My olden days

Claim: $\lim_{x \to 5} (3x-5) = 10$

Scratch: we reason from

$|f(x) - 10|$ to

$|x-5| <$ something

"Isolate the $|x-5|$."

want $|3x-5-10| < \varepsilon$

$|3x-15| < \varepsilon$

$3|x-5| < \varepsilon$

$|x-5| < \frac{\varepsilon}{3} \equiv \delta$

Pf

Let $\varepsilon > 0$ be given.

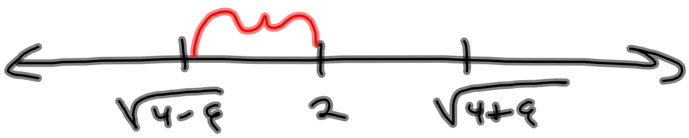
Define $\delta = \frac{\varepsilon}{3}$. Then $0 < |x-5| < \delta$, we have $|3x-15| = 3|x-5| < 3\delta = 3 \cdot \frac{\varepsilon}{3} = \varepsilon$ ∎

Show that $\lim_{x \to 2} (x^2) = 4$

want
$$|x^2 - 4| < \varepsilon$$

$$-\varepsilon < x^2 - 4 < \varepsilon$$
$$\sqrt{4-\varepsilon} < \sqrt{x^2} < \sqrt{4+\varepsilon}$$
$$\sqrt{4-\varepsilon} < |x| < \sqrt{4+\varepsilon}$$
$$\sqrt{4-\varepsilon} < x < \sqrt{4+\varepsilon}$$

Assume
$\delta \le 1$
Then
$|x-2| \le 1$
i.e.
$1 \le x \le 3$
Lets us drop the absolute value



$$\delta = \min\left\{2 - \sqrt{4-\varepsilon},\ \sqrt{4+\varepsilon} - 2\right\}$$

§2.3 questions? #35

$f(x) = \sqrt{1-5x}$ $\quad x_0 = -3, \ \varepsilon = 0.5$

$$\lim_{x \to -3} f(x) = 4$$

Want $\left|\sqrt{1-5x} - 4\right| < 0.5$

$$-.5 < \sqrt{1-5x} - 4 < .5$$

$$3.5 < \sqrt{1-5x} < 4.5$$

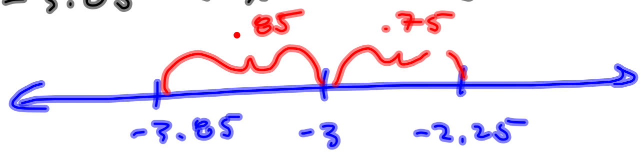
$$3.5^2 < 1-5x < 4.5^2$$

$$3.5^2 - 1 < -5x < 4.5^2 - 1$$

$$\frac{\frac{49}{4} - \frac{4}{4}}{-5} = \frac{3.5^2 - 1}{-5} > x > \frac{4.5^2 - 1}{-5} = \frac{\frac{81}{4} - \frac{4}{4}}{-5}$$

$$-2.25 = -\frac{9}{4} > x > -\frac{77}{20} = -3.85$$

$$-3.85 < x < -2.25$$



Let $\delta = .75$. Then $0 < |x - (-3)| < \delta$

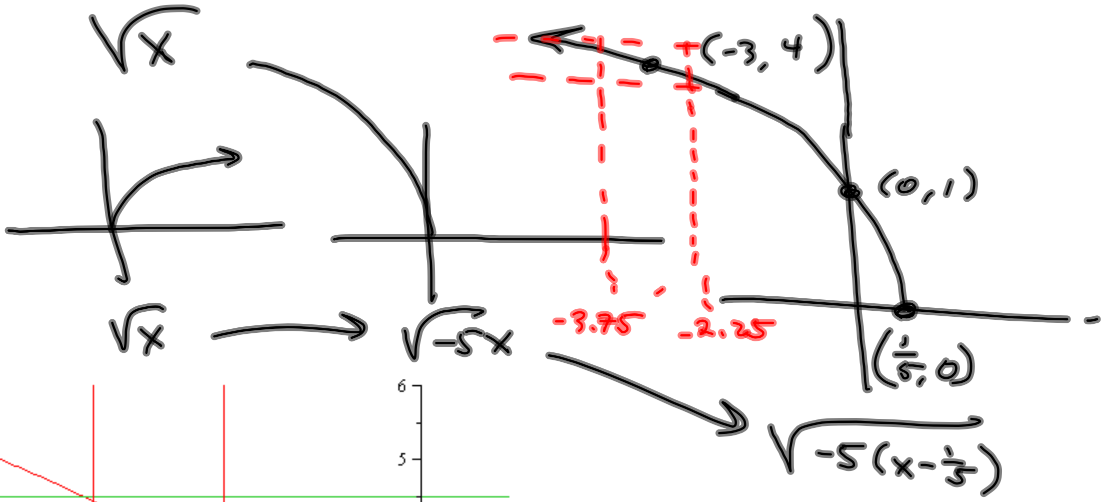
$\Rightarrow \left|\sqrt{1-5x} - 4\right| < 0.5$

