x^{2/9}-(-1)^{2/9}}{x-(-1)}$$

$$x^{2/9} = \left(x^{\frac{1}{9}}\right)^2$$

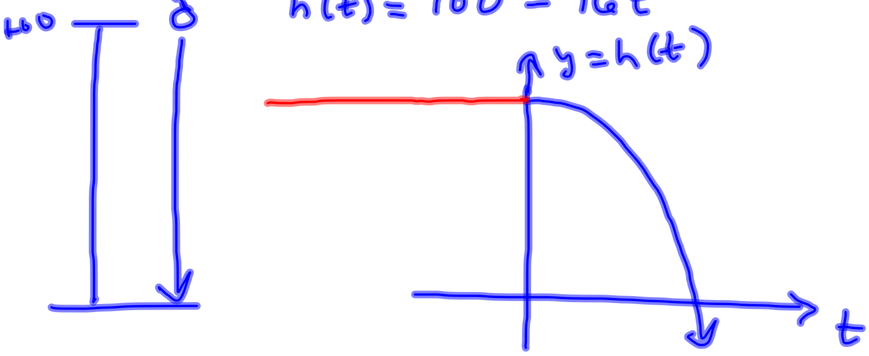
$$\left(\sqrt[9]{x}\right)^2$$

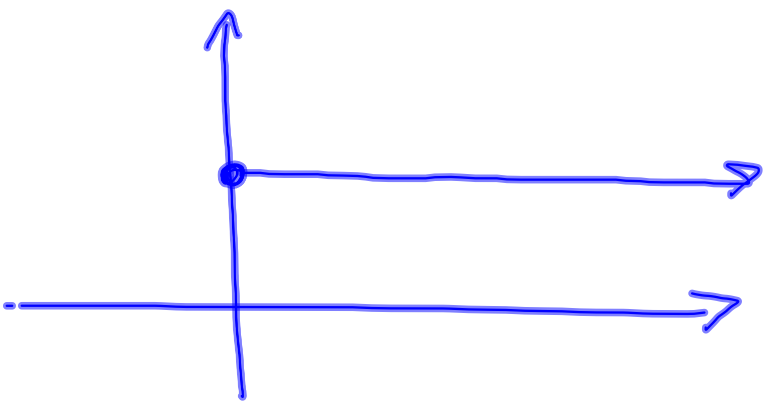


Sep 20-12:31 PM

FALLING APPLE as function of time

$h(t) = 100 - 16t^2$



Econ $C = C(x)$ cost as function of $x$ = the # of cars produced.

Marginal Cost $= \frac{dC}{dx} = C'(x)$

Economists say it's the cost of producing one more car:

$C(x+1) - C(x)$

$x = 100$

$C(101) - C(100)$ = cost of the $101^{st}$ car

$\frac{C(x+1) - C(x)}{1}$ = Average steepness of the cost curve between $x+1$ & $x$.

$\approx C'(x)$

$\boxed{\text{FACT}}$ Smooth curves are "locally linear!"

I will exploit this BIG time, later.

For now, it's enough to say

$C'(x) \approx \frac{C(x+1) - C(x)}{1}$

§ 3.4 Monday

§ 3.5 Tuesday

§ 3.6 ? Wednesday ? Mebbe

§ 3.5

$\sin(x+y) = \sin x \cos y + \sin y \cos x$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

$$\frac{\sin(x+h) - \sin x}{h} = \frac{\sin x \cos h + \sin h \cos x - \sin x}{h}$$

$$= \frac{\sin x\,(\cos h - 1)}{h} + \frac{\sin h \cos x}{h}$$

$$= (\sin x)\Big(\underbrace{\frac{\cos h - 1}{h}}_{\to 0}\Big) + \Big(\underbrace{\frac{\sin h}{h}}_{\to 1}\Big)\cos x$$

$$\xrightarrow{h \to 0} \cos x = \frac{d}{dx}[\sin x]$$

I haven't justified $\lim_{h \to 0} \frac{\cos h - 1}{h} = 0$

Sep 20-12:48 PM

$$\frac{\cos(x+h)-\cos x}{h} = \frac{\cos x\cos h - \sin x\sin h - \cos x}{h}$$

$$= \frac{\cos x\,(\cos h - 1)}{h} - (\sin x)\frac{\sin h}{h}$$

$\frac{\cos h - 1}{h} \to 0$ $\qquad$ $\frac{\sin h}{h} \to 1$

$$\frac{2+7}{11} = \frac{2}{11} + \frac{7}{11}$$

$\xrightarrow{h \to 0}$ $-\sin x = \frac{d}{dx}[\cos x]$

$$\frac{d}{dx}[\tan x] = \frac{d}{dx}\left[\frac{\sin x}{\cos x}\right] = \frac{\cos x\cdot\cos x - \sin x\cdot(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x = \frac{d}{dx}[\tan x]$$

$$\frac{d}{dx}[\sec x] = \sec x\tan x$$

$$\frac{d}{dx}[\csc x] = -\csc x\cot x$$

$$\frac{d}{dx}[\cot x] = -\csc^2 x$$

§3.4 #s 3, 4, 7, 10, 14, 15, 19

we did this one in physics.

§3.5 #s 2, 4, 7, 10, 17, 24, 30, 33, 35, 36, 39, 40, 44, 47, 48, 57 - 61, $\boxed{65, 66}$

Good theory.