§3.1 Tangents & Derivatives at a point.

$$\lim_{h\to 0} \frac{f(x_0+h)-f(x_0)}{h}$$ x_0 is fixed.

$x_0 = 3$

$$f'(x_0)$$

§3.2 Derivative as a function

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$ x is indeterminate.

$$= \boxed{\lim_{z\to x} \frac{f(z)-f(x)}{z-x}}$$ from book $f'(x)$

I've shown you this:

$$\lim_{x\to c} \frac{f(x)-f(c)}{x-c}$$ $f'(c)$

Sep 14-12:10 PM

#s 5 – 10 tan line @ (x_0, y_0)
graph f, & tan line together.

11 – 18 tan line @ (x_0, y_0)

#17 $f(x) = \sqrt{x}$ @ $(4, 2)$

Rather do it the §3.2 way & then plug in $x = 4$.

$$\frac{f(x+h) - f(x)}{h} = \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \frac{x + h - x}{h(\sqrt{x+h} + \sqrt{x})} = \frac{h}{h(\sqrt{x+h} + \sqrt{x})} = \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$\xrightarrow{h \to 0} \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}} = f'(x).$$

Tangent line @ $(4, 2) = (4, f(4)) = (4, \sqrt{4})$

$$y = m(x - x_1) + y_1$$

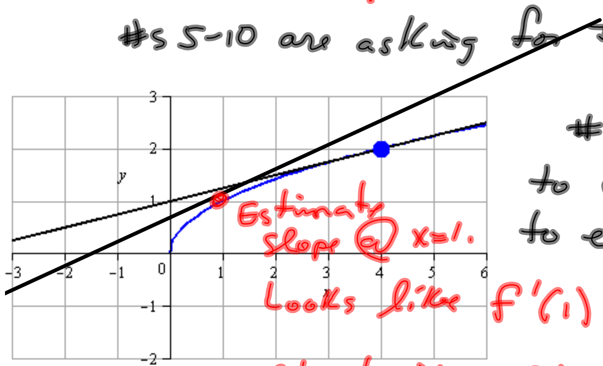
$$y = f'(4)(x - 4) + f(4)$$

$$f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{2 \cdot 2} = \frac{1}{4} = f'(4) \quad \text{ok!}$$

$$\boxed{y = \frac{1}{4}(x - 4) + 2}$$

$$= \frac{1}{4}x - 1 + 2 = \frac{1}{4}x + 1$$

#s 5-10 are asking for this picture.



Estimate slope @ $x = 1$.
Looks like $f'(1) \approx \frac{1}{2}$

#s 1-4 want you to use a straight edge to estimate slope.

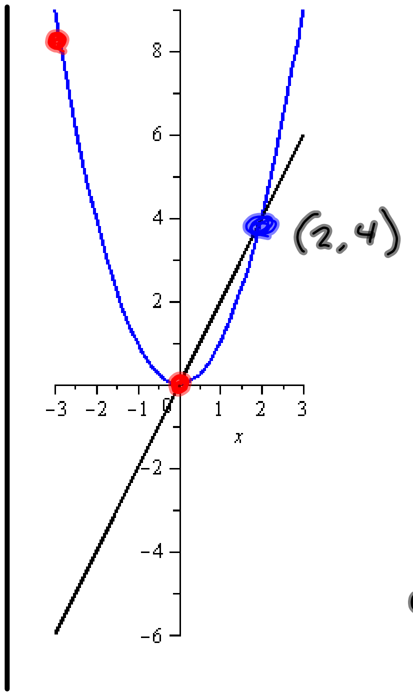
Check it: $f'(x) = \frac{1}{2\sqrt{x}}$, so

$f'(1) = \frac{1}{2}$ ✓

§ 3.2 is saying "View $f'(x)$ as a function."

$f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2\sqrt{x}}$ reports slope at any given $x \in \mathcal{D}(f)$



$f(x) = x^2$

$f'(x) = 2x$

what's special about it?

$f(2) = 2^2 = 4$

$f'(2) = 2(2) = 4$

Happy coincidence, where height = steepness

Sep 14-12:39 PM

Find eq'n of all lines with slope $m = -1$ that are tangent to $y = \frac{1}{x-1} = f(x)$

Find $f'(x)$

Set $f'(x) = -1$ & solve for x

Find $f(x)$ for those, giving $(x, f(x)) = (x_0, y_0)$ and build

line(s): $y = f'(x_0)(x - x_0) + f(x_0)$

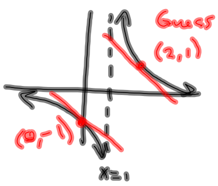
$y = f'(x_0)(x - x_0) + y_0$

$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{(x+h)-1} - \frac{1}{x-1}}{h}$$

$$= \frac{1}{h}\left[\frac{1}{x+h-1}\cdot\frac{x-1}{x-1} - \frac{1}{x-1}\cdot\frac{x+h-1}{x+h-1}\right]$$

$$= \frac{1}{h}\left[\frac{x-1-(x+h-1)}{(x-1)(x+h-1)}\right] = \frac{1}{h}\left[\frac{x-1-x-h+1}{(x-1)(x+h-1)}\right]$$

$$\frac{1}{\cancel{h}}\left[\frac{-\cancel{h}}{(x-1)(x+h-1)}\right] \xrightarrow{h \to 0} \frac{-1}{(x-1)(x-1)} = -\frac{1}{(x-1)^2} = f'(x)$$



$-\frac{1}{(x-1)^2} = -1$ LCD $= (x-1)^2$

$\frac{1}{(x-1)^2} = 1 \cdot \frac{(x-1)^2}{(x-1)^2}$

$\frac{1}{(x-1)^2} = \frac{(x-1)^2}{(x-1)^2}$

$1 = (x-1)^2$

Solve

$(x-1)^2 = 1^2$

$\sqrt{(x-1)^2} = \sqrt{1^2}$

$|x-1| = 1$

$x - 1 = \pm 1$

$x = 1 \pm 1$

$x = 0$ OR 2

$1 = x^2 - 2x + 1$

$x^2 - 2x + 1 = 1$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x \in \{0, 2\}$

$f(0) = \frac{1}{-1} = -1 \rightsquigarrow (0,-1) = (x_0, y_0)$

$f(2) = \frac{1}{2-1} = \frac{1}{1} = 1 \rightsquigarrow (2,1) = (x_0, y_0)$

Two lines $m = -1$

$y = m(x - x_0) + y_0$

$y = -1(x - 0) - 1$

$y = -1(x - 2) + 1$

For Now, Know I want to see #s 4, 7, 8, 26, 30, 33, 34

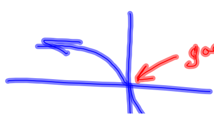
More to come. I'll post @ the wknd

The assignment

$f(x) = x^{\frac{2}{3}}$

limits differ from left & right.

$f(x) = -x^{\frac{1}{3}}$



goes vertical