201 6.3, 6.4

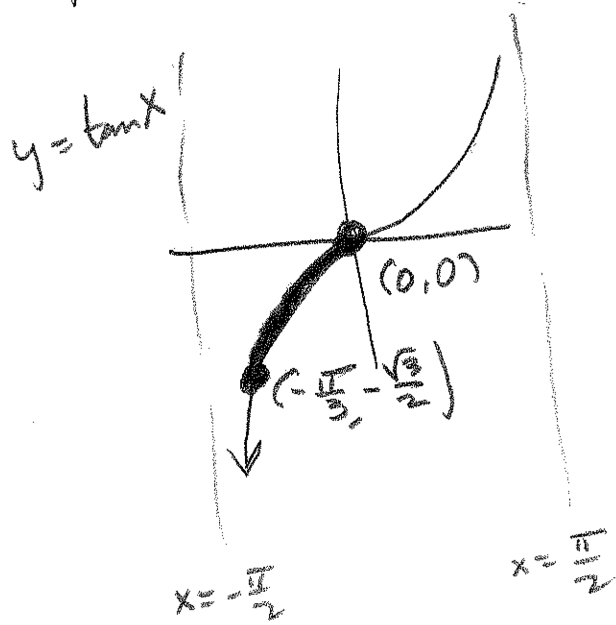
(1) $y = \tan x, \ -\frac{\pi}{3} \le x \le 0$

Arc Length $\displaystyle\int_a^b \sqrt{1 + f'(x)^2}\, dx = \int_{-\frac{\pi}{3}}^{0} \sqrt{1 + \sec^4 x}\, dx$

$y' = \sec^2 x$

$y'^2 = \sec^4 x$

$\boxed{\begin{array}{l} \approx 2.056999740 \\ \approx \text{Arc Length} \end{array}}$



L01 6.3, 6.4

2 $y = \int_0^x \sqrt{\sec^2 t - 1}\, dt \qquad -\frac{\pi}{3} \le x \le \frac{\pi}{4}$

$\Rightarrow \frac{dy}{dx} = \sqrt{\sec^2 x - 1} \Rightarrow \left(\frac{dy}{dx}\right)^2 = \sec^2 x - 1$

$\Rightarrow \int_{-\frac{\pi}{3}}^{\frac{\pi}{4}} \sqrt{1 + \sec^2 x - 1}\, dx = \int_{-\frac{\pi}{3}}^{\frac{\pi}{4}} |\sec x|\, dx = \int_{-\frac{\pi}{3}}^{\frac{\pi}{4}} \sec x\, dx$

≈ 2.20, by technology, or table (Clint).

3 a $L = \int_1^2 \sqrt{1 + \frac{1}{x^4}}\, dx \Rightarrow (y')^2 = \frac{1}{x^4} \Rightarrow$

$y' = \frac{1}{x^2} \Rightarrow y = -\frac{1}{x} + C$. We want

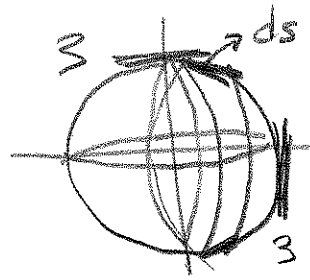
$y(1) = 0 \Rightarrow -\frac{1}{1} + C = 0 \Rightarrow C = 1 \Rightarrow \boxed{y = -\frac{1}{x} + 1}$

b There are infinitely many such curves, but $\boxed{\text{only } \underline{\text{ONE}}}$ has $y' = \frac{1}{x^2}$ and passes thru $(1,0)$.

201 S'6.3, 6.4

6.4 ① The surface area of a sphere of radius 3 is $4\pi r^2 = 4\pi \cdot 3^2 = 36\pi$.

