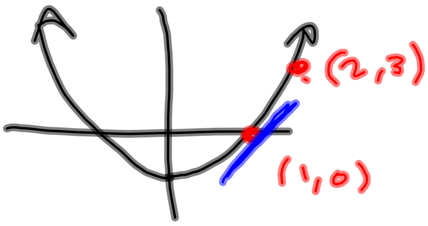
* Find the avg slope of $f(x) = x^2 - 1$ on $[1, 2]$

* what's the instantaneous slope of $f(x)$ at $x = 1$?

$$m_{avg} = \frac{f(2) - f(1)}{2-1} = \frac{(2^2-1) - (1^2-1)}{1}$$

$$= \frac{3-0}{1} = 3 = m_{avg}$$



$$\frac{f(1+h) - f(1)}{h} = \frac{(1+h)^2 - 1 - (1^2-1)}{h}$$

$$= \frac{1 + 2h + h^2 - 1 - 0}{h} = \frac{2h+h^2}{h}$$

$$= \frac{\cancel{h}(2+h)}{\cancel{h}} = 2 + h \xrightarrow{h \to 0} \boxed{2 = m_{tan}}$$

In general:

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - 1 - (x^2-1)}{h}$$

$$= \frac{x^2 + 2xh + h^2 - 1 - x^2 + 1}{h}$$

$$= \frac{2xh + h^2}{h} = \frac{h(2x+h)}{h} = 2x + h \xrightarrow{h \to 0} 2x$$

$$= \boxed{f'(x) = 2x}$$ and @ $x = 1$ $f'(1) = 2(1) = 2$

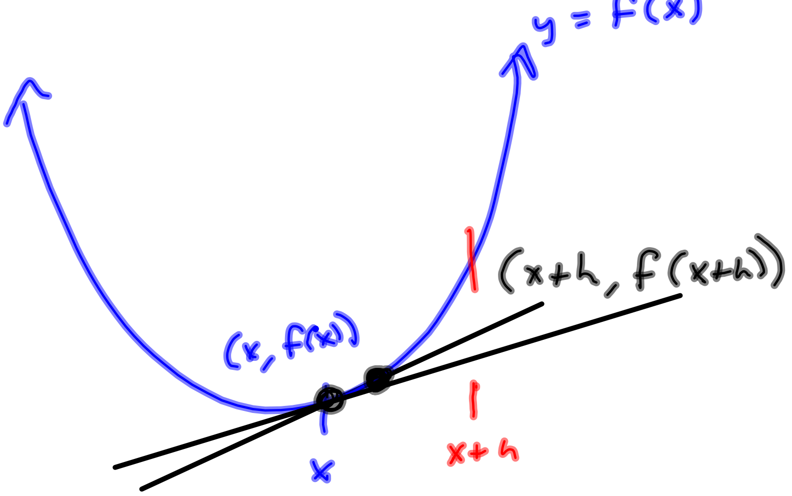
$f(x) = x^2 \longrightarrow 2x = f'(x)$

Aug 20-12:41 PM

Tangent is the limit of the slope of the secant line as the distance (h) between the two inputs approaches zero.

$h = 0$ Impossible

$h \to 0$ Calculus, Limits



Aug 21-12:13 PM

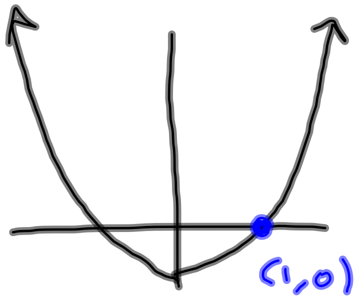
Example 1 §2.2 (Sort of)

Goal: Find m_{tan} for the function

$f(x) = x^2 - 1$ at $x = 1$.

Numerical / Tabular Approach

x	h	$x+h$	$\frac{f(x+h)-f(x)}{h}$
1	.5	1.5	2.5
1	.1	1.1	2.1
1	.01	1.01	2.01
1	.001	1.001	2.001
1	.0001	1.0001	2.0001



$Y_1 = x^2 - 1$

```
Plot1 Plot2 Plot3
\Y1=X^2-1
\Y2=(Y1(1+X)-Y1(1))/X
\Y3=
\Y4=
\Y5=
\Y6=
```

```
              2.01
Y2(.5)
               2.5
Y2(.1)
               2.1
Y2(.001
             2.001
```

want $\frac{f(x+h)-f(x)}{h}$

$\frac{f(1+h)-f(1)}{h}$

By our table, "it" appears to be approaching 2 as h approaches 0.

we say:

$$f'(1) = \lim_{h \to 0} \frac{f(1+h)-f(1)}{h} = 2$$

= Slope of the tangent line to $f(x)$

$= x^2 - 1$ @ $x = 1$.

