$f(x) = x^3 - 3x^2 + 4$, $P(2, 0)$

$(2+h)^3 =$

1 1
1 2 1
1 3 3 1

$2^3 + 3(2)^2 h + 3(2)(h^2) + h^3$

$2^3 - 3 \cdot 2^2 + 4$

$= 8 + 12h + 6h^2 + h^3$

$$\frac{f(2+h) - f(2)}{h} = \frac{(2+h)^3 - 3(2+h)^2 + 4 - 0}{h}$$

$$= \frac{8 + 12h + 6h^2 + h^3 - 3(2^2 + 2 \cdot 2h + h^2) + 4}{h}$$

$$= \frac{8 + 12h + 6h^2 + h^3 - 3(4 + 4h + h^2) + 4}{h}$$

$$= \frac{8 + 12h + 6h^2 + h^3 - 12 - 12h - 3h^2 + 4}{h}$$

$$= \frac{\cancel{12h} + 6h^2 + h^3 - \cancel{12h} - 3h^2}{h}$$

$\boxed{h \xrightarrow{h \to 0} 0}$

$$= \frac{3h^2 + h^3}{h} = \frac{\cancel{h}(3h + h^2)}{\cancel{h}} = 3h + h^2 \xrightarrow{h \to 0} 0$$

$\Longrightarrow f'(2) = 0$

$=$

$f(x) = x^3 - 3x^2 + 4$

$f'(x) = 3x^2 - 6x \longrightarrow$

$f'(2) = 3(4) - 12 = 0$ ✓

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$$\lim \frac{f}{g} = \frac{\lim f}{\lim g} \quad \text{, provided } \lim g \neq 0$$

If the limits of the pieces exist, then pretty much all arithmetic is respected.

$$\lim_{x \to 2} \frac{x+5}{x+3} = \frac{\lim_{x\to 2} (x+5)}{\lim_{x\to 2} (x+3)} = \frac{\lim_{x\to 2} x + \lim_{x\to 2} 5}{\lim_{x\to 2} x + \lim_{x\to 2} 3}$$

$$= \frac{2+5}{2+3} = \frac{7}{5}$$

$$3 \frac{d}{dx}\left[f(x)\right]$$

$$= \frac{d}{dx}\left[3f(x)\right]$$

§2.2 #s 11 – 42 All good test-type.
#s 1 – 4,

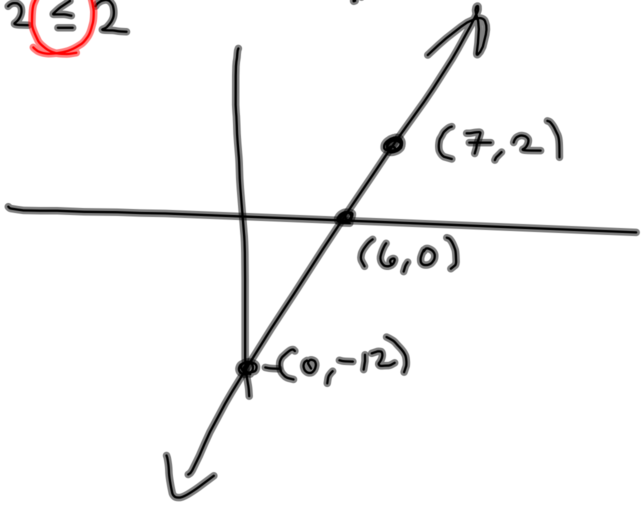
Sandwich Theorem
Squeeze Theorem

If I know $f(x) < 2$, always for $x < 7$. Then I know

$$\lim_{x\to 7} f(x) \le 2$$

$f(x) = 2x - 12 < 2$ if $x < 7$

$$\lim_{x\to 7} f(x) = 2 \le 2$$



If $f(x) \leq g(x) \leq h(x)$ for all $x \in (a,b)$, and if

$$\lim_{x\to b} f(x) = 7 = \lim_{x\to b} h(x), \text{ then}$$

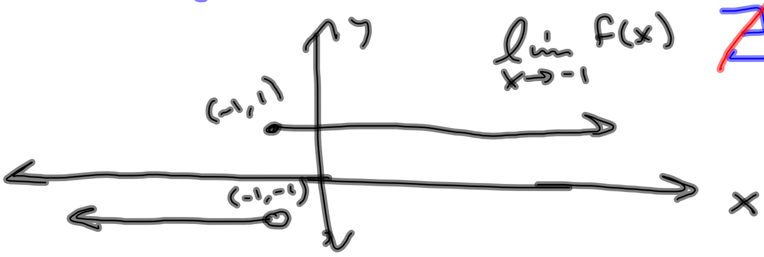
$$\lim_{x\to b} g(x) = 7.$$

$$f(x) = \frac{|x+1|}{x+1} =$$

$$\frac{|x+1|}{x+1} = \begin{cases} \frac{x+1}{x+1} & \text{if } x+1 \geq 0 \to x \geq -1 \\ \frac{-(x+1)}{x+1} & \text{if } x+1 < 0 \to x < -1 \end{cases}$$

$$= \begin{cases} 1 & \text{if } x \geq -1 \\ -1 & \text{if } x < -1 \end{cases}$$

