c, d constant

f, g difbl functions.

$(cf)' = cf'$ & $(f+g)' = f' + g'$

$\frac{d}{dx}[5x^2] = 5\frac{d}{dx}[x^2]$

Combine $(cf+dg)' = cf' + dg'$ $\quad \frac{d}{dx}$ is linear

$(c)' = 0$

$(fg)' = f'g + fg'$

$$\frac{d}{dx}\Big[\underbrace{(x^2-7)}_{f}\underbrace{\sqrt{x}}_{g}\Big] = \underbrace{(2x)}_{f'}\underbrace{\sqrt{x}}_{g} + \underbrace{(x^2-7)}_{f}\underbrace{\left(\frac{1}{2\sqrt{x}}\right)}_{g'}$$

scratch: $\frac{d}{dx}[\sqrt{x}] = \frac{d}{dx}[x^{\frac{1}{2}}] = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

24 $f(x) = x^2 - 3x + 4$

$$\frac{f(z)-f(x)}{z-x} = \frac{z^2-3z+4-(x^2-3x+4)}{z-x}$$

$$= \frac{z^2-x^2-3z+3x+4-4}{z-x}$$

$$= \frac{(z-x)(z+x)-3(z-x)}{z-x}$$

$$= \frac{(z-x)[z+x-3]}{z-x}$$

$$= z+x-3 \xrightarrow{z \to x} 2x-3$$

$$f(x) = 5x - \frac{1}{x}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{5(x+h) - \frac{1}{x+h} - \left(5x - \frac{1}{x}\right)}{h}$$

$$= \frac{1}{h}\left[\frac{5(x+h)^2 - 1}{x+h} - \frac{5x^2 - 1}{x}\right]$$

$$= \frac{1}{h}\left[\frac{(5(x+h)^2 - 1)x - (5x^2 - 1)(x+h)}{x(x+h)}\right]$$

$$= \frac{1}{h}\left[\frac{(5(x^2 + 2xh + h^2) - 1)x - (5x^3 + 5x^2h - x - h)}{x(x+h)}\right]$$

$$= \frac{1}{h}\left[\frac{(5x^2 + 10xh + 5h^2 - 1)x - 5x^3 - 5x^2h + x + h}{x(x+h)}\right]$$

$$= \frac{1}{h}\left[\frac{5x^3 + 10x^2h + 5xh^2 - x - 5x^3 - 5x^2h + x + h}{x(x+h)}\right]$$

$$= \frac{1}{h}\left[\frac{10x^2h + 5xh^2 - 5x^2h + h}{x(x+h)}\right]$$

$$= \frac{1}{h}\left[\frac{h(10x^2 + 5xh - 5x^2 + 1)}{x(x+h)}\right]$$

$$\xrightarrow{h \to 0} \frac{10x^2 - 5x^2 + 1}{x(x)} = \frac{5x^2 + 1}{x^2} = 5 + \frac{1}{x^2}$$

$$\frac{d}{dx}\left(5x - \frac{1}{x}\right) = \frac{d}{dx}\left[5x - x^{-1}\right] = 5 - (-1)x^{-2} = 5 + \frac{1}{x^2}$$

$$\frac{d}{dx}\left[x^n\right] = nx^{n-1} \quad (n \neq 0)$$

Proof of Product Rule

Want to prove

$$(fg)' = f'g + fg'$$

$$\frac{d}{dx}\left[f(x)\,g(x)\right] = \frac{d}{dx}\left[f(x)\right]g(x) + f(x)\cdot\frac{d}{dx}\left[g(x)\right]$$

Difference Quotient for $f(x)g(x)$ is

$$\frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

$$= \frac{f(x+h)g(x+h) - f(x)g(x+h) + f(x)g(x+h) - f(x)g(x)}{h}$$

$$= \frac{\left(f(x+h) - f(x)\right)g(x+h) \quad + f(x)\left(g(x+h) - g(x)\right)}{h}$$

$$= \frac{f(x+h) - f(x)}{h}\cdot g(x+h) + f(x)\cdot\frac{g(x+h) - g(x)}{h}$$

$$\xrightarrow{h \to 0} f'(x)g(x) \quad + f(x)g'(x) \quad \blacksquare$$

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Quotient Rule

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$$\frac{d}{dx}\left[\frac{x^2+7x}{5x-11}\right] = \frac{(2x+7)(5x-11) - (x^2+7x)(5)}{(5x-11)^2}$$

Recall

$$\frac{d}{dx}\left[\frac{1}{x^2}\right] = \frac{d}{dx}\left[x^{-2}\right] = -2x^{-3} = -\frac{2}{x^3}$$

$$\frac{d}{dx}\left[\frac{1}{x^2}\right] = \frac{f'g - fg'}{g^2} = \frac{0\cdot x^2 - 1\cdot 2x}{(x^2)^2} = -\frac{2x}{x^4} =$$

