201 §5.5, 5.6

① $\int_1^4 \frac{10\sqrt{x}}{(1+x^{3/2})^2}\,dx$

(a) Let $u = x^{3/2}+1$ $\quad u(1) = 1^{3/2}+1 = 2$

Then $du = \frac{3}{2}x^{\frac{1}{2}}dx = \frac{3\sqrt{x}}{2}dx \quad u(4) = 4^{3/2}+1 = 9$

$= 10 \cdot \frac{2}{3}\int_1^4 (x^{3/2}+1)^{-2} \cdot \frac{3}{2}\sqrt{x}\,dx$

$= \frac{20}{3}\int_2^9 u^{-2}du = \frac{20}{3}\left[\frac{u^{-1}}{-1}\right]_2^9 = -\frac{20}{3}\left[\frac{1}{9}-\frac{1}{2}\right]$

$= -\frac{20}{3}\left[\frac{2-9}{18}\right] = -\frac{20}{3}\left(-\frac{7}{18}\right) = \frac{(10)(7)}{3\cdot 9} = \frac{70}{27} \doteq 2.\overline{592}59$

(b) Same u, make the substitution $\ni$ we have

$10\cdot\frac{2}{3}\int (x^{3/2}+1)^{-2}\cdot\frac{3}{2}\sqrt{x}\,dx = \frac{20}{3}\frac{u^{-1}}{-1} + C = -\frac{20}{3}(x^{3/2}+1)^{-1}+C$

$\Rightarrow \left[-\frac{20}{3}(x^{3/2}+1)^{-1}\right]_1^4 = -\frac{20}{3}\left[\frac{1}{4^{3/2}+1} - \frac{1}{1^{3/2}+1}\right]$

$= -\frac{20}{3}\left[\frac{1}{9}-\frac{1}{2}\right] = \frac{70}{27} = 2.\overline{592}$, by previous wk.

201 §5.5, 5.6

(2) $\boxed{\#40, §5.5}$ $\displaystyle\int \frac{1}{x^3}\sqrt{\frac{x^2-1}{x^2}}\,dx$

$$= \int \sqrt{1-\frac{1}{x^2}} \cdot \frac{1}{x^3}\,dx = \int \left(1-\frac{1}{x^2}\right)^{\frac{1}{2}} \frac{1}{x^3}\,dx$$

Let $u = 1-x^{-2}$, then $du = 2x^{-3}dx = \frac{2}{x^3}dx$

This gives $\displaystyle \frac{1}{2}\int (1-x^{-2})^{\frac{1}{2}} \cdot \frac{2}{x^3}\,dx = \frac{1}{2}\int u^{\frac{1}{2}}du$

$$= \frac{1}{2}\,\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C = \boxed{\frac{1}{3}(1-x^{-2})^{\frac{3}{2}} + C}$$

or $\displaystyle \frac{1}{3}\sqrt{\frac{x^2-1}{x^2}}^{\,3} + C$ OR your favorite flavor.

$\boxed{\#42\ §5.5}$ $\displaystyle\int \sqrt{\frac{x^4}{x^3-1}}\,dx = \int \sqrt{x^4}\sqrt{\frac{1}{x^3-1}}\,dx$

$= \int x^2 (x^3-1)^{-\frac{1}{2}}dx$. Let $u = x^3-1 \Rightarrow du = 3x^2dx$

This gives $\displaystyle \frac{1}{3}\int (x^3-1)^{-\frac{1}{2}} \cdot 3x^2dx = \frac{1}{3}\int u^{-\frac{1}{2}}du$

$$= \frac{1}{3}\cdot\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C = \frac{2}{3}u^{\frac{1}{2}} + C = \boxed{\frac{2}{3}\sqrt{x^3-1} + C}$$

201 §5.5, 5.6

(3) Area between $f(x) = x^3 - 9x^2 + 9x + 10$ & $g(x) = x^2 - 4x + 4$

$f(x)$:

2	1	-9	9	10
		2	-14	-10
	1	-7	-5	0

$a=1, b=-7, c=-5$

$b^2 - 4ac = (-7)^2 - 4(1)(-5)$
$= 49 + 20 = 69$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{7 \pm \sqrt{69}}{2(1)}$

$= \dfrac{7 \pm \sqrt{69}}{2}$ → 7.6533119315, → -.6533119315

$g(x): x^2 - 4x + 4 = (x-2)^2$ is E.Z.

