201 §4.1 II #s 37, 38, 43, 46, 51, 58, 59, 67 a–c, 68, 71, 72

#s 37–40 Find abs. extremes and where they are "assumed" "achieved" "obtained"

(37) $f(x) = x^{\frac{4}{3}}$, $-1 \le x \le 8$

$f'(x) = \frac{4}{3} x^{\frac{1}{3}} \overset{SET}{=} 0 \implies x = 0$

$f(-1) = (-1)^{4/3} = ((-1)^4)^{\frac{1}{3}} = 1^{1/3} = 1$ Nada

$f(0) = 0^{4/3} = 0$ MIN

$f(8) = 8^{4/3} = (8^{1/3})^4 = 2^4 = 16$ MAX

Max of $y = 16$ @ $x = 8$ ($x = 8$ is where it "assumes" $y = 16$)

Min of $y = 0$ @ $x = 0$

(38) $f(x) = x^{5/3}$ $-1 \le x \le 8$

$f'(x) = \frac{5}{3} x^{2/3} \overset{SET}{=} 0 \implies x = 0$

$f(-1) = (-1)^{5/3} = ((-1)^5)^{1/3} = (-1)^{1/3} = -1$ Min

$f(0) = 0^{5/3} = 0$ Nada

$f(8) = 8^{5/3} = (8^{1/3})^5 = 2^5 = 32$ MAX

Max of $y = 32$ @ $x = 8$

Min of $y = -1$ @ $x = -1$

201 §4.1 II #s 43, 46, 51, 58, 59, 67 a–c, 68, 71, 72

#s 41–48 Find all critical pts

(43) $f(x) = x(4-x)^3 \longrightarrow$

$f'(x) = 1(4-x)^3 + x(3)(4-x)^2(-1)$

$= (4-x)^3 - 3x(4-x)^2$

$= (4-x)^2[4-x-3x]$

$= (4-x)^2(4-4x) \overset{\text{SET}}{=} 0 \longrightarrow$

$x = 4$ OR $x = 1$

CPS : $\boxed{x \in \{1, 4\}}$

(46) $f(x) = \dfrac{x^2}{x-2} \longrightarrow$

$f'(x) = \dfrac{2x(x-2) - x^2(1)}{(x-2)^2} = \dfrac{2x^2 - 4x - x^2}{(x-2)^2}$

$= \dfrac{x^2-4x}{(x-2)^2} = f'(x)$

$f'(x) = 0 \longrightarrow x^2 - 4x = 0 \longrightarrow x \in \{0, 4\}$

$f'(x) \nexists \longrightarrow (x-2)^2 = 0 \Rightarrow x = 2$, but $x = 2 \notin D$

$\longrightarrow$ CPS : $\boxed{x \in \{0, 4\}}$

#s 49–58 Find all $\underbrace{\text{extremes}}_{\hookrightarrow y\text{-values}}$ and $\underbrace{\text{where they occur}}_{\hookrightarrow x\text{-values}}$

(51) $y = x^3 + x^2 - 8x + 5 \longrightarrow$

$y' = f'(x) = 3x^2 + 2x - 8$

$= (3x-4)(x+2) \overset{\text{SET}}{=} 0 \longrightarrow x \in \{\frac{4}{3}, -2\}$

No ABS, EXT. $\boxed{f(\tfrac{4}{3}) = -\tfrac{41}{27} = -1.\overline{5185} \text{ is local min.} \quad f(-2) = 17 \text{ is local max.}}$

201 §4.1 II ~~#s~~ 58, 59, 67 a-c, 68, 71, 72

(58) $y = f(x) = \dfrac{x+1}{x^2+2x+2}$

$$\Longrightarrow f'(x) = \frac{x^2+2x+2 - (x+1)(2x+2)}{(x^2+2x+2)^2}$$

$$= \frac{x^2+2x+2 - [2x^2+4x+2]}{(\quad)^2}$$

$$= \frac{x^2+2x+2-2x^2-4x-2}{(\quad)^2}$$

$$= \frac{-x^2-2x}{(x^2+2x+2)^2}$$

$$f'(x) = 0 \Longrightarrow -x^2-2x = 0$$
$$-x(x+2) = 0$$

CPs: $x \in \{-2, 0\}$

$f'(x) \nexists \Longrightarrow x^2+2x+2 = 0$, but that makes $f(x) \nexists$ there (for now)

so waste of time.

Now

$$\boxed{f(0) = \tfrac{1}{2} \text{ is local max}}$$

$$\boxed{f(-2) = \frac{-2+1}{(-2)^2+2(-2)+2} = \frac{-1}{4-4+2} = -\frac{1}{2} \text{ is local min}}$$

$-$ | $+$ | $-$ $\quad f'$

$\quad -2 \quad\quad 0$

Now, are they absolute? Let's look a little farther.

