$$\frac{f(b)-f(a)}{b-a}$$ Same curve on test.

§3.1

Yes # 33 $f(x)=\begin{cases} x^2\sin\left(\frac{1}{x}\right) & \text{if } x\neq 0 \\ 0 & \text{if } x=0 \end{cases}$

No #34 $f(x)=\begin{cases} x\sin\left(\frac{1}{x}\right) & \text{if } x\neq 0 \\ 0 & \text{if } x=0 \end{cases}$

#33 away from $x=0$, it's all good.

@ 0, we have issues.

$$\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0}\frac{h^2\sin\left(\frac{1}{h}\right)-0}{h}$$

$$=\lim_{h\to 0}\frac{h^2\sin\left(\frac{1}{h}\right)}{h}=\lim_{h\to 0}h\sin\left(\frac{1}{h}\right)=0 \text{ Exists!}$$

$h>0$: Likewise for $h<0$.

$$-h\le h\sin\left(\tfrac{1}{h}\right)\le h$$

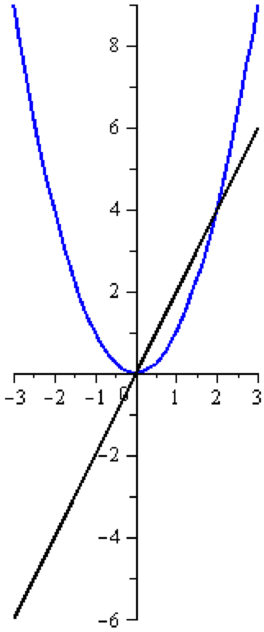
$$\begin{array}{ccc} \downarrow & & \downarrow \\ 0 & & 0 \end{array}$$

#34 $\lim_{h\to 0}\frac{f(h)-f(0)}{h}=\lim_{h\to 0}\frac{h\sin\left(\frac{1}{h}\right)}{h}$

$$=\lim_{h\to 0}\sin\left(\tfrac{1}{h}\right)\ \nexists$$

§3.1 #s 13, 14, & Read about vertical tangents & cusps

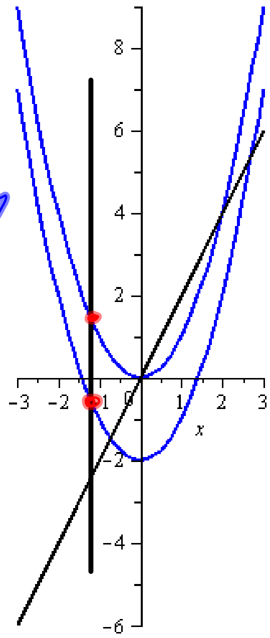
Discussion preceding #s 35, 36.



$f(x) = x^2$ &

$f'(x) = 2x$ graphed together

NOTE: $y = 2x$ is the derivative (slope function) for the entire family $g(x) = x^2 + \text{constant}$



This is why, in differential equations, we usually employ a condition on $f(x)$ to get a unique answer.

Solve $y' = 2x$

$f'(x) = 2x$

Then $f(x) = x^2 + C$, for any $c \in \mathbb{R}$

If I told you $f(0) = -2$, then

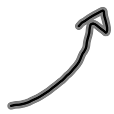
$f(0) = (0)^2 + C = -2 \Rightarrow$

$C = -2 \Rightarrow$

$f(x) = x^2 - 2$

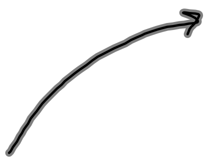
$y'' + y' = 7$

Planting Seeds.



$f' > 0$

$f'' > 0$



$f' > 0$

$f'' < 0$

§3.2

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x)$$

$$\lim_{z \to x} \frac{f(z) - f(x)}{z - x} = f'(x)$$

$$= \frac{dy}{dx} = \frac{d}{dx}[y] = Dy = y' = D(f)(x)$$

$$= D_x f(x) = D[f(x)] = \frac{d}{dx}[f(x)]$$

$$= \frac{df}{dx}$$

Liebniz Notation

Differentiable — this means $f'(x)$ exists. At endpoints the limit may be one-sided.

Sep 17-12:33 PM

Differentiable - SMOOTH

is a higher requirement

Thm 1 in §3.2

If f is difb$^{\underline{l}}$, then f is cnt$^{\underline{s}}$

Proof. To be continuous, we need

$\lim_{x \to c} f(x) = f(c)$ want to show.

Read Thm 1.

§3.2 #s 1, 4, 8, 13, 14, 17,
20[1], 23, 24, 27-30[2], 31, 32[3], 37, 54[4]

[1] $\left.\frac{dy}{dx}\right|_{x=\sqrt{3}}$ means $f'(\sqrt{3})$

[2] Graph of $f(x)$ & $f'(x)$

[3] your 1st differential eq'n.

[4] Hard puzzle

Sep 17-12:43 PM

§3.3 At last!

$$\frac{d}{dx}\left[x^n\right] = n x^{n-1}$$ $\boxed{n \neq 0}$

See proof.
$x^n - c^n$ always factors like in proof

$f(x) = x^7 \Rightarrow f'(x) = 7x^6$

$$\frac{d}{dx}\left[x^7\right] = 7x^6$$

Rules: "$\frac{d}{dx}$ is a linear operator."

1. It respects multiplication by a constant

$$\frac{d}{dx}\left[35x^7\right] = 35\frac{d}{dx}\left[x^7\right] = 35 \cdot 7x^6 = 245x^6$$

2. It respects sums

$$\frac{d}{dx}\left[x^7 + x^5\right] = \frac{d}{dx}\left[x^7\right] + \frac{d}{dx}\left[x^5\right]$$
$$= 7x^6 + 5x^4$$

Product Rule

$$\frac{d}{dx}\left[f(x)g(x)\right] = f'(x)g(x) + f(x)g'(x)$$

$f(x) = x^2 - 2$

$g(x) = x^4$

$$\frac{d}{dx}\left[f(x)g(x)\right] = (2x)(x^4) + (x^2-2)(4x^3)$$

$$= 2x^5 + 4x^5 - 8x^3$$

$$= 6x^5 - 8x^3$$

$$f(x)g(x) = (x^2-2)(x^4) = x^6 - 2x^4$$

$$\frac{d}{dx}\left(f(x)g(x)\right) = 6x^5 - 8x^3$$

$$\frac{d}{dx}\left[(x^3+5x^2-3x+2)(5x^6-12x^3+7x)\right]$$

$$= \overset{f'}{(3x^2+10x-3)}\overset{g}{(5x^6-12x^3+7x)}$$

$$+ \overset{f}{(x^3+5x^2-3x+2)}\overset{g'}{(30x^5-36x^2+7)}$$ DONE.

§3.3 #s 1, 4, 7, 10, 14, 17, 20, 30, 33, 42abc, 43-45, 51, 56, 58

Sep 17-1:02 PM