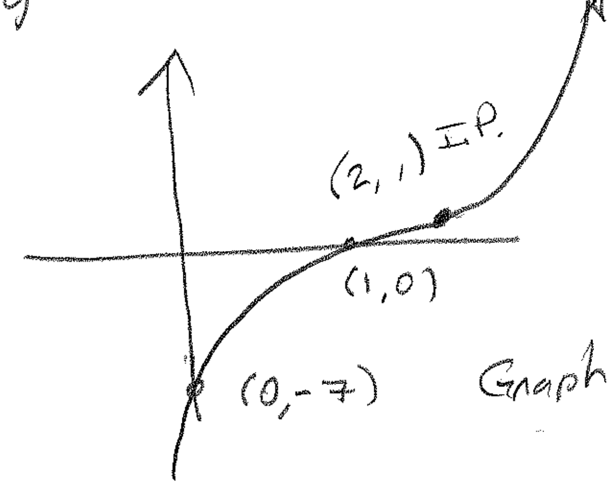
301 §4.4 #s 15, 20, 23, 25, 26, 29, 30, 34, 37, 44, 45

#s 9-48 Graph. Find all local extremes & I.P.s.

(15) $y = (x-2)^3 + 1$



(2,1) is also a terrace pt.
$y' = 0$ @ $x = 2$

Graph from Mat 121.

(20) $y = x^4 + 2x^3 = x^3(x+2) \overset{SET}{=} 0 \longrightarrow x \in \{-2, 0\}$

$y' = 4x^3 + 6x^2 = 2x^2(2x+3) \overset{SET}{=} 0 \longrightarrow$

$x \in \{-\frac{3}{2}, 0\}$

$y(-\frac{3}{2}) = -\frac{27}{16} \longrightarrow (-\frac{3}{2}, -\frac{27}{16})$ poss. min

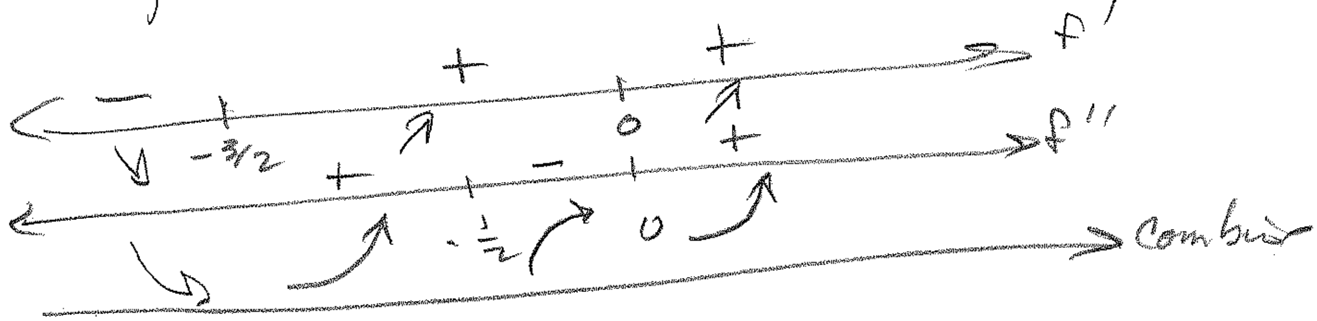
$y(0) = 0 \longrightarrow (0,0)$ poss max

$y'' = 12x^2 + 6x = 6x(2x+1) \overset{SET}{=} 0 \longrightarrow$

$x \in \{-\frac{1}{2}, 0\}$

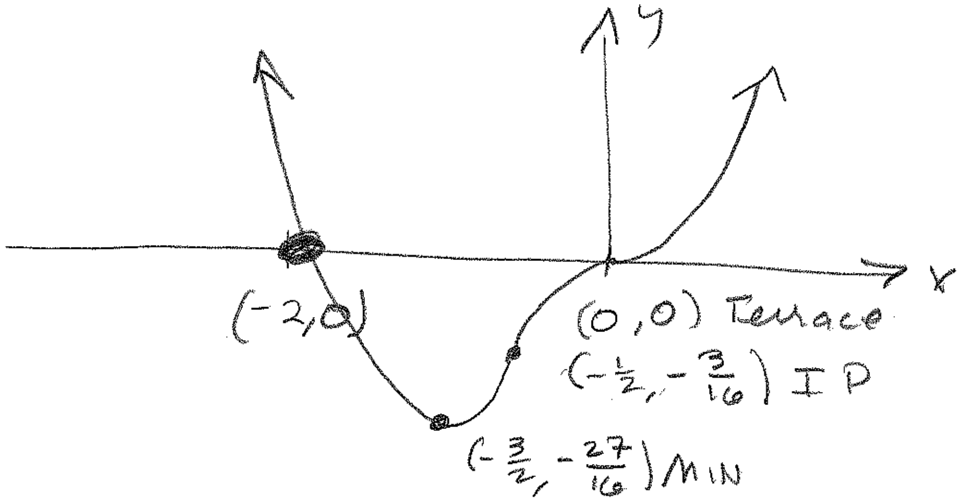
$y(-\frac{1}{2}) = -\frac{3}{16} \longrightarrow (-\frac{1}{2}, -\frac{3}{16})$

$y(0) = 0$ is possible I.P.



