THEOREM 7

$$\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1 \qquad (\theta \text{ in radians}) \qquad (1)$$

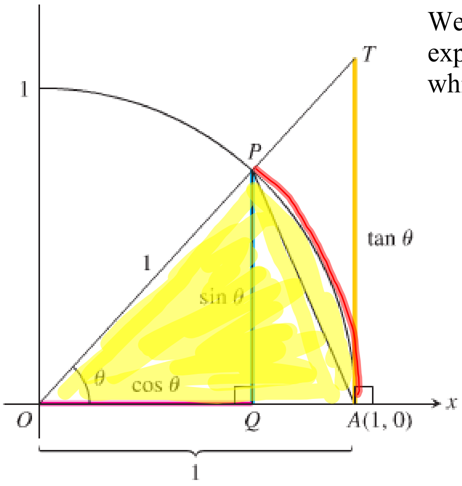


FIGURE 2.33 The figure for the proof of Theorem 7. By definition, $TA/OA = \tan\theta$, but $OA = 1$, so $TA = \tan\theta$.

We're going to squeeze (sandwich) this expression between two functions, both of which approach 1 as theta approaches 0.

Prove $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

$\frac{1}{2}$ base · height

$\frac{1}{2} r^2 \theta$ = area of sector, when θ is given in radians.

$r\theta$ = arc length.

$$\frac{1}{2} \cdot 1 \cdot \sin\theta < \frac{1}{2}(1^2)\theta < \frac{1}{2}\tan\theta$$

$$1 < \frac{\theta}{\sin\theta} < \frac{\tan\theta}{\sin\theta} = \sec\theta$$

$$1 > \frac{\sin\theta}{\theta} > \cos\theta \quad \forall\, \theta \in (0, \tfrac{\pi}{2})$$

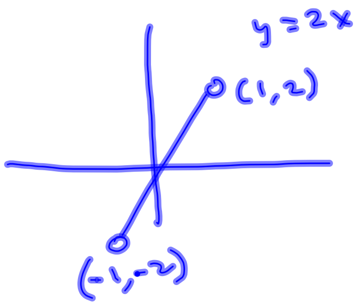
Take the limit as $\theta \to 0$

$$\lim_{\theta \to 0} 1 \geq \lim_{\theta \to 0} \frac{\sin\theta}{\theta} \geq \lim_{\theta \to 0} \cos\theta$$

$$1 \geq \lim_{\theta \to 0} \frac{\sin\theta}{\theta} \geq 1$$

i.e. $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

"Basically $\sin\theta \to 0$ about as quickly as $\theta \to 0$."



for $x \in (-1, 1)$

$2x < 2$ if $x < 1$

But $\lim_{x \to 1} 2x = 2 \leq 2$

§2.4 #29, 23, 33

(23)

$$\frac{\sin(3x)}{4x} = \frac{3}{4}\,\frac{\sin(3x)}{3x} \xrightarrow{x \to 0} \frac{3}{4}\cdot 1 = \frac{3}{4}$$

$$\frac{3}{4}\cdot\frac{\sin(3x)}{3x}$$

$$\frac{3}{3}\,\frac{\sin(3x)}{4x}$$

(29)

$$\frac{x(1+\cos x)}{\sin x\,\cos x} = \underbrace{\frac{x}{\sin x}}_{\to 1}\cdot\underbrace{\frac{1+\cos x}{\cos x}}_{\to 2}$$

$$\xrightarrow{x \to 0} 1\cdot\frac{2}{1} = 2$$

$\lim(fg) = \lim f \lim g$, provided $\lim f$ & $\lim g$ both exist.

(33) $$\lim_{t\to 0} \frac{\sin(1-\cos t)}{1-\cos t} = \lim_{u\to 0} \frac{\sin u}{u} = 1$$

Let $\boxed{u = 1-\cos t.}$ Then

$\lim_{t\to 0} u = 0$, right?

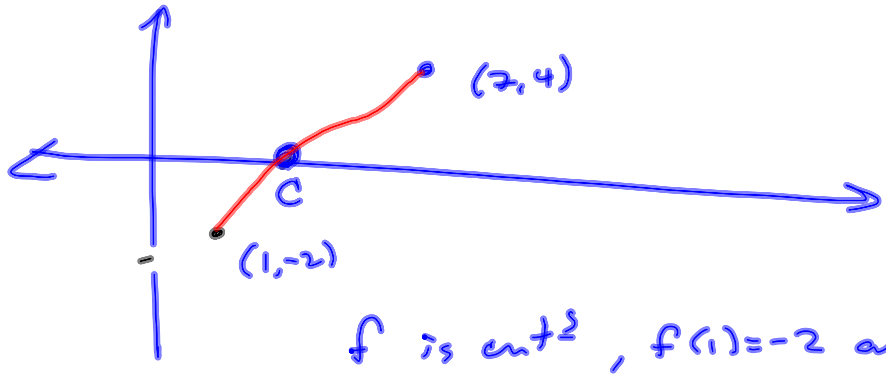
Change of variable

u-substitution.

$$\frac{\sin(3x)}{4x} = \frac{3}{3}\cdot\frac{\sin(3x)}{4x} = \frac{3\cdot\sin(3x)}{3\cdot 4x} =$$

$$= \frac{3\cdot\sin(3x)}{4\cdot 3x} = \frac{3}{4}\cdot\frac{\sin(3x)}{3x}$$

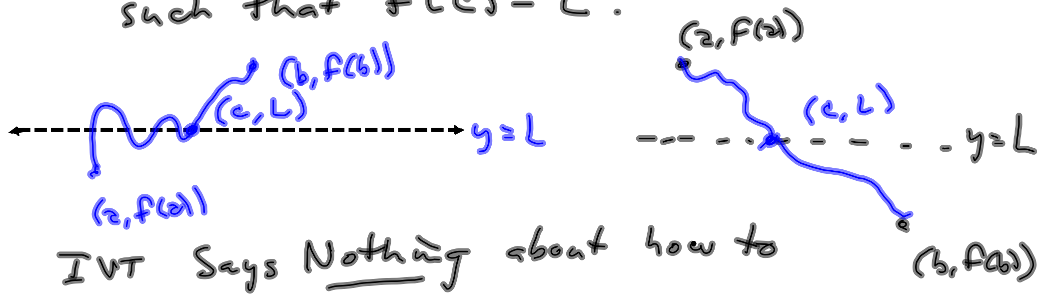
If $f(x)$ is cnt^s



f is cnt^s, $f(1)=-2$ and $f(7)=4$. Then there's a number c between $x=-2$ & $x=4$ such that $f(c)=0$.

IVT - Intermediate Value Theorem

If $f(x)$ is continuous on $[a,b]$, and $f(a) \neq f(b)$, let L be any number between $f(a)$ & $f(b)$. Then there's a $c \in (a,b)$ such that $f(c) = L$.



IVT Says Nothing about how to find c.

$$f(x)=\frac{x^5+x^2-5x+3}{x+100}$$

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              2.001
Y2(.0001
             2.0001
Y1(-3)
       -2.226804124
Y1(3)
        2.330097087
```

$f(-3) \approx -2.2268$

$f(3) \approx 2.3301$

Show that $\exists\ c \in (-3,3) \ni f(c)=0$.

and f is continuous everywhere, except @ $x=-100$

So IVT Says there's a zero in there!