201 §4.2 #s 1, 4, 7, 10, 11, 13, 14, 18, 19, 24, 27, 31, 34, 37, 42, 46

#s 1–6 Find c $\ni f'(c) = \frac{f(b)-f(a)}{b-a}$

① $f(x) = x^2 + 2x - 1$ on $[0, 1]$

$$\frac{f(1)-f(0)}{1-0} = \frac{1^2+2(1)-1-(0^2+2(0)-1)}{1} = 3$$

$$f'(x) = 2x + 2 \overset{\text{SET}}{=} 3 \implies$$

$$2x = 1$$

$$\boxed{x = \frac{1}{2}} = c$$

④ $f(x) = \sqrt{x-1}$ $[1, 3]$

$$\frac{f(3)-f(1)}{3-1} = \frac{\sqrt{3-1} - \sqrt{1-1}}{2} = \frac{\sqrt{2}}{2}$$

$$f'(x) = \frac{1}{2}(x-1)^{-\frac{1}{2}}(1) \overset{\text{SET}}{=} \frac{\sqrt{2}}{2} \implies$$

$$\frac{1}{2\sqrt{x-1}} = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{x-1}}{\sqrt{x-1}} \implies$$

$$1 = \sqrt{2}\sqrt{x-1} \implies$$

$$1 = 2(x-1) = 2x - 2 = 1 \implies$$

$$2x = 3$$

$$\boxed{x = \frac{3}{2} = c}$$

201 §4.2 #s 7, 10, 11, 13, 14, 18, 19, 24, 27, 31, 34, 37, 42, 46

#s 7–12 Which func. satisfies MVT? Which don't?

(7) $f(x) = x^{2/3}$ on $[-1, 8]$

f is cnt^s $\forall \mathbb{R}$ ✓

$f'(x) = \frac{2}{3}x^{-\frac{1}{3}}$ is defined $\forall x \neq 0$

But $0 \in (-1, 8)$, so MVT DNA.

(10) $f(x) = \begin{cases} \frac{\sin x}{x} & -\pi \le x < 0 \\ 0 & x = 0 \end{cases}$ on $[-\pi, 0]$

$\lim_{x \to 0} \frac{\sin x}{x} = 1 \neq 0 = f(0) \Longrightarrow$ not cnt^s on $[-\pi, 0]$.

(11) $f(x) = \begin{cases} x^2 - x & -2 \le x \le -1 \\ 2x^2 - 3x - 3 & -1 < x \le 0 \end{cases}$ on $[-2, 0]$

cnt^s on pieces ✓

cnt^s @ suture?

$f(x) \xrightarrow{x \to -1^-} (-1)^2 - (-1) = 2$

$f(x) \xrightarrow{x \to -1^+} 2(-1)^2 - 3(-1) - 3 = 2 + 3 - 3 = 2$ ✓

cnt^s ✓ on $[-2, 0]$

$f'(x) = \begin{cases} 2x - 1 & -2 \le x \le -1 \\ 4x - 3 & -1 < x \le 0 \end{cases}$

$\boxed{f'_-(-1) = -3 \quad f'_+(-1) = -7 \quad \text{Nope} \quad \text{MVT DNA}}$

201 §4.2 #s 13, 14, 18, 19, 24, 27, 31, 34, 37, 42, 46

(13) $f(x) = \begin{cases} x & 0 \le x < 1 \\ 0 & x = 1 \end{cases}$

is zero @ $x = 0$ & $x = 1$, but $f'(x) \neq 0$ on $(0,1)$.

The issue, here, is that f isn't cnt^s on $[0,1]$, so Rolle's Theorem DNA.

(14) For what values of a, m, & b does f(x) on $[0,2]$ satisfy the hypotheses of MVT?

$$f(x) = \begin{cases} 3 & x = 0 \\ -x^2 + 3x + a & 0 < x < 1 \\ mx + b & 1 \le x \le 2 \end{cases}$$

continuity: @ $x = 0$

$3 = -0^2 + 3(0) + a \Rightarrow \boxed{a = 3}$

Differentiability: @ $x = 1$

$(-2x+3)\Big|_{x=1} = -2(1) + 3 = \boxed{1 = m}$

continuity @ $x = 1$

$-(1)^2 + 3(1) + 3 = 1 + b$

$-1 + 6 = 1 + b$

$5 = 1 + b$

$\boxed{4 = b}$

201 § 4.2 #s 18, 19, 24, 27, 31, 34, 37, 42, 46

(18) A cubic polynomial can have at most 3 zeros, because for 3 zeros you need at least 2 turning points, which means at least 2 zeros of its derivative and a cubic polynomial has a quadratic polynomial as its derivative and quadratic polynomials have at most 2 real zeros. ■