201 §4.4 #s 20, 23, 25, 26, 29, 30, 34, 37, 44, 45

(20)



(23) $y = x + \sin x,\ 0 \le x \le 2\pi$

x-intercepts require something like Newton's:

$y' = f'(x) = 1 + \cos x$ NOTE ALWAYS ≥ 0

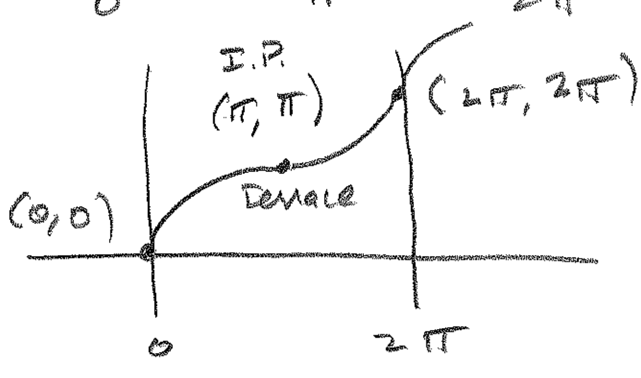
$f'(x) = 0$ when $\cos x = -1 \Longrightarrow x = \pi$

$x_0 =$ Wait! @ $x = 0$, $x = \sin x = 0$. Ahhh...

$$\overset{+}{\longleftarrow\!\!-\!\!-\!\!-}\underset{\pi}{|}\overset{+}{-\!\!-\!\!-\!\!\longrightarrow}\ y' = 1 + \cos x$$

$y'' = -\sin x$ set $= 0 \Longrightarrow x = 0, \pi, 2\pi$, so $x = 0, \pi, 2\pi$ are IPs.

$$\longleftarrow\underset{0}{|}\overset{-}{-\!\!-\!\!-}\underset{\pi}{|}\overset{+}{-\!\!-\!\!-}\underset{2\pi}{|}\overset{-}{-\!\!-\!\!\longrightarrow}\ y'' = -\sin x$$



201 §4.4 #s 25, 26, 29, 30, 34, 37, 44, 45

(25) $y = \sqrt{3}\,x - 2\cos x \qquad x \in [0, 2\pi]$

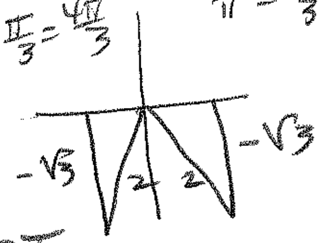
THIS one might need Newton's to find any x-int's.

$f'(x) = \sqrt{3} + 2\sin x \overset{SET}{=} 0$

$2\sin x = -\sqrt{3}$

$\sin x = -\frac{\sqrt{3}}{2}$

$\pi + \frac{\pi}{3} = \frac{4\pi}{3}$ $\qquad \pi - \frac{\pi}{3} = \frac{5\pi}{3}$



$x = \frac{4\pi}{3}, \frac{5\pi}{3}$

$f''(x) = 2\cos x \overset{SET}{=} 0$

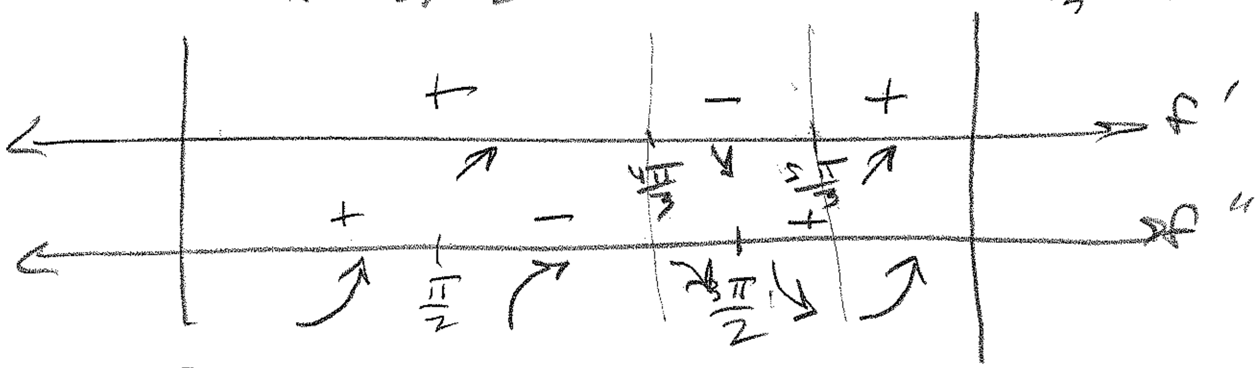
$\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\frac{3\pi}{2} = \frac{9\pi}{6}$