We prove this by find the surface area of a circle rotated about the x-axis



$ds = \sqrt{1+y'^2}\,dx$

$r = \sqrt{9-x^2}$ = y-value @ x

$$2\pi \int_a^b y\,ds = 2\pi \int \sqrt{9-x^2}\,ds$$

$$ds = \sqrt{1+y'^2}\,dx$$

$$= \sqrt{1+\left(\tfrac{1}{2}(9-x^2)^{-\frac{1}{2}}(-2x)\right)^2}\,dx = \sqrt{1+(9-x^2)^{-1}x^2}\,dx$$

$$= \sqrt{\frac{9-x^2+x^2}{9-x^2}}\,dx = \frac{\sqrt{9}}{\sqrt{9-x^2}}\,dx = \frac{3}{\sqrt{9-x^2}}\,dx, \text{ so}$$

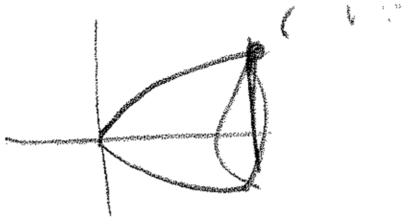
$$\text{Surface Area} = 2\pi \int_{-3}^{3} \sqrt{9-x^2} \cdot \frac{3}{\sqrt{9-x^2}}\,dx = 2\pi \cdot 3 \int_{-3}^{3} dx =$$

$$2\pi \cdot 3 \left[\, x \,\right]_{-3}^{3} = 2\pi \cdot 3\left[3-(-3)\right] = 2\pi \cdot 3 \cdot 2 \cdot 3$$

$$= 4\pi \cdot 3^2$$

201 6.3, 6.4

6.4 #2 $y=\sqrt{x+1}$, $1 \le x \le 5$ about x-axis



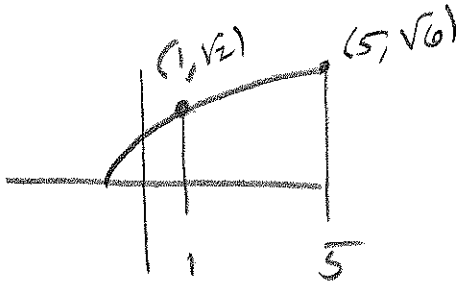
$$2\pi \int_0^5 r\, ds$$

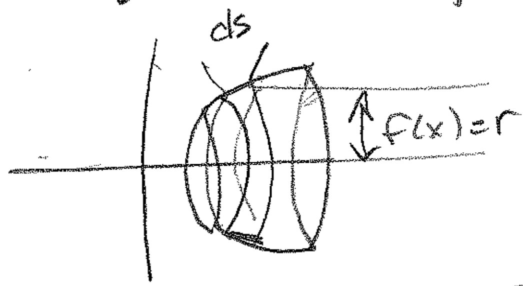
$y = (x+1)^{\frac{1}{2}}$

$y' = \frac{1}{2}(x+1)^{-\frac{1}{2}}$

$y'^2 = \frac{1}{4}(x+1)^{-1} = \frac{1}{4(x+1)}$

$1 + y'^2 = \frac{4x+4+1}{4(x+1)}$





$$2\pi \int_1^5 f(x)\sqrt{1+(y'^2)}\,dx = 2\pi \int_1^5 \sqrt{x+1}\sqrt{\frac{4x+5}{4(x+1)}}\,dx$$

Let $u = x+1 \Rightarrow du = dx$

$x=5 \Rightarrow u=6$

$x=1 \Rightarrow u=2$

$$2\pi \int_2^6 \sqrt{u}\sqrt{1+\frac{1}{4u}}\,du$$

$$= 2\pi \int_2^6 \sqrt{u}\sqrt{\frac{4u+1}{4u}}\,du = \frac{2\pi}{2}\int_2^6 \sqrt{u\left(\frac{4u+1}{u}\right)}\,du$$

$$= \pi \int_2^6 \sqrt{4u+1}\,du = \frac{\pi}{4}\int_9^{25} v^{\frac{1}{2}}\,dv = \frac{\pi}{4}\cdot\frac{2}{3}v^{\frac{3}{2}}\Big]_9^{25} = \frac{\pi}{6}\left[125-27\right]$$

$$= \frac{98\pi}{6} = \boxed{\frac{49\pi}{3}}$$

$v = 4u+1$ $u=6 \Rightarrow v=25$

$dv = 4du$ $u=2 \Rightarrow v=9$