$f(x) = g(x) \Longrightarrow f(x) - g(x) = 0$ → Much of this work relates to 2nd part

$x^3 - 9x^2 + 9x + 10 - (x^2 - 4x + 4) = 0$

$x^3 - 10x^2 + 13x + 6 = 0$

2	1	-10	13	6
		2	-16	-6
	1	-8	-3	0

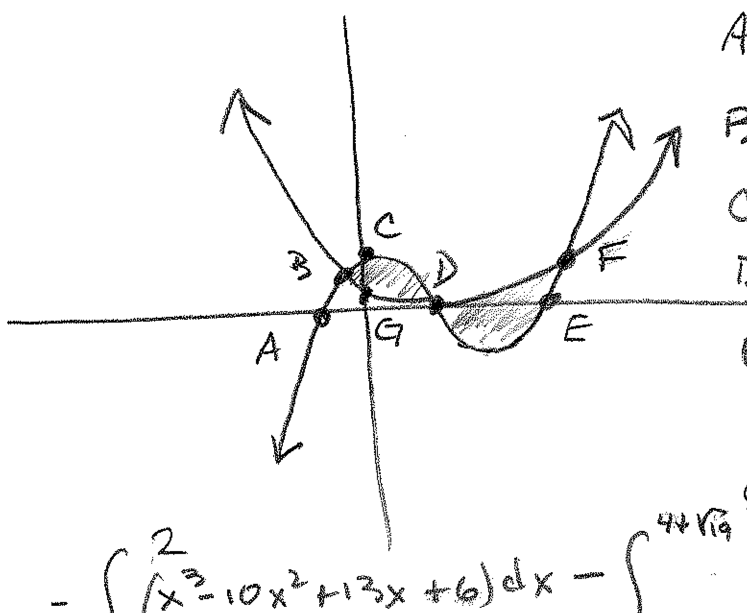
$a=1, b=-8, c=-3$

$b^2 - 4ac = (-8)^2 - 4(1)(-3)$
$= 64 + 12 = 76$

$2 \lfloor 76$, $2 \lfloor 38$, 19

$x = \dfrac{8 \pm \sqrt{76}}{2(1)} = \dfrac{8 \pm 2\sqrt{19}}{2} = 4 \pm \sqrt{19}$

8.358898944 OR -.358898944



$A \approx (-.359, 0)$
$B \approx (-.359, 5.564)$
$C \approx (0, 10)$
$D \approx (2, 0)$
$E \approx (7.653, 0)$
$F \approx (8.359, 40.436)$
$G \approx (0, 4)$

$$\text{Area} = \int_{4-\sqrt{19}}^{2} (x^3 - 10x^2 + 13x + 6)\,dx - \int_{2}^{4+\sqrt{19}} (x^3 - 10x^2 + 13x + 6)\,dx$$

201 §§5.5, 5.6

#3 cnt'd

$$= \left[\frac{1}{4}x^4 - \frac{10}{3}x^3 + \frac{13}{2}x^2 + 6x \right]_{4-\sqrt{19}}^{2}$$

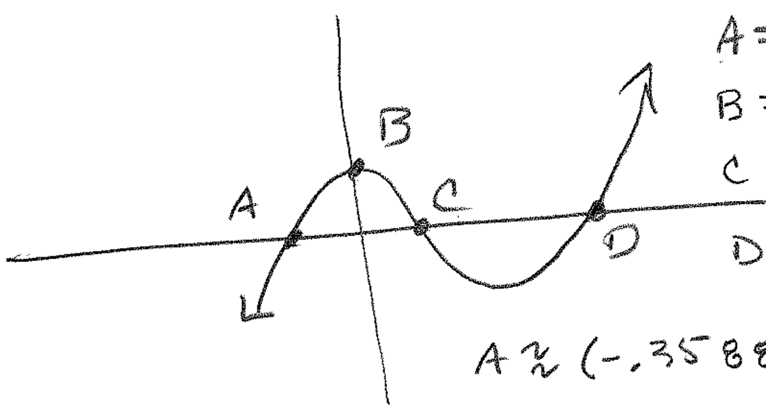
$$- \left[\frac{1}{4}x^4 - \frac{10}{3}x^3 + \frac{13}{2}x^2 + 6x \right]_{2}^{4+\sqrt{19}}$$

BONUS

$$= \frac{1523}{12} - \frac{76}{3}\sqrt{19} - \left(-\frac{1523}{12} - \frac{76}{3}\sqrt{19} \right)$$

$$= \frac{1523}{6} = 253.8\bar{3}$$

Integral is the same for 2nd part.
The picture for $f(x) - g(x)$ is different:



$A = (4-\sqrt{19}, 0)$
$B = (0, 6)$
$C = (2, 0)$
$D = (4+\sqrt{19}, 0)$

$A \approx (-.358898944, 0)$
$D \approx (8.358898944, 0)$