$\frac{5\pi}{3} = \frac{10\pi}{6}$

$\frac{4\pi}{3} = \frac{8\pi}{6}$



$f(0) = -2$

$f\left(\frac{\pi}{2}\right) = \frac{\sqrt{3}\pi}{2} \approx 2.7207 \rightsquigarrow \left(\frac{\pi}{2}, 2.7207\right)$

$f\left(\frac{4\pi}{3}\right) = \sqrt{3}\cdot\frac{4\pi}{3} - 2\cos\left(\frac{4\pi}{3}\right) = \frac{4\pi\sqrt{3}}{3} - 2\left(-\frac{1}{2}\right) = \frac{4\pi\sqrt{3}}{3} + 1$

≈ 8.2552

$f\left(\frac{3\pi}{2}\right) = \frac{3\sqrt{3}\pi}{2} \approx 8.162097$

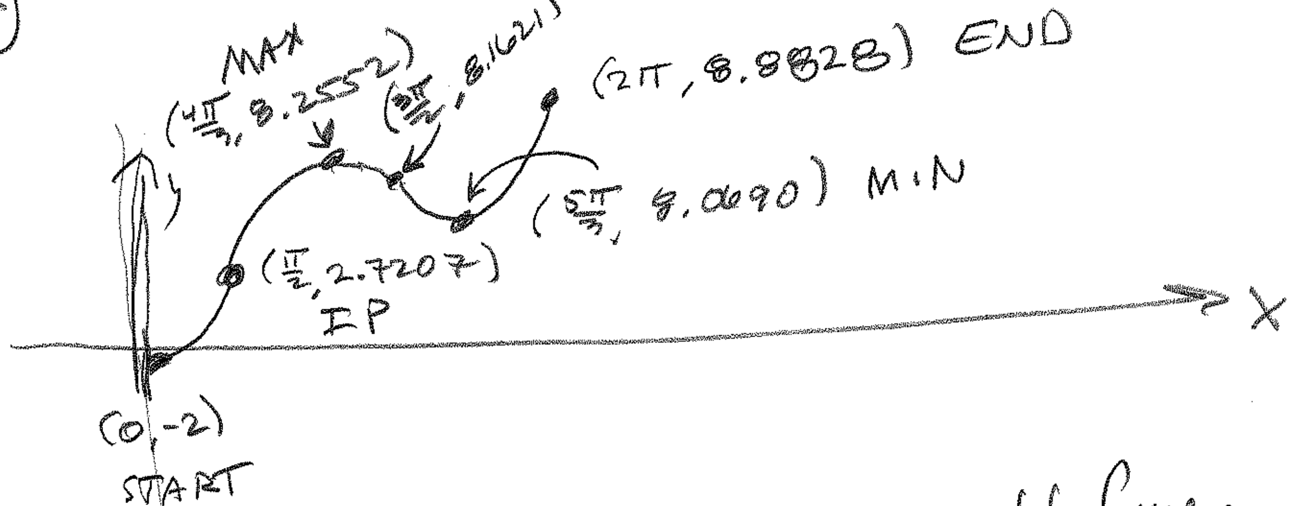
$f\left(\frac{5\pi}{3}\right) = \sqrt{3}\cdot\frac{5\pi}{3} - 2\cos\left(\frac{5\pi}{3}\right) = \frac{5\pi\sqrt{3}}{3} - 1 \approx 8.068997$

$(0, -2)$ start, $\left(\frac{\pi}{2}, 2.7207\right)$ I.P., $\left(\frac{4\pi}{3}, 8.2552\right)$ MAX, $\left(\frac{3\pi}{2}, 8.1621\right)$ IP, $\left(\frac{5\pi}{3}, 8.0690\right)$ MIN

$(2\pi, 8.8828)$ END

201 § 4.4 # 25, 26, 29, 30, 34, 37, 44, 45

(25)



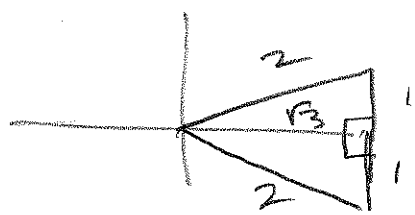
(26) $y = \frac{4}{3}x - \tan x, \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$ — Is odd func. undefined when $\cos x = 0$

clearly $f(x) = 0$ when $x = 0$.

