$\lim_{x\to c} f(x) = L$ means, given any $\varepsilon > 0$, I can find a $\delta > 0$ such that $0 < |x-c| < \delta \Rightarrow |f(x)-L| < \varepsilon$.

$\boxed{E}$ Find $\delta > 0$ so that $0 < |x-3| < \delta$ implies $|f(x)-11| < \varepsilon = .1$ for $f(x) = 2x+5$.

want $f(x)$ between 10.9 & 11.1

Want $|f(x)-11| < \varepsilon$

$|2x+5-11| < .1$ Solve for x.

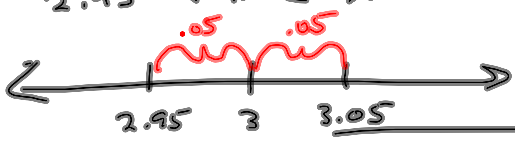
$|2x-6| < .1$

$-.1 < 2x-6 < .1$

$5.9 < 2x < 6.1$

$\frac{5.9}{2} < x < \frac{6.1}{2}$ $x \to 3$

$2.95 < x < 3.05$



Define (Let) $\boxed{\delta = .05}$ More discussion.

This means $3 - .05 < x < 3 + .05$

$2.95 < x < 3.05$

$f(2.95) = 2(2.95)+5 = 10.9$

$f(3.05) = 2(3.05)+5 = 11.1$

So it's exactly .1 above or .1 below if I plug in the endpoints. and anything between (anything in (2.95, 3.05) will give $f(x)$ in (10.9, 11.1) which was the goal.

$|2x-6| < .1 \implies$

$2x-6 < .1$ AND $2x-6 > -.1$

$|2x-6| > .1$

Reverse the sense of the inequality and change signs.

$2x-6 > .1$ OR $2x-6 < -.1$

$-.1 > 2x-6 > .1$ BAD !

$-.1 < 2x-6 < .1$ OK

$\boxed{E}$ $f(x) = 2x+5$. Find $\delta \ni$

$0 < |x-3| < \delta \Longrightarrow |f(x) - 11| < \varepsilon = .01$

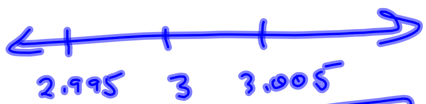
$|2x+5-11| < .01$

$-.01 < 2x-6 < .01$

$+\ 5.99 < 2x < 6.01$

$\frac{5.99}{2} < x < \frac{6.01}{2}$

$2.995 < x < 3.005$



So $\boxed{\delta = .005}$ ought to work.

Pattern:

$\varepsilon = .1 \Longrightarrow \delta = .05$ works

$\varepsilon = .01 \Longrightarrow \delta = .005$

$\varepsilon = \varepsilon \Longrightarrow \frac{\varepsilon}{2}$.

Claim: $\lim_{x \to 3} (2x+5) = 11$

Proof: Let $\varepsilon > 0$ be given. Define $\delta = \frac{\varepsilon}{2}$.
Then for any x such that $0 < |x-3| < \delta$,
we have

$$
\begin{aligned}
&|f(x) - 11| \\
&= |2x+5-11| \\
&= |2x-6| \\
&= 2|x-3| \\
&< 2\delta \\
&= 2 \cdot \frac{\varepsilon}{2} \\
&= \varepsilon \quad \blacksquare
\end{aligned}
$$

Scratch Work

$$
\begin{aligned}
|2x+5-11| &< \varepsilon \\
|2x-6| &< \varepsilon \\
-\varepsilon < 2x-6 &< \varepsilon \\
6-\varepsilon < 2x &< 6+\varepsilon
\end{aligned}
$$

$$3 - \frac{\varepsilon}{2} = \frac{6-\varepsilon}{2} < x < \frac{6+\varepsilon}{2} = 3 + \frac{\varepsilon}{2}$$

Let $\delta = \frac{\varepsilon}{2}$

My olden days

Claim: $\lim_{x \to 5} (3x-5) = 10$

Scratch: we reason from

$|f(x) - 10|$ to

$|x-5| <$ something

"Isolate the $|x-5|$."

want $|3x-5-10| < \varepsilon$

$|3x-15| < \varepsilon$

$3|x-5| < \varepsilon$

$|x-5| < \frac{\varepsilon}{3} \equiv \delta$

Pf

Let $\varepsilon > 0$ be given.

Define $\delta = \frac{\varepsilon}{3}$. Then $0 < |x-5| < \delta$, we have $|3x-15| = 3|x-5| < 3\delta = 3 \cdot \frac{\varepsilon}{3} = \varepsilon$ ∎

Aug 24-12:44 PM

Show that $\lim_{x \to 2} (x^2) = 4$

want

$$|x^2 - 4| < \varepsilon$$

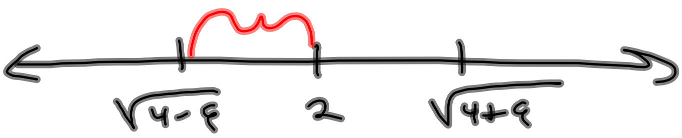
$$-\varepsilon < x^2 - 4 < \varepsilon$$

$$\sqrt{4-\varepsilon} < \sqrt{x^2} < \sqrt{4+\varepsilon}$$

$$\sqrt{4-\varepsilon} < |x| < \sqrt{4+\varepsilon}$$

$$\sqrt{4-\varepsilon} < x < \sqrt{4+\varepsilon}$$

Assume
$\delta \le 1$
Then
$|x-2| \le 1$
i.e.
$1 \le x \le 3$
Lets us drop the absolute value



$$\delta = \min\{2 - \sqrt{4-\varepsilon},\ \sqrt{4+\varepsilon} - 2\}$$