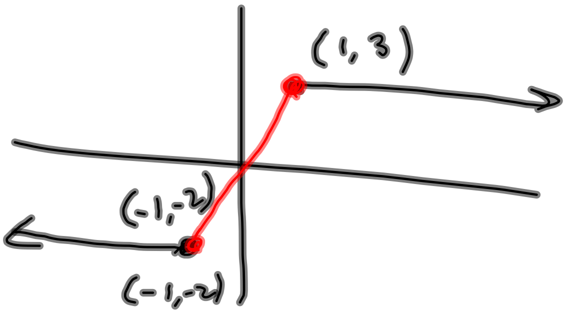
23. 45

45

$$f(x) = \begin{cases} -2 & x \le -1 \\ ax - b & -1 < x < 1 \\ 3 & x \ge 1 \end{cases}$$



$(-1,-2) = (x_1, y_1)$

$(1,3)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3+2}{1+1} = \frac{5}{2}$$

$$y = m(x - x_1) + y_1$$

$$= \frac{5}{2}(x+1) - 2$$

$$= \frac{5}{2}x + \frac{5}{2} - 2$$

$$= \underbrace{\frac{5}{2}}_{a}x + \underbrace{\frac{1}{2}}_{b}$$

Things are mostly cnt's on their domains:

23 ① Polynomials
Rational Functions
Root Functions
Trig Functions

99% of the work is just finding their domains

① Poly : $\mathcal{D} = \mathbb{R}$ $P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$

$3x^2+5x-2$

② Rat'l func:

$R(x) = \frac{P(x)}{Q(x)}$, P, Q are polys.

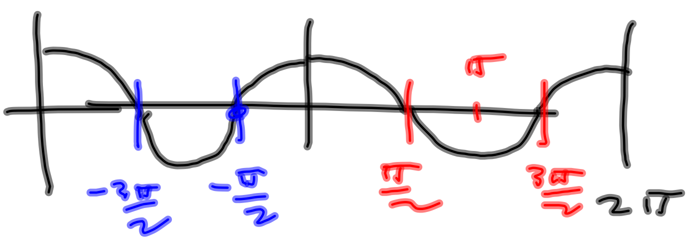
$$\mathcal{D} = \{ x \mid Q(x) \neq 0 \}$$

$$R(x) = \frac{3x^2+5x-2}{x+1}$$

$$\mathcal{D}(R) = \mathbb{R} \setminus \{-1\} = (-\infty,-1) \cup (-1,\infty)$$
$$= \{x \mid x \neq -1\}$$

$$\frac{x \tan x}{x^2+1} = \boxed{\frac{x\left(\frac{\sin x}{\cos x}\right)}{x^2+1} = \frac{x \sin x}{(x^2+1)\cos x}}$$

$$\mathcal{D} = \{x \mid \cos x \neq 0\} = \{x \mid x \neq \frac{(2n+1)\pi}{2}\}$$



$\frac{n\pi}{2}$

(27) $y = (2x-1)^{\frac{1}{3}}$ $\mathcal{D} = \mathbb{R}$ (roots cont's on their domain)

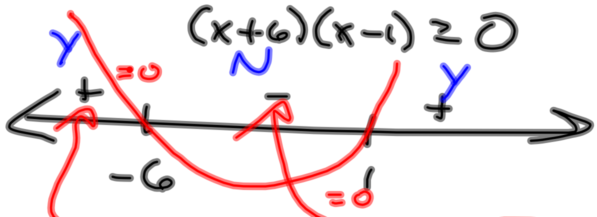
Roots
- Even index – Need argument ≥ 0
- odd index – $\mathcal{D} = \mathbb{R}$

Continuous on their domains.

$f(x) = \sqrt{x^2+5x-6}$

Need $x^2+5x-6 \geq 0$

$(x+6)(x-1) \geq 0$



	Test	
$(-\infty, -6)$	$-7 = x$	$(-7+6)(-7-1) = (-1)(-8)$ +
$(-6, 1)$	$x = 0$	$(0+6)(0-1) = (6)(-1)$ −
$(1, \infty)$	$x = 2$	

Interpret : $\mathcal{D} = (-\infty, -6] \cup [1, \infty)$
$= \{x \mid x \leq -6 \text{ OR } x \geq 1\}$

where's $\sqrt{\dfrac{(x+3)(x+2)}{x-1}}$ cont^s?

Need $\dfrac{(x+3)^1(x+2)^1}{(x-1)^1} \geq 0$ AND $x-1 \neq 0$

$x \neq 1$

N Y N Y

$-$ $+$ $-$ $+$

-3 -2 1

$=0$ $=0$ ☆

AND

1

$(-\infty, 1) \cup (1, \infty)$

$[-3,-2] \cup [1, \infty)$ AND

The "AND" means intersect the two.