$f'(x) = \frac{4}{3} - \sec^2 x \overset{SET}{=} 0 \Longrightarrow$ Is Even funct. undefined when $\cos x = 0$

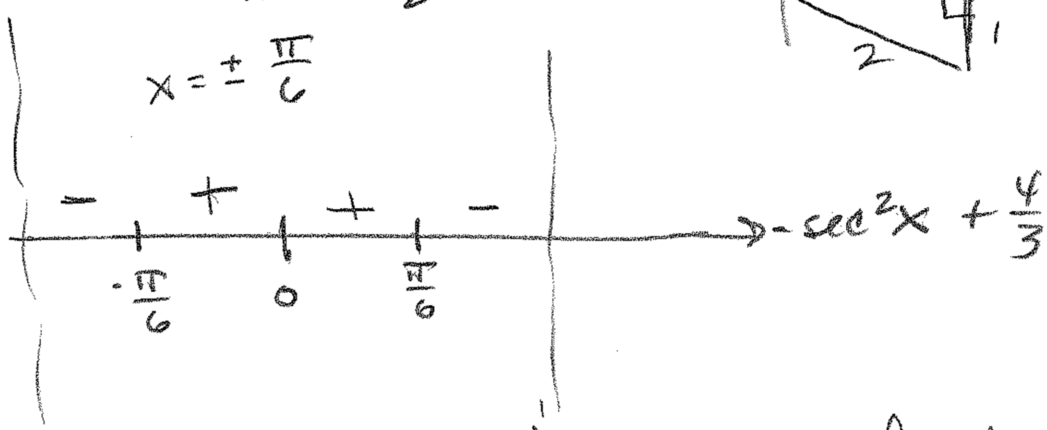
$\sec^2 x = \frac{4}{3}$

$\sec x = \pm \frac{2}{\sqrt{3}} \Longleftrightarrow$



$\cos x = \pm \frac{\sqrt{3}}{2}$

$x = \pm \frac{\pi}{6}$



$f''(x) = -2\sec x\,(\sec x \tan x)$ — undefined when $\cos x = 0$

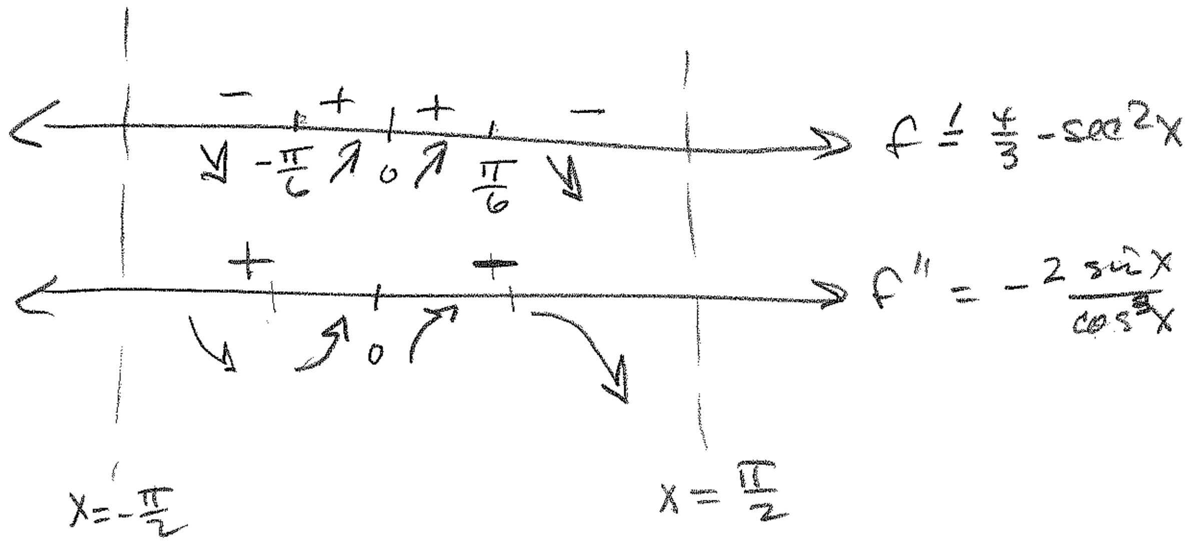
$= -2\sec^2 x \tan x$

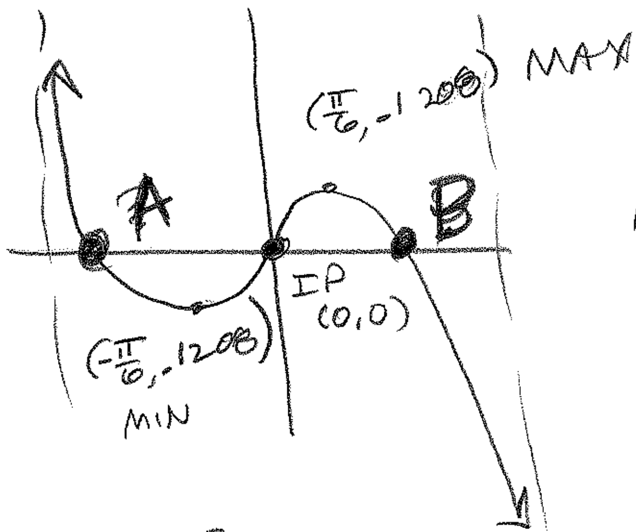
$= -\frac{2}{\cos^2 x} \cdot \frac{\sin x}{\cos x} = -\frac{2\sin x}{\cos^3 x}$ — Is ODD func.

$\overset{SET}{=} 0 \Rightarrow x = 0$ is I.P. location

201 §4.4 #s 26, 29, 30, 34, 37, 44, 45

26





x-intercepts Newton's Method or graphing tech.