$f = 1$

$g = x^2$

Find derivatives of all orders

$f(x) = x^5 - 2x^3 = y$

$y' = 5x^4 - 6x^2$

$y'' = 20x^3 - 12x$

$y''' = 60x^2 - 12$

$y^{(4)} = 120x$

$y^{(5)} = 120$

$y^{(6)} = 0 = y^{(7)} = y^{(8)} = \cdots$

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$y = \frac{1}{x} = x^{-1}$

$y' = -x^{-2}$ $n=1$ $(-1)x^{-2}$

$y'' = 2x^{-3}$ $n=2$ $2x^{-3}$

$y''' = -6x^{-4} = -3\cdot 2\cdot x^{-4}$ $n=3$ $(-1)3\cdot 2\cdot x^{-4}$

$4! = n!$

$y^{(4)} = 24x^{-5} = 4\cdot 3\cdot 2\cdot x^{-5}$

$5! = n!$

$y^{(5)} = -120x^{-6} = -5\cdot 4\cdot 3\cdot 2\, x^{-6}$

$\vdots$

$$\boxed{y^{(n)} = (-1)^n \cdot n!\, x^{-(n+1)}}$$

§3.3 # 43

$y = x^3 - 4x + 1$ Perpendicular Normal @ (2,1)

$y' = 3x^2 - 4$

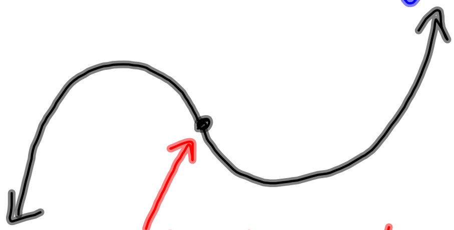
$y'\Big|_{x=2} = 3(2)^2 - 4 = 12 - 4 = 8 = m \Longrightarrow m_{\perp} = -\frac{1}{8}$

$y = -\frac{1}{8}(x-2) + 1$

b. "Smallest Slope" on the curve?

↳ 0 if slope ≥ 0.

"Greatest negative slope"



Slope.

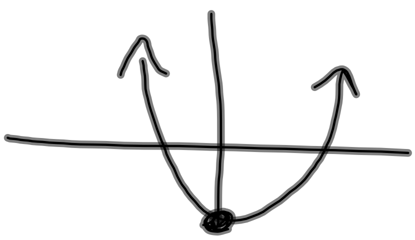
Inflection pt is where it is its most negative

Let's look @ $y' = 3x^2 - 4$ & find its low point:

$3x^2 - 4$

Minimized @ (0, -4)

"max negative" slope



$y = x^3 - 4x + 1$

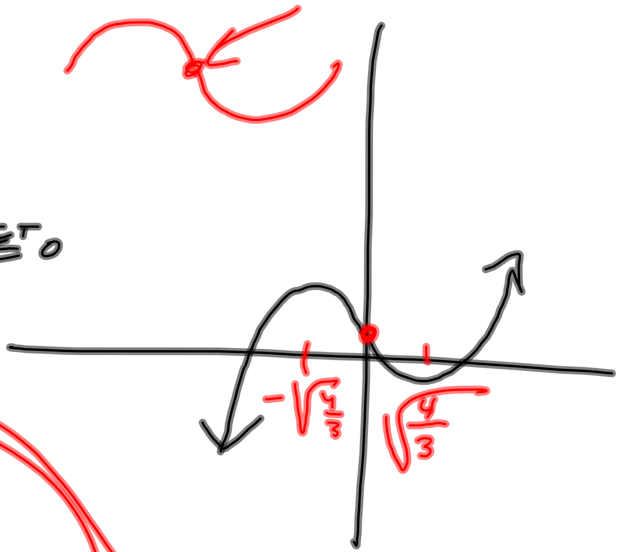
$y' = 3x^2 - 4$ set $= 0$

$y'' = 6x$

$3x^2 = 4$

$x^2 = \frac{4}{3}$

$x = \pm\sqrt{\frac{4}{3}}$



$6x = 0$

$x = 0$

3.2 #31

a

$$y = \begin{cases} \frac{1}{2}x + 2 & \text{if } x < 0 \\ -4x + 2 & \text{if } x \geq 0 \end{cases}$$

$$y' = \begin{cases} \frac{1}{2} & \text{if } x < 0 \\ -4 & \text{if } x > 0 \end{cases}$$

But @ $x=0$,

we need $\lim\limits_{h \to 0} \frac{f(x+h) - f(x)}{h}$ to exist.

But $\lim\limits_{h \to 0^-} \frac{f(x+h) - f(x)}{h} = \frac{1}{2} \neq -4 = \lim\limits_{h \to 0^+} \frac{f ----}{h}$

so the derivative $\nexists$

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