This gives $\mathcal{D} = [-3,-2] \cup (1, \infty)$

Domain: $\sqrt{\text{Negative}}$ bad

$\dfrac{\text{stuff}}{0}$ bad

Everything else is good

§2.6 Two kinds of limits involving ∞:

$\lim_{x\to\infty} f(x) = L$

Horizontal/oblique asymptotes. Relates to End Behavior

$\lim_{x\to 5} f(x) = \infty$

$\lim_{x\to\infty} \frac{1}{x} = 0$

want $-\varepsilon < \frac{1}{x} < \varepsilon$ for every x beyond a certain value. (M)

Proof:

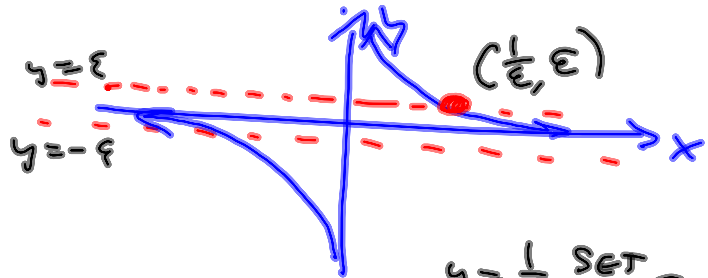
we find $M > 0$ so that $x > M$ implies

$\left|\frac{1}{x} - 0\right| < \varepsilon$

$\left|\frac{1}{x}\right| < \varepsilon$

$\frac{1}{x} < \varepsilon$

$x > \frac{1}{\varepsilon}$



$y = \frac{1}{x} \overset{\text{SET}}{=} \varepsilon$

$\Rightarrow x = \frac{1}{\varepsilon}$

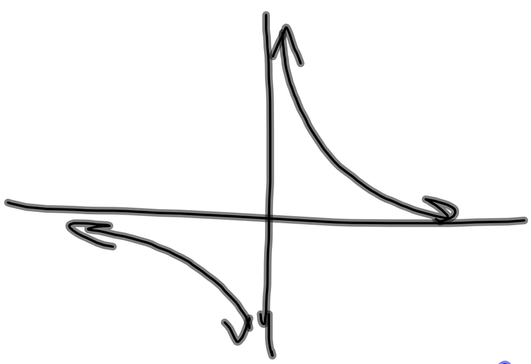
For x to the right of $\frac{1}{\varepsilon}$, we have $\frac{1}{x} < \varepsilon$

Let $M = \frac{1}{\varepsilon}$. Then $\left|\frac{1}{x} - 0\right| < \varepsilon$ for all $x > M$.

$\lim_{x \to 0} \frac{1}{x} \nexists$

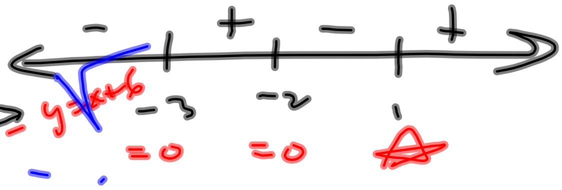
$\lim_{x \to 0^-} \frac{1}{x} = -\infty$

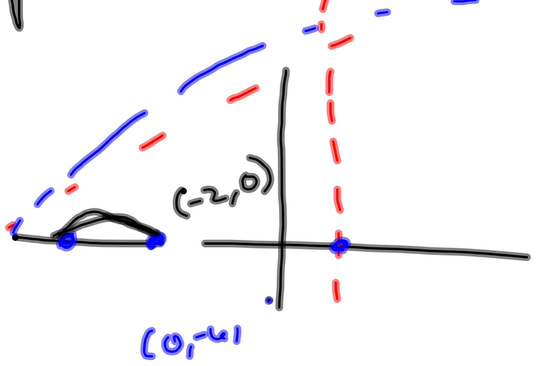
$\lim_{x \to 0^+} \frac{1}{x} = +\infty$



$\lim_{x \to 0} \frac{1}{x^2} = \infty$

$$\sqrt{\frac{(x+3)(x+2)}{x-1}}$$





$$\frac{(3)(2)}{-1} = -6$$

$$\frac{(x+3)(x+2)}{x-1}$$

$$= \frac{x^2+5x+6}{x-1}$$

$$\begin{array}{r|rrr} 1 & 1 & 5 & 6 \\ & & 1 & 6 \\ \hline & 1 & 6 & 12 \end{array}$$

says $\frac{x^2+5x+6}{x-1} = x+6 + \frac{12}{x-1}$

$\to 0$
as $x \to \pm\infty$