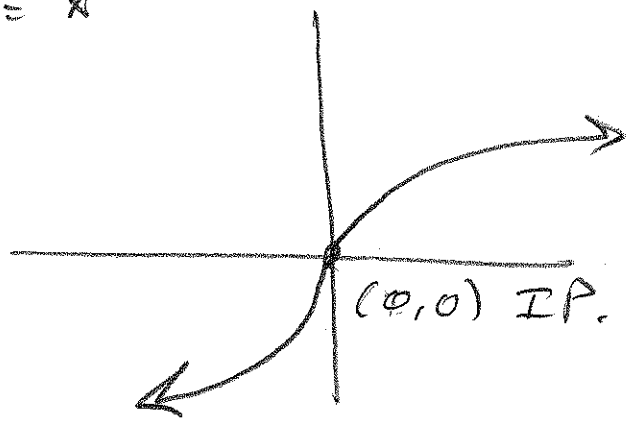
$f(-\frac{\pi}{6}) \approx -.1208$

$f(\frac{\pi}{6}) \approx +.1208$

$A \approx (-.8447308, 0)$
$B \approx (+.8447308, 0)$
$\Big\}$ Graphing Calculator

201 §4.4 #s 29, 30, 34, 37, 44, 45

29 $y = x^{\frac{1}{5}}$



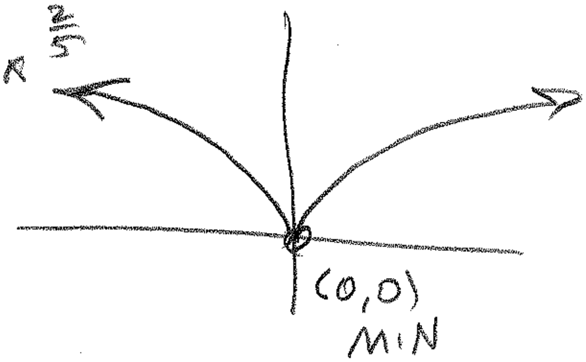
ODD,

$f' \geq 0 \ \forall x$

$f'' < 0 \ \forall x > 0$

$f'' > 0 \ \forall x < 0$

30 $y = x^{\frac{2}{5}}$



$f' > 0 \ \forall x > 0$

$f' < 0 \ \forall x < 0$

$f'' < 0 \ \forall x \neq 0$

34 $y = 5x^{\frac{2}{5}} - 2x = x^{\frac{2}{5}}(5 - 2x^{\frac{3}{5}})$

$= 0 \Rightarrow x = 0$ OR $x^{\frac{3}{5}} = \frac{5}{2} \longrightarrow x = \left(\frac{5}{2}\right)^{\frac{5}{3}} \approx 4.60504$

$f'(x) = 2x^{-\frac{3}{5}} - 2 \overset{\text{SET}}{=} 0$

$x^{-\frac{3}{5}} = 1$

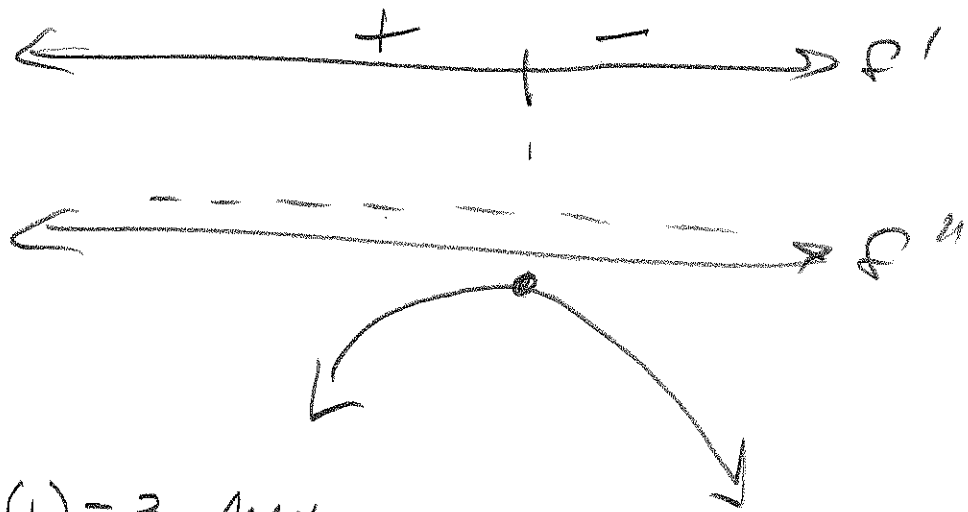
$x = 1$

$f''(x) = -\frac{6}{5} x^{-\frac{8}{5}} \neq 0$

Always negative ("8" is even)