201 §4.1 II #s 58, 59, 67a–c, 68, 71, 72

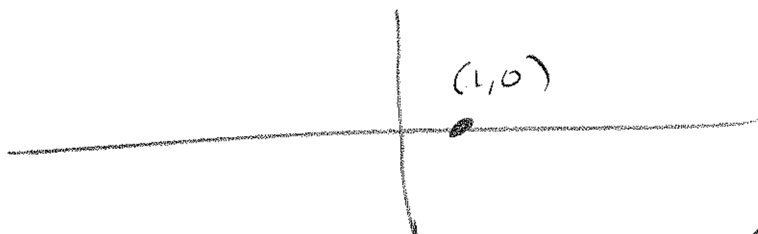
58 What's $\dfrac{x+1}{x^2+2x+2}$ look like?

$\mathcal{D}$: Set $x^2+2x+2=0$ as only possible problem.

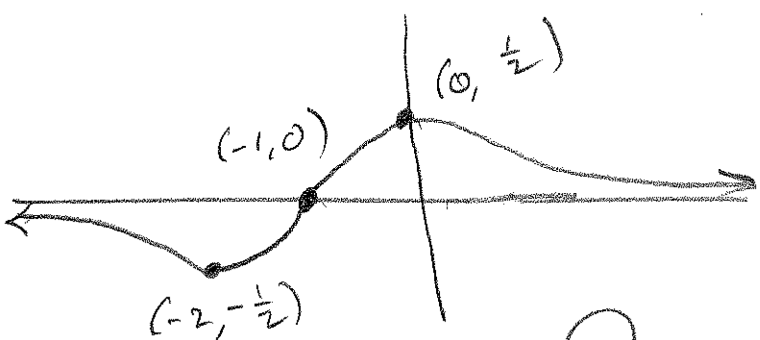
$b^2-4ac = 2^2-4(1)(2) = 4-8 < 0 \Longrightarrow$ No real zeros $\longrightarrow$ No vertical asymptotes.

$\Longrightarrow$ No sign changes contributed by vertical asymptotes.

Now $x+1=0$ @ $x=-1$ & so, we have



Horizontal asymptote: $y=0$ (It's "proper.")



So Abs. Min of $y=-\frac{1}{2}$ @ $x=-2$

Abs Max of $y=\frac{1}{2}$ @ $x=0$

#s 59–66 Find CP's, $\mathcal{D}$ Endpts, and all extrema

201 §4.1 II #s 59, 67 a–c, 68, 71, 72

(59) $y = x^{\frac{2}{3}}(x+2)$ $\qquad \mathcal{D} = \mathbb{R}$

$y' = \frac{2}{3}x^{-\frac{1}{3}}(x+2) + x^{\frac{2}{3}}(1)$

$= \dfrac{2x+4}{3x^{\frac{1}{3}}} + x^{\frac{2}{3}}$

$= \dfrac{2x+4+3x}{3x^{1/3}} = \dfrac{5x+4}{3x^{1/3}}$

f': $+$ $\quad -\frac{4}{5} \quad$ $-$ $\quad 0 \quad$ $+$

$f'(x) = 0 \longrightarrow 5x+4=0$
$\Longrightarrow x = -\frac{4}{5}$

$f'(x) \nexists \Longrightarrow x = 0$

CP's $x = -\frac{4}{5}$, $x = 0$

$f(0) = 0 \leadsto (0,0)$

$f(-\frac{4}{5}) = \left(-\frac{4}{5}\right)^{\frac{2}{3}}\left(-\frac{4}{5}+2\right) = \left(\frac{2^2}{5}\right)^{\frac{2}{3}}\left(\frac{6}{5}\right)$

$= \left(\frac{2^4}{5^2}\right)^{\frac{1}{3}}\left(\frac{6}{5}\right) = \dfrac{(2^4)^{\frac{1}{3}}}{(5^2)^{1/3}}\left(\frac{6}{5}\right) = \dfrac{(2^3 2^1)^{1/3}}{5^{2/3}} \cdot \frac{6}{5}$

$= \dfrac{2 \cdot 2^{\frac{1}{3}}}{5^{2/3}} \cdot \frac{6}{5} = \frac{12}{5}\sqrt[3]{\frac{2}{25}} = \frac{12}{5}\,\frac{\sqrt[3]{10}}{5} = \frac{12}{25}\sqrt[3]{10}$

≈ 5.17064325

$$\boxed{\begin{array}{l}\text{Local Max of } \frac{12}{25}\sqrt[3]{10} \text{ @ } x = -\frac{4}{5} \\ \text{Local Min of } 0 \text{ @ } x = 0\end{array}}$$

201 §4.1 II ~~#s~~ 67 a-c, 68, 71, 72

67 Give reasons

$f(x) = (x-2)^{2/3}$

(a) Does $f'(2)$ exist? No

$f'(x) = \frac{2}{3}(x-2)^{-\frac{1}{3}} = \frac{2}{3\sqrt[3]{x-2}}$ $\nexists$ @ $x=2$

(b) The only local extreme occurs @ $x=2$
b/c $x=2 \in \mathcal{D}(f)$ and $x=2 \notin \mathcal{D}(f') \Rightarrow$
$x=2$ is critical pt. Also $x=2$ is the ONLY critical point, since $\frac{2}{3}(x-2)^{-\frac{1}{3}} \neq 0$.

