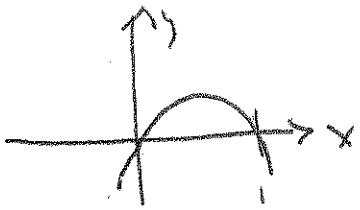
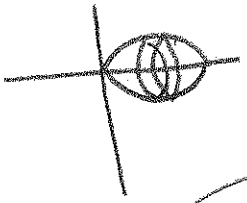
201 §6.1, 6.2

① $y = -x^2 + x$ & $y = 0$

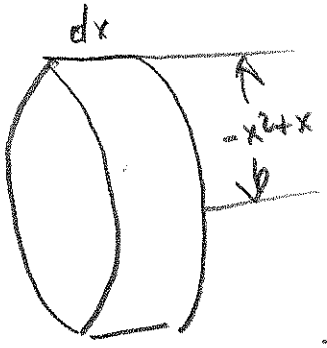
$= -x(x-1)$



② about x-axis:



$\pi r^2 \Delta x$

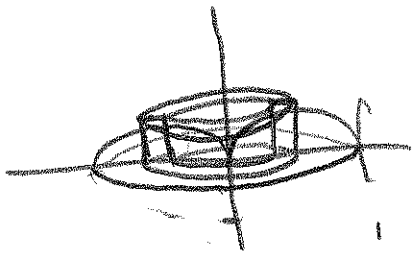


$$\text{Volume} = \pi \int_0^1 (-x^2+x)^2 \, dx$$

$$= \pi \int_0^1 (x^4 - 2x^3 + x^2)\, dx$$

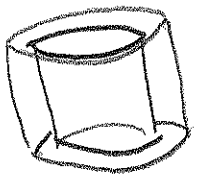
$$= \pi \left[\frac{x^5}{5} - \frac{2x^4}{4} + \frac{x^3}{3} \right]_0^1 = \pi \left[\frac{1}{5} - \frac{1}{2} + \frac{1}{3} \right] = \frac{6-15+10}{30}\pi = \boxed{\frac{\pi}{30}}$$

③ about y-axis



$2\pi r h \Delta x$

$2\pi x f(x) \Delta x$



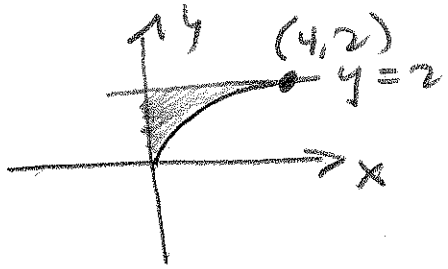
$$V = 2\pi \int_0^1 x(-x^2+x)\, dx$$

$$= 2\pi \int_0^1 (-x^3 + x^2)\, dx = 2\pi \left[-\frac{x^4}{4} + \frac{x^3}{3} \right]_0^1$$

$$= 2\pi \left[-\frac{1}{4} + \frac{1}{3} \right] = 2\pi \left[\frac{-3+4}{12} \right] = \boxed{\frac{\pi}{12}}$$

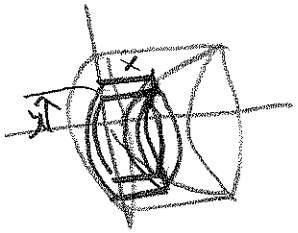
201 §6.1, 6.2

(2) $y = 2\sqrt{x}$, $y = 2$, $x = 0$



(a) x-axis

Easier w/ shells



$2\pi rh\Delta y$

$2\pi y \cdot \frac{y^2}{4} \cdot \Delta y$

$y = 2\sqrt{x}$

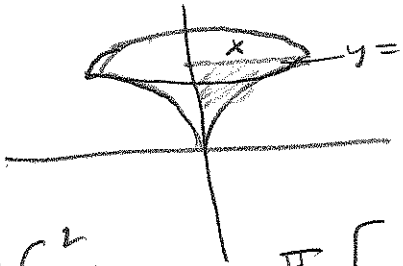
$\frac{y}{2} = \sqrt{x}$

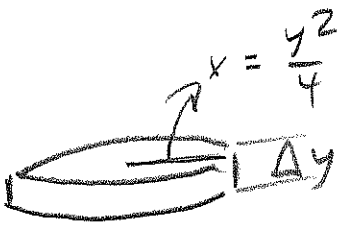
$\frac{y^2}{4} = x$

$$\text{Volume} = 2\pi \int_0^2 \frac{y^3}{4}\,dy = \frac{\pi}{2}\int_0^2 y^3\,dy$$

$$= \frac{\pi}{2}\left[\frac{y^4}{4}\right]_0^2 = \frac{\pi}{8}\left[2^4\right] = \boxed{2\pi}$$

(b) About y-axis



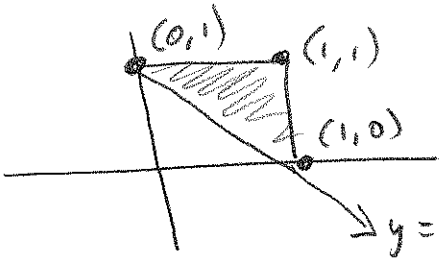


Disk method easier.

$$\pi \int_0^2 \left(\frac{y^2}{4}\right)^2 dy = \frac{\pi}{16}\int_0^2 y^4\,dy = \frac{\pi}{16}\left[\frac{y^5}{5}\right]_0^2 = \frac{\pi}{16}\left[\frac{32}{5}\right] = \boxed{\frac{2\pi}{5}}$$

201 §6.1, 6.2

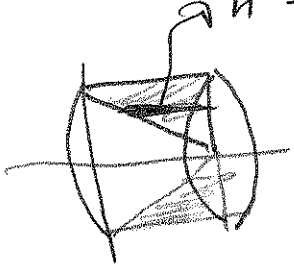
(3)



$x = -y + 1$

$h = 1 - (1 - y) = y$

(a) About x-axis



Shells: $2\pi r h \Delta y$

$= 2\pi y h \Delta y$

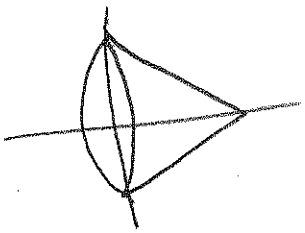
$= 2\pi y \cdot y \Delta y$

$$\text{Volume} = 2\pi \int_0^1 y^2 dy = 2\pi \left[\frac{y^3}{3}\right]_0^1 = \boxed{\frac{2\pi}{3}}$$

Disks: (washers)

Outer = cylinder of radius 1, ht 1

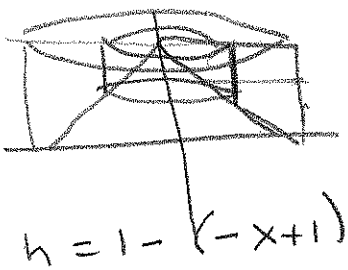
Inner = cone of height 1, radius 1



outer $\pi r^2 h = \pi$

inner $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (1)(1) = \frac{\pi}{3}$

Vol $= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ ✓



$h = 1 - (-x + 1)$

$h = x$

(b) About y-axis

Shells: $2\pi r h \Delta x = 2\pi x h \Delta x$

$$\text{Vol} = 2\pi \int_0^1 x^2 dx = \frac{2\pi}{3}\left[x^3\right]_0^1 = \frac{2\pi}{3}$$