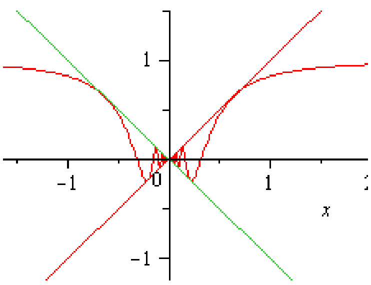
§2.3 # 53, 57

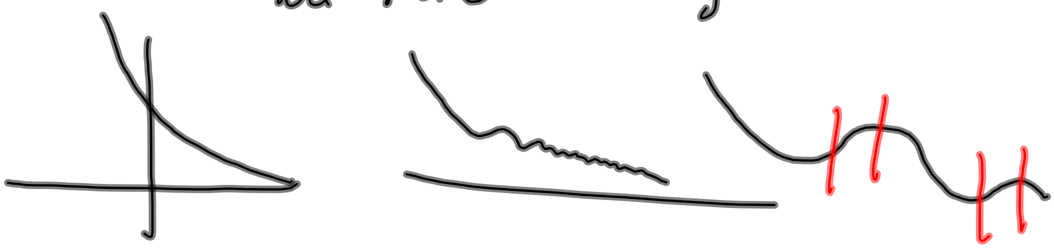
53 Show by example that this is wrong!

The # L is the limit of f(x) as $x \to x_0$ if f(x) gets closer to L as $x \to x_0$.



$f(x) = x \sin(\frac{1}{x})$ is oscillating so it's not just dwindling. You can put a floor & a ceiling on it, though, & that's what limit means!

Any oscillations "eventually" are "small enough."

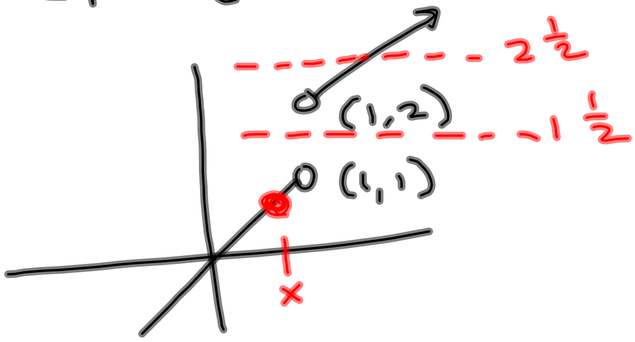


#57 Proving the limit does Not exist.

$$f(x) = \begin{cases} x & x < 1 \\ x+1 & x > 1 \end{cases}$$

Let $\varepsilon = \frac{1}{2}$. Show that $\nexists\ \delta > 0 \ni$

$|f(x) - 2| < \varepsilon$ whenever $0 < |x - 1| < \delta$



$1 - \delta/2$ in here.

Let $\delta > 0$ be given.

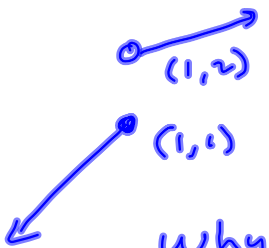
Consider $\boxed{x = 1 - \frac{\delta}{2}} \in (1-\delta, 1+\delta)$

is inside the "δ-band" or

"δ-neighborhood" of $x_0 = 1$.

But $f(1 - \frac{\delta}{2}) < 1$ & that's more

than $\frac{1}{2}$ units away from $y = 2$.



why doesn't $\lim\limits_{x\to 1} f(x) = 1$?

$$\lim_{x\to 1^-} f(x) = 1 \neq 2 = \lim_{x\to 1^+} f(x)$$

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Bonus Prove $\lim_{x\to 9} \sqrt{x-5} = 2$

Scratch: want $|\sqrt{x-5} - 2| < \varepsilon$

Goal is to extract $|x-9| <$ something from this.

$$|\sqrt{x-5} - 2| = \left| \frac{(\sqrt{x-5} - 2)(\sqrt{x-5} + 2)}{(\sqrt{x-5} + 2)} \right|$$

$$= \frac{|x-5-4|}{\sqrt{x-5} + 2} = \frac{|x-9|}{\sqrt{x-5} + 2}$$

Assume $\delta \le 1$ then $8 \le x \le 10$

and so $\sqrt{8-5} < \sqrt{x-5} < \sqrt{10-5}$

$\sqrt{3} < \sqrt{x-5} < \sqrt{5}$

$3 = 1+2 < \sqrt{3} + 2 \le \sqrt{x-5} + 2 \le \sqrt{5} + 2$

So $\frac{|x-9|}{\sqrt{x-5} + 2} < \frac{|x-9|}{3} \overset{\text{want}}{<} \varepsilon$

Look: $\frac{|x-9|}{3} < \frac{\delta}{3} = \varepsilon \longrightarrow$

$\boxed{\delta = 3\varepsilon}$ will do it.

