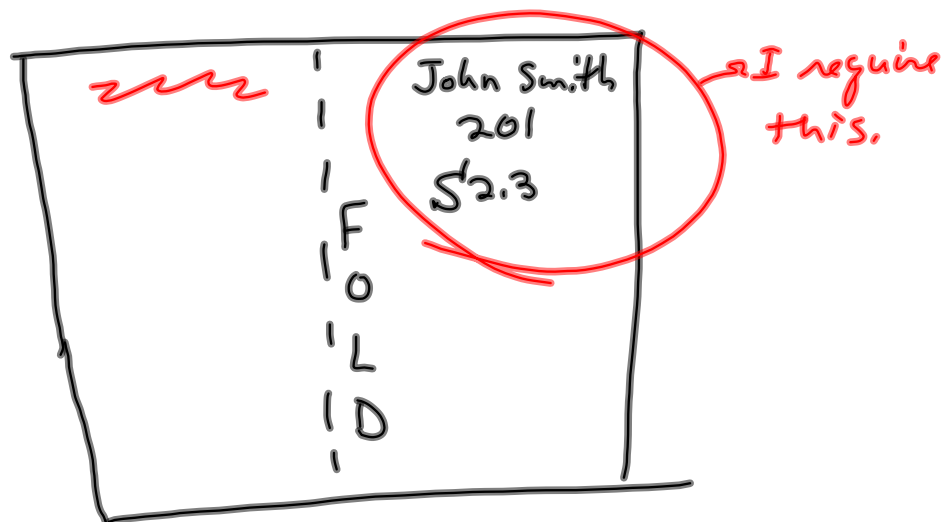


Some Homeworks are looking like answer keys. I require written work & context.

See Solutions Online for what I mean.



2.4 Tues

2.5 Wed

2.6 Thurs

Test 1 Early the following week

§2.5 Continuity.

Very closely tied to §2.2

Theorem 1 \longleftrightarrow Theorem 8
§2.2 \longleftrightarrow §2.5-

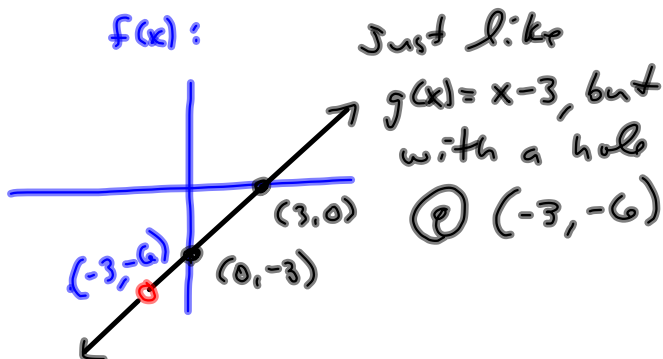
Removable Discontinuity means you can create a CONTINUOUS EXTENSION.

 $f(x) = \frac{x^2 - 9}{x + 3}$ is cont \pm everywhere, except① $x = -3$. We've seen $\lim_{x \rightarrow -3} \frac{x^2 - 9}{x + 3}$

$$= \lim_{x \rightarrow -3} \frac{(x-3)(x+3)}{x+3} = \lim_{x \rightarrow -3} (x-3) = -6$$

One way to extend $f(x)$ to all of \mathbb{R} :

$$f^*(x) = \begin{cases} \frac{x^2 - 9}{x + 3} & \text{if } x \neq -3 \\ -6 & \text{if } x = -3 \end{cases}$$



$f^*(x)$: Just replace $f(-3)$ with $\lim_{x \rightarrow -3} f(x)$

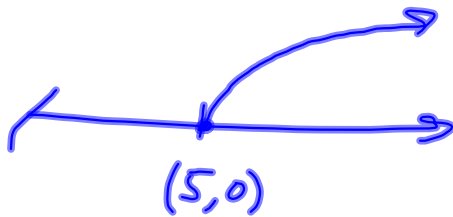
$f(x) = \frac{\sin x}{x}$ is cont^d everywhere except
① $x = 0$.

$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$, so Define $f^*(x)$ by

$$f^*(x) = \begin{cases} \frac{\sin x}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$

Jump to § 2.4 #47: $\sqrt{x-5}$

Given $\epsilon > 0$, find interval $I = (5, 5+\delta)$, $\delta > 0$ such that if $x \in I$, then $\sqrt{x-5} < \epsilon$.
 What limit is being discussed, here?
 What's its value.



$$\lim_{x \rightarrow 5^+} \sqrt{x-5} = 0$$

You
 you're
 They're
 Their
 There

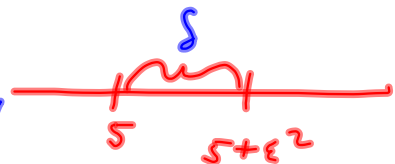
still need δ :

$$|\sqrt{x-5} - 0| < \epsilon$$

$$\sqrt{x-5} < \epsilon$$

$$x-5 < \epsilon^2 \equiv \delta$$

$$x < \epsilon^2 + 5$$



is principal square root. Always ≥ 0 ,

if it's real. If it's nonreal,

then it's the root with the smallest argument (angle measured counterclockwise from positive real axis.)

S 2.4 #52 b.

$$|\sin(\frac{1}{x})| \leq 1$$

$$f(x) = \begin{cases} x^2 \sin(\frac{1}{x}) & , x < 0 \\ \sqrt{x} & , x > 0 \end{cases}$$

$$\lim_{x \rightarrow 0^-} f(x) = 0 \quad y \leq 5$$

$$|xy| \leq |x||y| \leq |x| \cdot 5 = 5|x|$$

want $|x^2 \sin(\frac{1}{x}) - 0| < \epsilon$

STEP 1 $|x^2 \sin(\frac{1}{x})|$

$$= |x^2| |\sin(\frac{1}{x})| \leq |x^2| \cdot 1$$

$$|-1| = 1$$

$$= -(-1)$$

$$\leq |x^2|$$

$$= x^2 \quad \text{want } < \epsilon, \text{ i.e., want}$$

$$\sqrt{x^2} < \sqrt{\epsilon}$$

$$|x| < \sqrt{\epsilon}$$

$$-x < \sqrt{\epsilon}$$

$$x > -\sqrt{\epsilon}$$

STEP 2

$$|x - 0| < \delta$$

$$|x| < \delta$$

$$-x < \delta$$

$$x > -\delta$$

$$\text{Let } \delta = \sqrt{\epsilon}$$