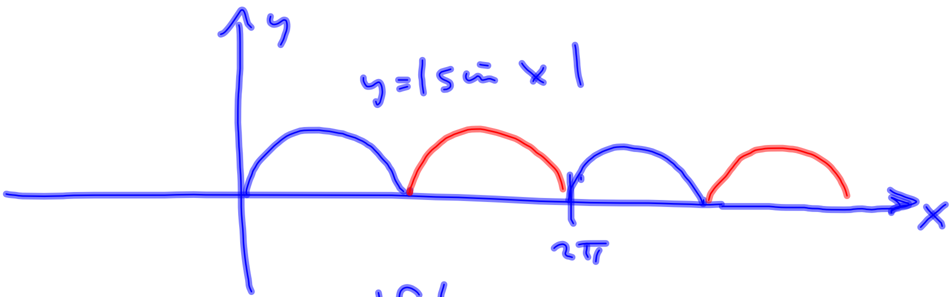
$$\lim_{x\to -1} f(x) \quad \nexists$$

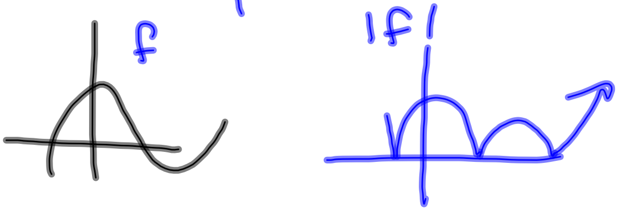


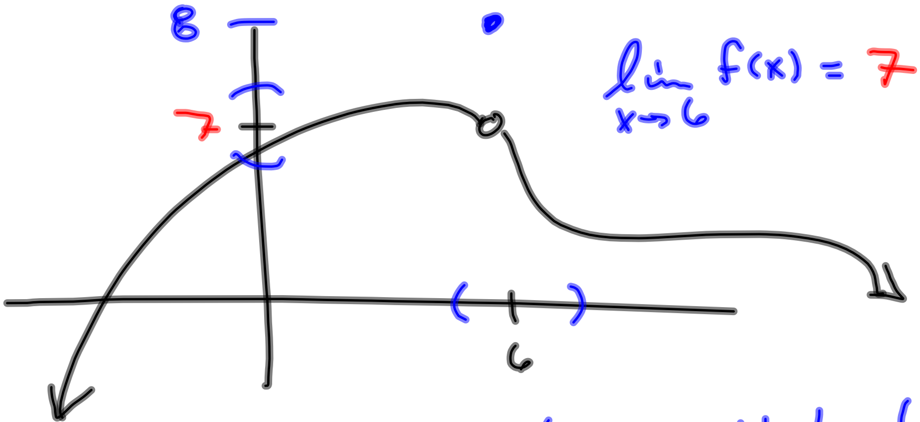
| ☺ | =

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$|\sin x| = \begin{cases} \sin x & \text{if } \sin x \geq 0 \\ -\sin x & \text{if } \sin x < 0 \end{cases}$$







$$\lim_{x\to 6} f(x) = 7$$

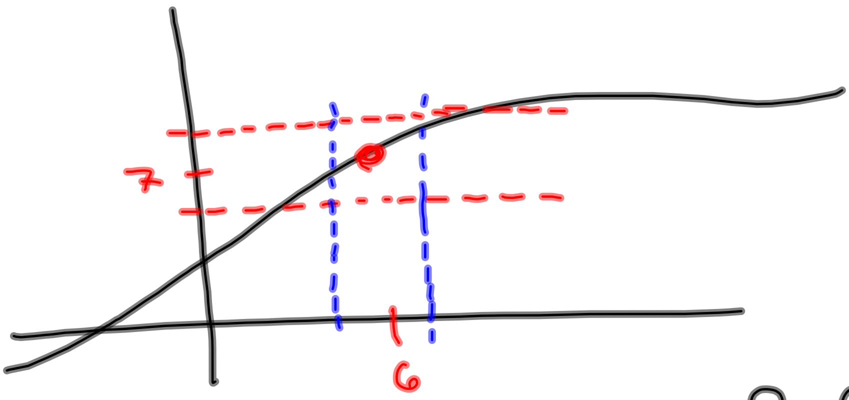
If x is in the neighborhood of x=6, then y is close to y = 7.

$0 < |x-6| <$ small $\Longrightarrow$

$|f(x)-7| <$ small

$\varepsilon > 0$ is how close you want $f(x)$ to stay to $y=7$.

$\delta > 0$ is how close x has to be to $x=6$ to keep $f(x)$ within ε of $y=7$.



we say that $\lim_{x\to 6} f(x) = 7$ if, for any $\varepsilon > 0$, there exists a $\delta > 0$ such that any x such that $0 < |x-6| < \delta$ satisfies

$$|f(x) - 7| < \varepsilon.$$

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