Claim: $\lim_{x\to 9} \sqrt{x-5} = 2$

Proof:

Let $\varepsilon > 0$. Define $\boxed{\delta = \min\{1, 3\varepsilon\}}$. Then

$0 < |x-9| < \delta \implies$

$$|\sqrt{x-5} - 2| = \left| \frac{(\sqrt{x-5} - 2)(\sqrt{x-5} + 2)}{\sqrt{x-5} + 2} \right|$$

$$= \frac{|x-5-4|}{\sqrt{x-5} + 2} \le \frac{|x-9|}{\sqrt{3} + 2} < \frac{|x-9|}{3}$$

$$< \frac{\delta}{3} \le \frac{3\varepsilon}{3} = \varepsilon \quad ■$$

<u>Claim</u> $\lim_{x \to 3} x^2 = 9$

<u>Scratch</u> want $|x^2-9| < \varepsilon$

$$|x^2-9| = |(x-3)(x+3)|$$

$$= \overset{\delta}{|x-3|}\,|x+3| \leq$$

Need a handle on $|x+3|$

Assume $\delta \leq 1$ then

$$2 \leq x \leq 4$$

$$5 \leq x+3 \leq 7 \implies$$

$$|x+3| \leq 7$$

So $|x-3||x+3| \leq 7|x-3| < 7\delta \equiv \varepsilon$

$\implies \delta = \frac{\varepsilon}{7}$ ought to work.

<u>Proof</u> Let $\varepsilon > 0$ be given.

Define $\delta = \min\{1, \frac{\varepsilon}{7}\}$. Then

$0 < |x-3| < \delta \implies$

$$|x^2-9| = |x-3||x+3| \leq 7|x-3|$$

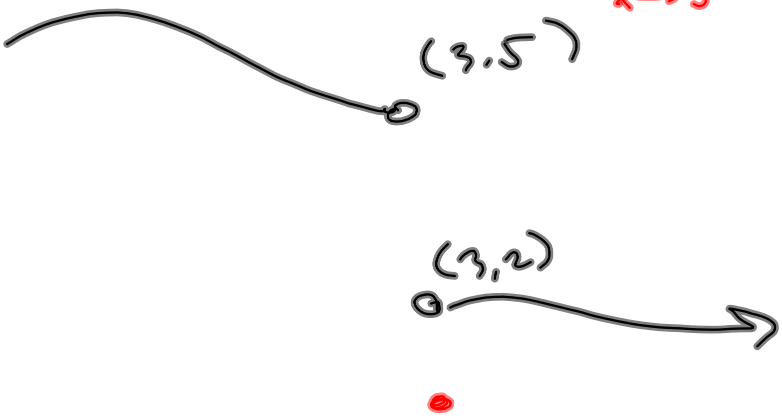
$$< 7\delta \leq 7\frac{\varepsilon}{7} = \varepsilon \quad \blacksquare$$

§ 2.4 one-sided limits.

$$\lim_{x\to 3^-} f(x) = 5 =$$

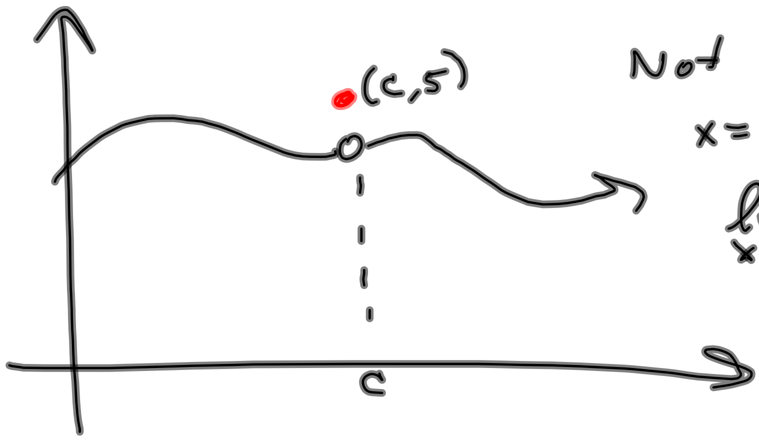
$$\lim_{x\to 3^+} f(x) = 2$$

Since $\lim_{x\to 3^-} f(x) \neq \lim_{x\to 3^+} f(x)$, we know $\lim_{x\to 3} f(x) \nexists$



§2.5 Continuity. cnt$^{\underline{s}}$, difb$^{\underline{l}}$

We say $f(x)$ is continuous at $x = c$ if $\lim_{x \to c} f(x) = f(c)$



Not cnt$^{\underline{s}}$ @ $x = c$, b/c $\lim_{x \to c} f(x) = 5 \neq f(c)$

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