(c) Part b does Not contradict EVT
This is a function with one minimum @ $x=2$.

$- \quad + \quad | \quad - \quad + $
$-3 \quad 0 \quad 3$
$x(x-3)(x+3)$

68 $f(x) = |x^3 - 9x| = |x|\,|x^2-9|$
$= |x|\,|x-3|\,|x+3|$

$\boxed{\text{(a) Does } f'(0)\ \exists?}$ No

$$f(x) = \begin{cases} -x(x-3)(x+3) & \text{if } x < -3 & -x^3+9x \\ x(x-3)(x+3) & \text{if } -3 \le x \le 0 & x^3-9x \\ -x(x-3)(x+3) & \text{if } 0 < x < 3 & -x^3+9x \\ x(x-3)(x+3) & \text{if } 3 \le x & x^3-9x \end{cases}$$

$$f'(x) = \begin{cases} -3x^2+9 & \text{if } x < -3 \\ 3x^2-9 & \text{if } -3 < x < 0 \\ -3x^2+9 & \text{if } 0 < x < 3 \\ 3x^2-9 & \text{if } x \ge 3 \end{cases}$$

(b) $f'(3)\ \exists$? No
(c) $f'(-3)\ \exists$?
(d) Extrema of f

201 §4.1 II #s 68, 71, 72

(68) How to think of f' at these key spots:

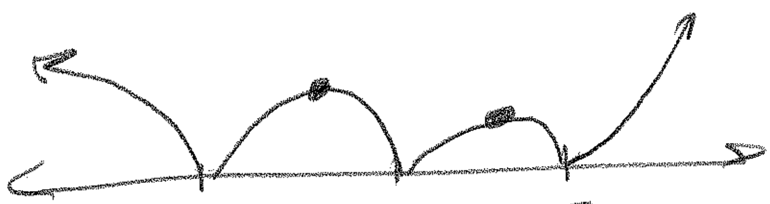
$f'_-(0) = 3(0)^2 - 9 = -9 = \lim_{h \to 0^-} \frac{f(0+h) - f(0)}{h}$

$f'_+(0) = -3(0)^2 + 9 = +9 = \lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h}$

$f'_-(3) = -3(3)^2 + 9 = -27 + 9 = -18$

$f'_+(3) = 3(3)^2 - 9 = 27 - 9 = +18$

Picture of $f(x)$:



Extrema: Look at $f'(x)$ in the subintervals

$x < -3$: $-3x^2 + 9 = 0$

Also $0 < x < 3$ has this def'n

$-3x^2 = -9$

$x^2 = 3$

$x = \pm\sqrt{3}$

$-3 < x < 0$ $\cup$ $x \geq 3$: $3x^2 - 9 = 0$

$\Rightarrow x = \pm\sqrt{3}$, again.

So, find $f(\pm\sqrt{3})$:

$f(+\sqrt{3}) = |\sqrt{3}|\,|\sqrt{3}^2 - 9| = \sqrt{3}(9-3) = 6\sqrt{3}$

$f(-\sqrt{3}) = |-\sqrt{3}|\,|(-\sqrt{3})^2 - 9| = 6\sqrt{3}$

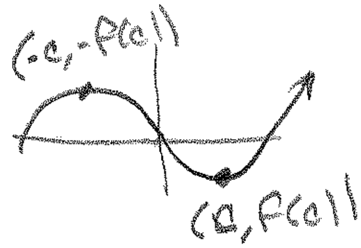
(d) Min of $y = 0$ @ $x = -3, 0, 3$

Max of $y = 6\sqrt{3}$ @ $x = \pm\sqrt{3}$

201 §4.1 II #s 71, 72

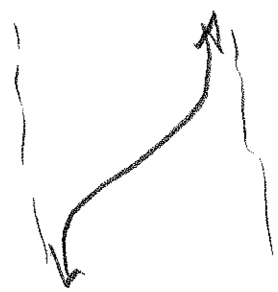
(71) odd funcs

If g is odd, w/ local min @ $x=c$,
then g has a local MAX @ $x=-c$.



$f(-c)=-f(c)$

(72)



unless f is constant, I just don't see it having extreme values

If it is constant, then every point is both a max and a min!