Algebraically: $\frac{f(1+h)-f(1)}{h} = \frac{(1+h)^2 - 1 - (1^2 - 1)}{h}$

$$= \frac{1 + 2h + h^2 - 1 - 0}{h} = \frac{2h + h^2}{h} = \frac{h(2+h)}{h}$$

$$= 2 + h \xrightarrow{h \to 0} 2 = f'(1)$$

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2-1-(x^2-1)}{h}$$

$$= \frac{x^2+2xh+h^2-1-x^2+1}{h} = \frac{2xh+h^2}{h}$$

$$= \frac{h(2x+h)}{h} = 2x+h \xrightarrow{h\to 0} 2x = f'(x)$$

will give us the slope of the tangent line to $f(x)$ @ any x we choose.

$f(1) = 2(1) = 2$

$f(3) = 2(3) = 6$ = tan @ $x=3$!

Do it for $f(x) = x^3$

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^3 - x^3}{h}$$

$$= \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$$

$$= \frac{h(3x^2 + 3xh + h^2)}{h} = 3x^2 + 3xh + h^2 \xrightarrow{h \to 0} 3x^2 = f'(x)$$

Pascal's Triangle.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

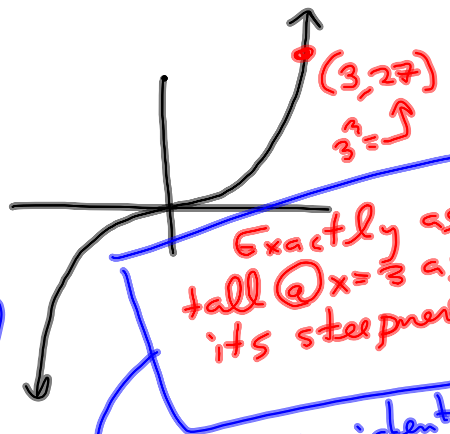
So $(x+1)^5$ is
$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$

$(x+h)^5$ is
$x^5 + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5$

Expanding Powers of a Binomial

BINOMIAL THEOREM

$f'(1) = 3$
$f'(3) = 27$



Exactly as tall @ $x=3$ as its steepness!

Pretty Coincidental

ONLY ONE FUNCTION is Exactly as steep as it is tall.

Differential Equations.

$f(x) = e^x$

Natural Exponential

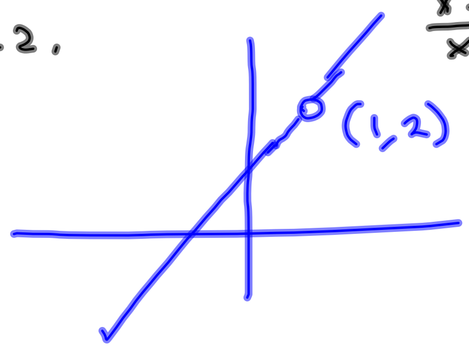
$f'(x) = f(x)$!

$$\boxed{\lim_{x\to 1} \frac{x^2-1}{x-1} = \lim_{x\to 1} \frac{(x+1)\cancel{(x-1)}}{\cancel{x-1}} = \lim_{x\to 1}(x+1) = 2}$$

Rose.

This is a removable discontinuity. There's just a single hole @ $x=1$, but the function behaves in a neighborhood of $x=2$.

$$\frac{x^2-1}{x-1} = x+1, \text{ if } x \neq 1.$$



Find $f'(x)$ for $f(x) = \sqrt{x}$

$$\frac{f(x+h)-f(x)}{h} = \left(\frac{\sqrt{x+h} - \sqrt{x}}{h}\right)\left(\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}\right)$$

$$= \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})} = \frac{h}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \frac{1}{\sqrt{x+h} + \sqrt{x}} \xrightarrow{h \to 0} \frac{1}{\sqrt{x}+\sqrt{x}} = \frac{1}{2\sqrt{x}} = f'(x)$$

$h \to 0$

$$(a-b)(a+b) = a^2 - b^2$$

Aug 21-12:59 PM