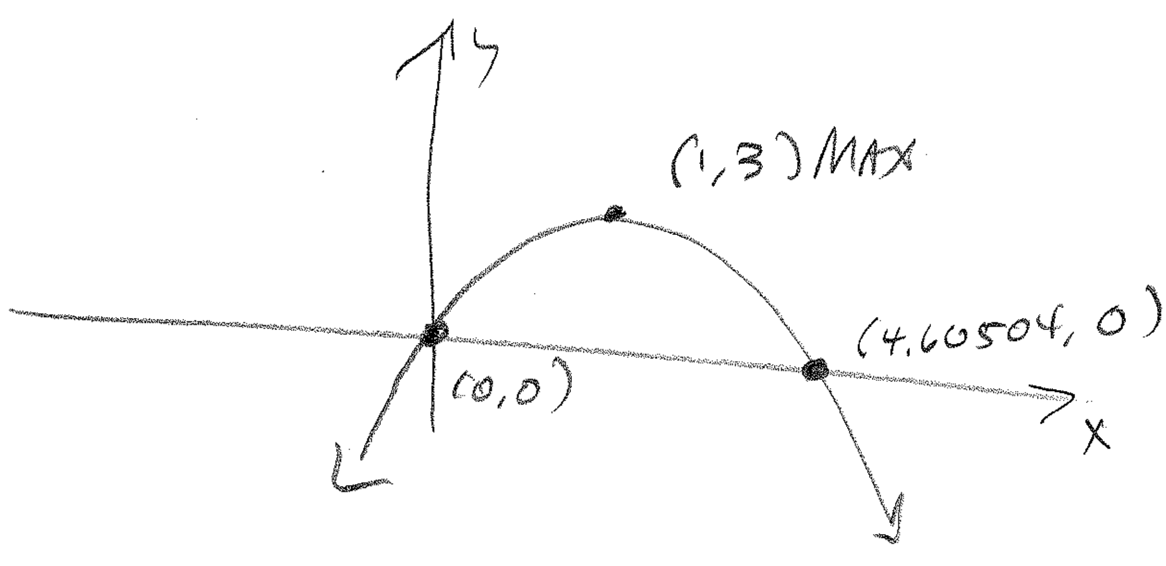
201 §4.4 #s 34, 37, 44, 45

34



$f(1) = 3$ MAX

$f(x) = 0 \Rightarrow x = 0$ OR $x \approx 4.60504$



201 §4.4 #s 37, 44, 45

(37) $f(x) = x\sqrt{8-x^2} = x(8-x^2)^{\frac{1}{2}}$

$= 0$ when $x = 0, \pm 2\sqrt{2}$

It's ODD.

$\mathcal{D} = [-2\sqrt{2}, 2\sqrt{2}]$

$$f'(x) = (8-x^2)^{\frac{1}{2}} + x\left(\tfrac{1}{2}(8-x^2)^{-\frac{1}{2}}\right)(-2x)$$

$$= (8-x^2)^{\frac{1}{2}} \cdot \frac{(8-x^2)^{\frac{1}{2}}}{(8-x^2)^{\frac{1}{2}}} - \frac{x^2}{(8-x^2)^{\frac{1}{2}}}$$

$$= \frac{8-x^2-x^2}{(8-x^2)^{\frac{1}{2}}} = \frac{8-2x^2}{(8-x^2)^{1/2}} \overset{\text{SET}}{=} 0 \Longrightarrow$$

$$2x^2 = 8 \Longrightarrow x^2 = 4 \Longrightarrow x = \pm 2$$

$$f(2) = 2\sqrt{8-4} = 2\sqrt{4} = 4 \rightsquigarrow (2,4) \longrightarrow$$

$$f(-2) = -4 \longrightarrow (-2,-4)$$

$$f''(x) = \frac{-4x(8-x^2)^{\frac{1}{2}} - (8-2x^2)\left(\tfrac{1}{2}(8-x^2)^{-\frac{1}{2}}(-2x)\right)}{\left((8-x^2)^{1/2}\right)^2}$$

$$= \frac{1}{8-x^2}\left[\frac{-4x(8-x^2)}{(8-x^2)^{1/2}} + \frac{x(8-2x^2)}{(8-x^2)^{1/2}}\right]$$

$$= \frac{1}{(8-x^2)^{3/2}}\left[-32x + 4x^3 + 8x - 2x^3\right] = \frac{1}{(8-x^2)^{3/2}}\left[2x^3 - 24x\right]$$

