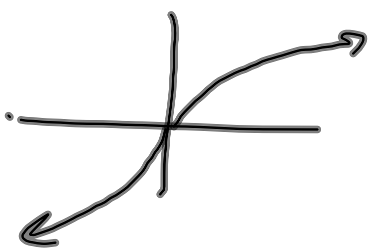
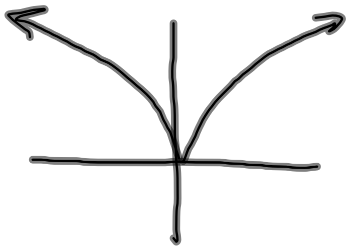
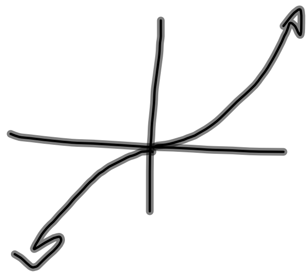
$x^{\frac{1}{3}}$



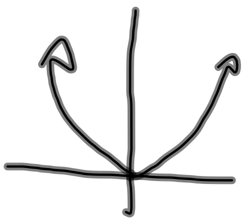
$x^{\frac{2}{3}}$



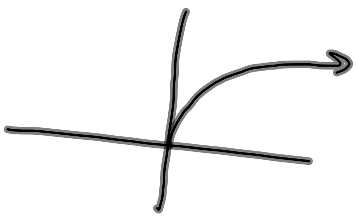
x^{3}



x^{2}



$x^{\frac{1}{2}}$



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$$R(x)=\frac{x^2+5x+6}{x-2} \qquad \text{O.A.}$$