~~#s~~ 19–26 Show $f(x)$ has exactly one zero on $[a,b]$

(19) $f(x) = x^4 + 3x + 1$ on $[-2,-1]$

$f(-2) = (-2)^4 + 3(-2) + 1 = 16 - 6 + 1 = 11$ $\longrightarrow$

$f(-1) = (-1)^4 + 3(-1) + 1 = -1$

@ least one root, by IVT.

$f'(x) = 4x^3 + 3 \overset{\text{SET}}{=} 0 \Longrightarrow x^3 = -\frac{3}{4} \longrightarrow$

$x = \sqrt[3]{-\frac{3}{4}} \notin [-2,-1]$, so At MOST one root, since no turning pts in $[-2,-1]$

(24)

201 §4.2 #s 24, 27, 31, 34, 37, 42, 46

(24) $r(\theta) = 2\theta - \cos^2\theta + \sqrt{2}$ on $(-\infty, \infty)$

Clearly $r(-100) < 0$ & $r(100) > 0$.

Now $r'(\theta) = 2 - 2\cos\theta \overset{\text{SET}}{=} 0 \Rightarrow$

$$2\cos\theta = 2$$

$$\cos\theta = 1$$

$$\Rightarrow \theta = 2n\pi,\ n \in \mathbb{Z}$$

But since $\cos\theta \le 1$, $r'(\theta)$ never changes sign at any of these critical points,

$r'(\theta) = 2 - 2\cos\theta$

$$\cdots + \quad | \quad + \quad | \quad + \quad | \quad + \quad | \quad + \quad | \quad + \cdots$$
$$\quad -4\pi \qquad -2\pi \qquad 0 \qquad 2\pi \qquad 4\pi$$

$\therefore$ $r(\theta)$ has <u>exactly</u> one real zero.

(27) $f(-1) = 3$ & $f'(x) = 0\ \forall x \Rightarrow$

$f(x) = 3\ \forall x$. B/C $f'(x) = 0 \Rightarrow f(x)$ is constant and $f(-1) = 3$ tells us $y = 3$ is that constant.

#s 31 – 36 Find all possible funcs with the given derivative.

201 Sec #s 31, 34, 37, 42, 46

31 (a) $y' = x \longrightarrow$

$y = \frac{1}{2}x^2 + c \quad \forall c \in \mathbb{R}$

(b) $y' = x^2 \longrightarrow$

$y = \frac{x^3}{3} + c \quad \forall c \in \mathbb{R}$

(c) $y' = x^3 \longrightarrow$

$y = \frac{x^4}{4} + c \quad \forall c \in \mathbb{R}$

34 (a) $y' = \frac{1}{2\sqrt{x}} = \frac{1}{2}x^{-\frac{1}{2}} \Longrightarrow$

$y = x^{\frac{1}{2}} + c \quad \ldots$

(b) $y' = \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}} \Longrightarrow$

$y = 2x^{\frac{1}{2}} + c \quad \ldots$

(c) $y' = 4x - \frac{1}{\sqrt{x}} \longrightarrow$

$y = 2x^2 - 2\sqrt{x} + c \quad \ldots$

#s 41–44 $v = \frac{ds}{dt}$ is given. Find s.

42 $v = 32t - 2, \quad s(0.5) = 4$

$\boxed{s = 16t^2 - 2t + c}$

$s(0.5) = 16(\frac{1}{2})^2 - 2(\frac{1}{2}) + c = 4 - 1 + c = 3 + c = 4$

$\Longrightarrow \boxed{c = 1}$

201 §4.2 #46

#s 45–48 $a = \frac{d^2s}{dt^2}$ is given. Find s.

#46 $a = 9.8$, $v(0) = -3$, $s(0) = 0$

$\boxed{v(t) = 9.8t + C}$ $v(t) = 9.8t - 3$

$v(0) = \boxed{C = -3}$

$\boxed{s(t) = 9.8\frac{t^2}{2} - 3t + C}$

$= 4.9t^2 - 3t + C$

$\boxed{s(t) = 4.9t^2 - 3t}$

$\boxed{s(0) = 0 \Longrightarrow C = 0}$