201 §4.4 #s 37, 44, 45

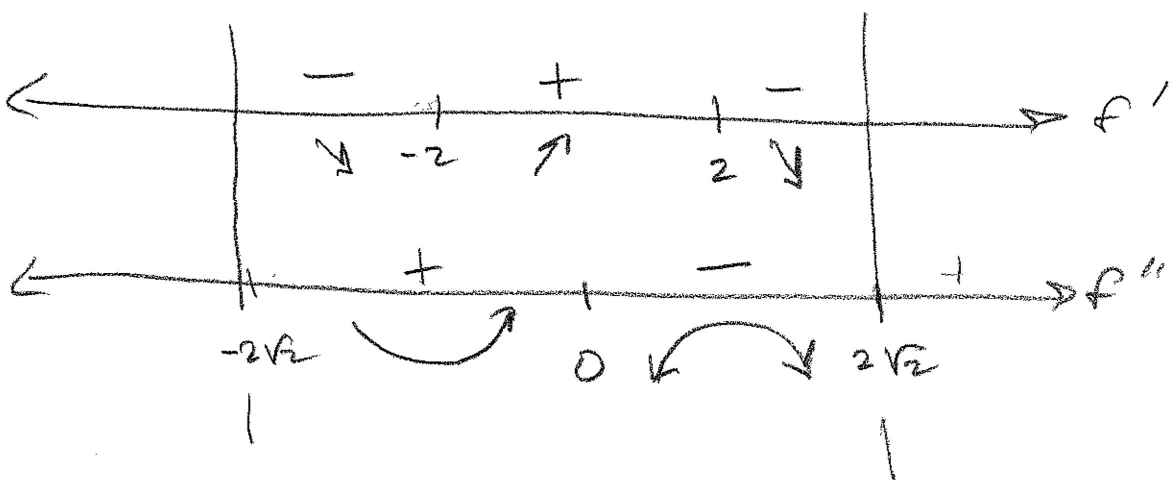
37 SET $= 0 \longrightarrow$

$2x^3 - 24x = 0 \longrightarrow$

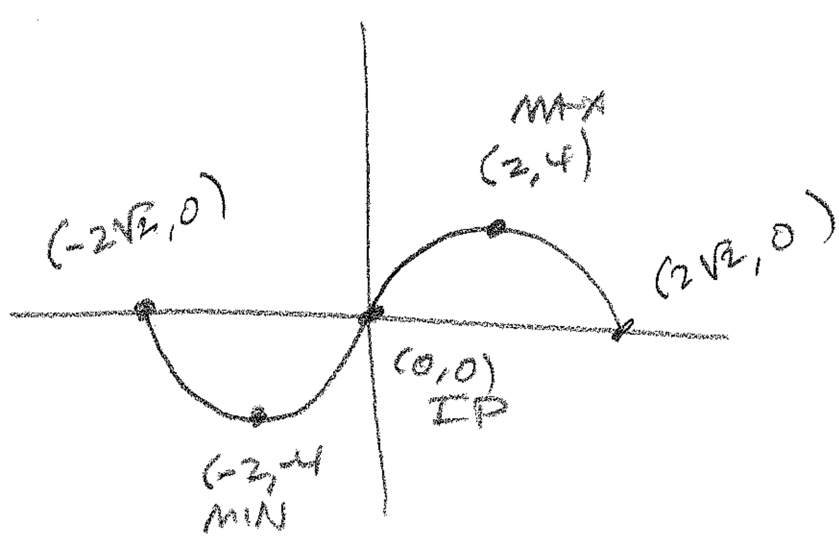
$2x(x^2 - 8) = 0 \longrightarrow$

$x \in \{0, \pm 2\sqrt{2}\}$ ~~so $f' = 0$ & $f'' = 0$ are happening in sort of~~

$f'' = 0$ @ x-intercepts (or 2 of 'em)



$f(-2) = -2\sqrt{8-4} = -2 \cdot 2 = -4$



201 §4.4 #s 44, 45

(44) $y = \dfrac{8x}{x^2+4}$ $\quad \mathcal{D} = \mathbb{R}$

It's ODD

$f(x) = 0$ @ $x = 0 \rightsquigarrow (0,0)$

$$f'(x) = \frac{8(x^2+4) - 8x(2x)}{(x^2+4)^2} = \frac{8x^2+32-16x^2}{(\ )^2}$$

$$= \frac{-8x^2+32}{(\ \ )^2} \quad \underset{\text{SET}}{=} 0 \longrightarrow 8x^2 = 32$$

$$x^2 = 4$$

$$= -\frac{8(x^2-4)}{(x^2+4)^2} \qquad x = \pm 2$$

$$f'' = -8 \; \frac{x^2-4}{(x^2+4)^2} \longrightarrow$$

$$f''(x) = -8\left[\frac{2x(x^2+4)^2 - (x^2-4)(2(x^2+4))(2x)}{(x^2+4)^4}\right]$$

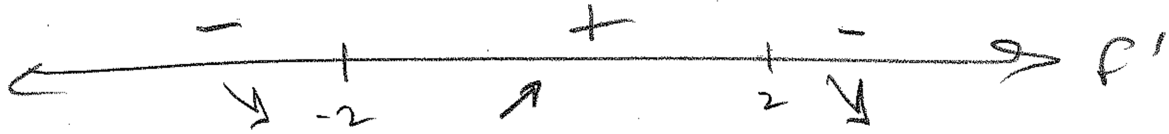
$$= -8\left[\frac{2x(x^2+4) - 4x(x^2-4)}{(x^2+4)^3}\right]$$