$$x-2\,\overline{)\,x^2+5x+6}$$

2	1	5	6
		2	14
	1	7	20

$$R(x)=x+7+\frac{20}{x-2}$$

$\frac{20}{x-2} \to 0$ as $x \to$ BIG

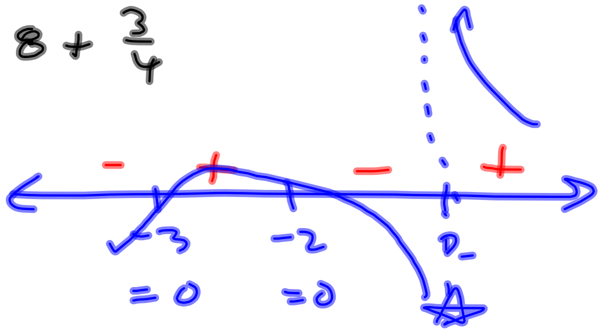
$$\frac{35}{4}=8+\frac{3}{4}$$

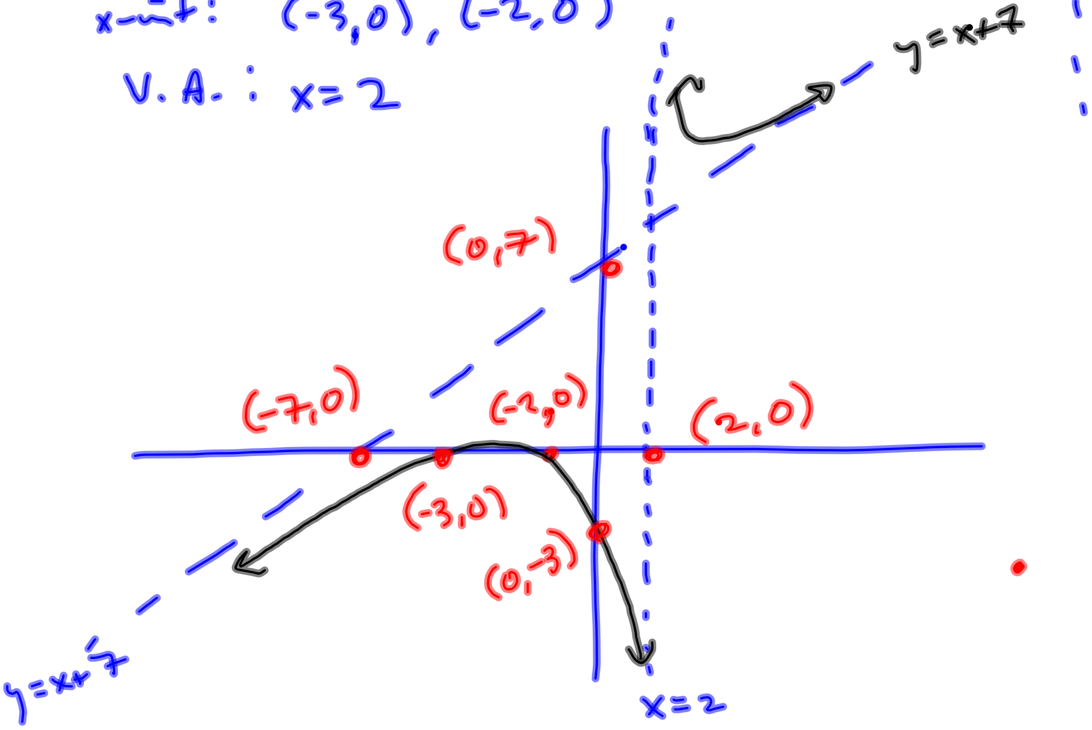
$$R(x)=\frac{(x+3)^1(x+2)^1}{x-2}$$

O.A.: $y=x+7$

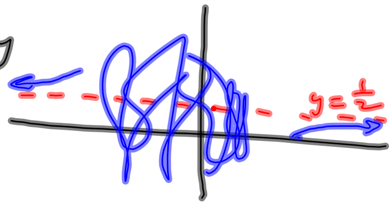
x-int: $(-3,0)$, $(-2,0)$

V.A.: $x=2$





$$\lim_{x\to\infty} \frac{x^3+7x-5}{2x^3+11x} = \frac{1}{2} = y$$



Book Way (Sledgehammer)

$$\frac{x^3+7x-5}{2x^3+11x} = \frac{x^3\left(1+\frac{7}{x^2}-\frac{5}{x^3}\right)}{x^3\left(2+\frac{11}{x^2}\right)}$$

$$= \frac{1+\frac{7}{x^2}-\frac{5}{x^3}}{2+\frac{11}{x^2}} \xrightarrow{x\to\infty} \frac{1+0-0}{2+0} = \frac{1}{2}$$

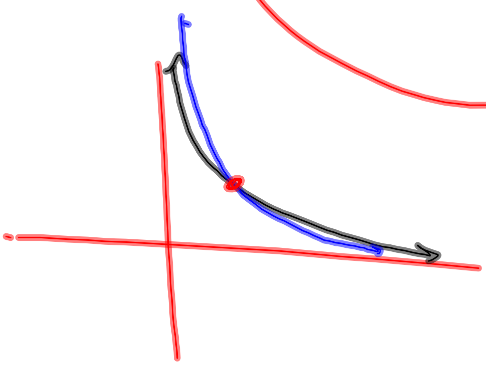
$$\lim_{x\to\infty} \frac{x^3+2x-5}{2x^3+11}$$

$$\frac{x^3+\text{smaller}}{2x^3+\text{smaller}} \xrightarrow{x\to\infty} \frac{x^3}{2x^3} = \frac{1}{2}$$

$$\lim_{x\to-\infty} \frac{x^3+7x^2-5}{25x^2+2} = -\infty$$

$$\frac{\cancel{x^3}\left(1+\frac{7}{x}-\frac{5}{x^3}\right)}{\cancel{x^3}\left(\frac{25}{x}+\frac{2}{x^3}\right)} \xrightarrow{x\to-\infty} \infty$$

eventually smaller than $\frac{25}{x}$ in absolute value (magnitude)



$$\frac{\cancel{x^3}\left(1+\frac{7}{x}-\frac{5}{x^3}\right)}{\cancel{x^3}\left(\frac{25}{x}+\frac{2}{x^3}\right)} \xrightarrow{x \to -\infty} + \infty$$

$\frac{25}{x} + \frac{2}{x^3} = \frac{25x^2+2}{x^3}$ so the $\frac{25}{x}$ must be dominant. & $\frac{25}{x} < 0$

So, it's $-\infty$, not $+\infty$ in the limit.

$$\lim_{x \to -\infty} \sqrt{x^2+3} + x$$

$$\left(\frac{\sqrt{x^2+3} + x}{\sqrt{x^2+3} - x} \right) \left(\sqrt{x^2+3} - x \right)$$

$$= \frac{x^2+3-x^2}{\sqrt{x^2+3} - x} = \frac{3}{\sqrt{x^2+3} - x} = \frac{3}{\sqrt{x^2(1+\frac{3}{x})} - x}$$

$$= \frac{3}{|x|\sqrt{1+\frac{3}{x}} - x} = \frac{3}{-x\sqrt{1+\frac{3}{x}} - x} = \frac{3}{-x\left(\sqrt{1+\frac{3}{x}} + 1\right)}$$

$$\xrightarrow{x \to -\infty} \frac{3}{-(-\infty)} = 0$$

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2.3 on test. $\varepsilon = .1$

Find δ for given ε Any function.

ε given concretely. $\sqrt{x}$, x^2

Abstract:

Prove $\lim_{x \to 3} (5x-2) = 13$

Proof Let $\varepsilon > 0$ be given. Define $\delta = \frac{\varepsilon}{5}$.

Then $0 < |x-3| < \delta \Longrightarrow$

$|(5x-2)-13| = |5x-15| = 5|x-3| < 5\delta = 5 \cdot \frac{\varepsilon}{5}$

$= \varepsilon$ ■

Old-School / Grad School

want $\boxed{|(5x-2)-13| < \varepsilon}$ Want to get an $|x-3|$ to pop up.

$|5x-15| < \varepsilon$

$5|x-3| < \varepsilon$

$5\boxed{|x-3|} < 5\delta < \varepsilon \Longrightarrow$

$\delta < \frac{\varepsilon}{5}$ is where the $\frac{\varepsilon}{5}$ pops out.

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