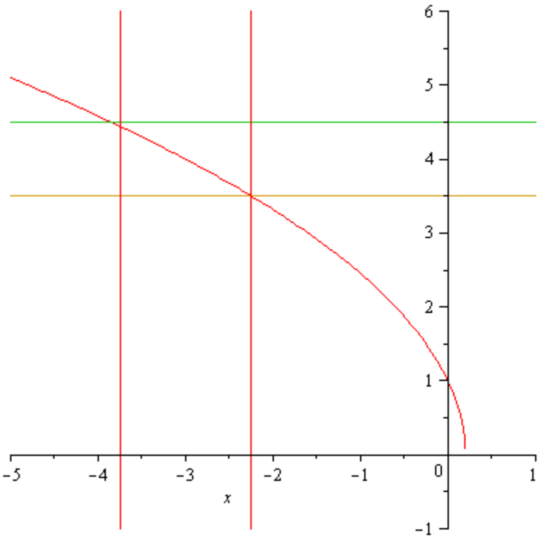
Scratch $\frac{45}{(4)(5)}$

$= -\frac{9}{4}$

$-\frac{77}{(20)} = -\frac{77}{20}$

$\sqrt{1-5x}$ in the neighborhood of $x=-3.$ $\sqrt{-5(x-\frac{1}{5})}$





Claim $\lim_{x\to 3} 5x-2 = 13$

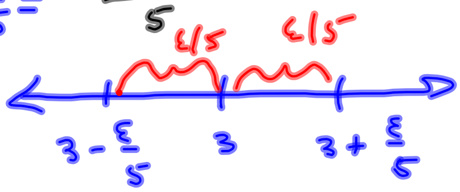
Scratch want

$$|5x-2-13| < \varepsilon$$

$$-\varepsilon < 5x-15 < \varepsilon$$

$$15-\varepsilon < 5x < 15+\varepsilon$$

$$3-\frac{\varepsilon}{5} = \frac{15-\varepsilon}{5} < x < \frac{15+\varepsilon}{5} = 3+\frac{\varepsilon}{5}$$



Let $\boxed{\delta = \frac{\varepsilon}{5}}$

Formal write up <u>Claim</u> $\lim_{x\to 3}(5x-2) = 13$

<u>Proof</u> Let $\varepsilon > 0$ be given. Define $\delta = \frac{\varepsilon}{5}$.

Then $\underline{0 < |x-3| < \delta} \Longrightarrow$

$$|5x-2-13| = |5x-15| =$$

$$5|x-3| < 5\delta = 5\frac{\varepsilon}{5} = \varepsilon \ \blacksquare$$

$$|AB| = |A||B|$$

$$\underline{|5x-15|} = |5(x-3)| = |5|\,|x-3| = \underline{5|x-3|}$$

Ask about #23 Mañana if there's a question.

#39 claim: $\lim_{x \to 9} \sqrt{x-5} = 2$

(scratch)

want $|\sqrt{x-5} - 2| < \varepsilon$

1

$(2-\varepsilon)^2 = ((-1)(\varepsilon-2))^2 = (\varepsilon-2)^2$

$-\varepsilon < \sqrt{x-5} - 2 < \varepsilon$

$2-\varepsilon < \sqrt{x-5} < 2+\varepsilon$

$(\varepsilon-2)^2 = (2-\varepsilon)^2 < x-5 < (2+\varepsilon)^2 = (\varepsilon+2)^2$

$\varepsilon^2 - 4\varepsilon + 4 < x-5 < \varepsilon^2 + 4\varepsilon + 4$

$\varepsilon^2 - 4\varepsilon + 9 < x < \varepsilon^2 + 4\varepsilon + 9$

2

$|x-9| < \delta$

$-\delta < x-9 < \delta$

$9-\delta < x < 9+\delta$

I'm a way, this work is proof, but I'll give you a formal writeup, tomorrow.

$9-\delta = \varepsilon^2 - 4\varepsilon + 9$ $\qquad$ $9+\delta = \varepsilon^2 + 4\varepsilon + 9$

$-\delta = \varepsilon^2 - 4\varepsilon$ $\qquad$ $\delta = \varepsilon^2 + 4\varepsilon = 4\varepsilon + \varepsilon^2$

$\boxed{\delta = 4\varepsilon - \varepsilon^2}$ Make this your δ.

Proof Let $\varepsilon > 0$ be given. Define $\delta = 4\varepsilon - \varepsilon^2$.

(Assume $\varepsilon < 4$ to keep $\delta > 0$)

Then $0 < |x-9| < \delta = 4\varepsilon - \varepsilon^2$

want to end here.

$|\sqrt{x-5} - 2| < \varepsilon$

$\delta = 4\varepsilon - \varepsilon^2$

$0 < |x - a| < \delta \Rightarrow$

$|\sqrt{x-5} - 2|$

$\varepsilon^2 - 4\varepsilon < x - a < 4\varepsilon - \varepsilon^2$

$\varepsilon^2 - 4\varepsilon + 4 < x - 5 < -\varepsilon^2 + 4\varepsilon + 4$

$|\varepsilon - 2| = \sqrt{\varepsilon^2 - 4\varepsilon + 4} < \sqrt{x-5} < \sqrt{-\varepsilon^2 + 4\varepsilon + 4}$

$\underbrace{|\varepsilon - 2| - 2}_{\varepsilon} < |\sqrt{x-5} - 2| < \underbrace{\sqrt{} - 2}_{\varepsilon}$