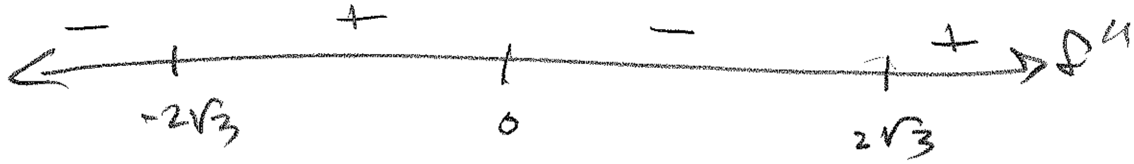
$$= -8\left[\frac{2x^3+8x-4x^3+16x}{(x^2+4)^3}\right] = -8\left[\frac{-2x^3+24x}{(\ \ )^3}\right]$$

$$\underset{\text{SET}}{=} 0 \longrightarrow -2x(x^2-12) = 0 \longrightarrow$$

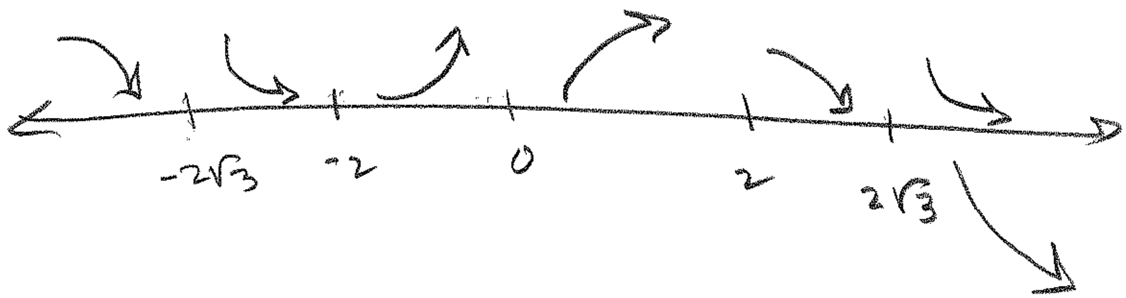
$$x = 0, \pm\sqrt{12} = \pm 2\sqrt{3}$$

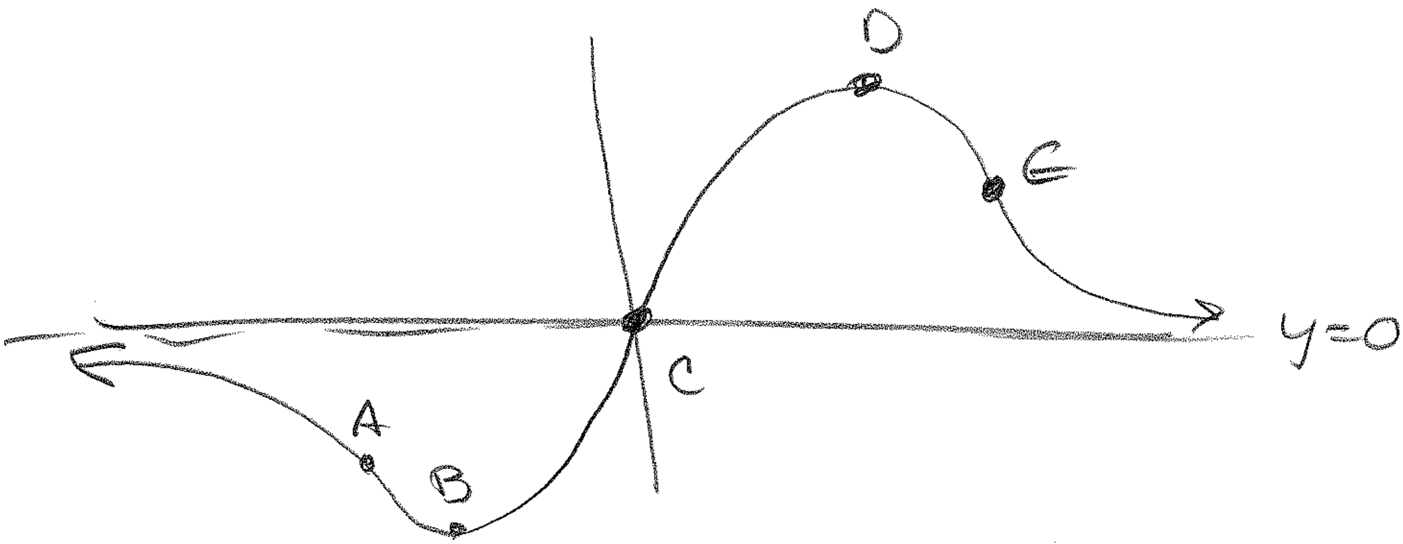
201 § 4.4 #s 44, 45





COMBINE:





$f(-2\sqrt{3}) = -\sqrt{3}$

$f(-2) = -2$

$f(0) = 0$

$f(2) = 2$

$f(2\sqrt{3}) = \sqrt{3}$

$A = (-2\sqrt{3}, -\sqrt{3})$ IP

$B = (-2, -2)$ MIN

$C = (0, 0)$ IP

$D = (2, 2)$ MAX

$E = (2\sqrt{3}, \sqrt{3})$ IP