201 §3.9 #s 2, 8, 13, 15, 18, 22, 29, 30, 41, 44, 48, 54

#s 1-5 Find $L_a(x)$

2) $f(x) = \sqrt{x^2+9}$, $a = -4$

$f(a) = \sqrt{(-4)^2+9}$
$= \sqrt{25}$
$= 5$

$(-4, 5) = (a, f(a))$

$f'(x) = \frac{1}{2}(x^2+9)^{-1/2}(2x)$

$f'(-4) = \frac{1}{2}(25)^{-1/2}(2(-4))$

$= \frac{-8}{2(5)} = -\frac{4}{5} = m = f'(a)$

$$\boxed{y = -\frac{4}{5}(x+4) + 5}$$

$= -\frac{4}{5}x - \frac{16}{5} + \frac{25}{5} = -\frac{4}{5}x + \frac{9}{5}$

#s 7-12 Choose "suitable" linearization integer near x_0 ∋ given func. & derivative are "easy."

8) $f(x) = x^{-1}$, $x_0 = 0.9$

$a = 1$,

$f'(x) = -x^{-2}$

$f'(1) = -1 = f'(a)$

$f(1) = 1 = f(a)$

$L_1(x) = -1(x-1) + 1$
$= f'(1)(x-1) + f(1)$

13) Show that $L_0(x)$ for $(1+x)^k$ is

$L_0(x) = kx + 1$

$f'(0) = k(1+x)^{k-1}\Big|_{x=0} = k$

$f(0) = 1$

So $L_0(x) = k(x-0) + 1 = kx + 1$ ∎

201 § 3.9 #s 15, 18, 22, 29, 30, 41, 44, 48, 54

(15) use #13 to estimate the following:

(a) $(1.0002)^{50} \approx 50(.0002) + 1 = 1.01$

b) $\sqrt[3]{1.009} = (1.009)^{\frac{1}{3}} \approx \frac{1}{3}(.009) + 1 = 1.003$

#s 17-28 Find dy

(18) $y = x(1-x^2)^{\frac{1}{2}} \longrightarrow$

$$\boxed{dy = \left((1-x^2)^{\frac{1}{2}} + x\left(\tfrac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x)\right)\right)dx}$$

$$\sqrt{1-x^2} - \frac{x^2}{\sqrt{1-x^2}} = \frac{1-x^2-x^2}{\sqrt{1-x^2}} = \frac{1-2x^2}{\sqrt{1-x^2}}$$

(22) $xy^2 - 4x^{3/2} - y = 0$

$$y^2 + 2xyy' - 6x^{\frac{1}{2}} - y' = 0$$

$$y' = \frac{6x^{\frac{1}{2}} - y^2}{2xy - 1} \longrightarrow$$

$$dy = \left(\frac{6x^{\frac{1}{2}} - y^2}{2xy-1}\right)dx$$

201 § 3.9 #s 29, 30, 41, 44, 48, 54

#s 29-34 Find

(a) $\Delta f = f(x_0 + dx) - f(x_0)$

(b) $df = f'(x_0)dx$

(c) error of approximation $= |\Delta f - df|$

29 $f(x) = x^2 + 2x,\ x_0 = 1,\ dx = 0.1$

(a) $f(1.1) - f(1) = 1.21 + 2.2 - (1^2 + 2(1))$

$= 3.41 - 3 = \boxed{.41 = \Delta f}$

$df = (2x+2)dx \longrightarrow$

(b) $df\Big|_{\substack{x=1\\ dx=.1}} = (2(1)+2)(.1) = 4(.1) = \boxed{.4 = df}$

(c) $\boxed{|\Delta f - df| = 0.01}$

30 $f(x) = 2x^2 + 4x - 3,\ x_0 = -1,\ dx = 0.1$

(a) $f(-1+.1) - f(-1) = \Delta f = 2(-0.9)^2 + 4(-0.9) - 3$

$- (2(-1)^2 + 4(-1) - 3)$

$= 2(.81) - 3.6 - 3 - 2 + 4 + 3 = \boxed{0.02 = \Delta f}$ (a)

(b) $df\Big|_{\substack{x=-1\\ dx=.1}} = (4x+4)dx\Big|_{\substack{x=-1\\ dx=.1}} = (-4+4)(.1) = \boxed{0 = df}$ (b)

(c) $\boxed{|\Delta f - df| = .02}$

201 § 3.9 #s 41, 44, 48, 54

(41) radius of circle increases from 2.00 to 2.02 cm $A = \pi r^2$

(a) Change in Area, $A = \pi r^2$

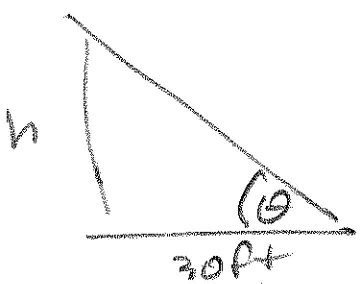
$$= \Delta A \approx dA = 2\pi r \, dr$$
$$= 2\pi (2)(.02) = \boxed{.08\pi} \approx .2513274123$$

(b) As % of original area:

$$\left(\frac{.08\pi}{\pi (2)^2}\right)(100\%) = \left(\frac{.08}{4}\right)(100\%) = (.02)(100\%) = \boxed{2\%}$$

(44) Estimate ht. of building (Done in class)

$\theta \approx 75^\circ$, 1 measurement.



Then $h = 30 \tan\theta$
$$= 30 \tan(75^\circ)$$
$$= 30 \tan\left(\frac{7\pi}{12}\right) \approx 111.9615242$$

What's max error in θ that will keep h within 4% of actual?

$h = 30 \tan\theta$

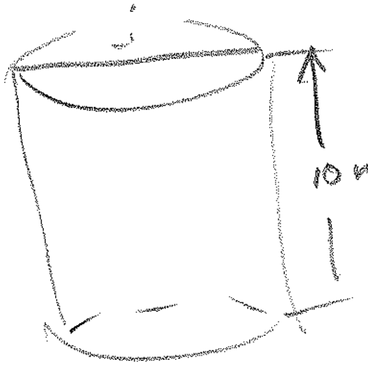
$$|dh| = |30 \sec^2\theta \, d\theta| \overset{\text{want}}{<} \left|.04\left(30 \tan\frac{7\pi}{12}\right)\right| \Longrightarrow$$

$$d\theta < \left|.04 \tan\left(\frac{7\pi}{12}\right)\cos^2\left(\frac{7\pi}{12}\right)\right| = \left|.04 \sin\frac{7\pi}{12}\cos\frac{7\pi}{12}\right|$$

$$\approx \boxed{.01 \text{ radians}} \text{ or } \approx \boxed{.57^\circ}$$

201 §3.9 #s 48, 54

(48) How accurate must interior diameter (D) of a 10-m-high cylindrical storage tank be measured to calculate its volume w/in 1% of its true value?



$V = \pi r^2 h$

$= \pi \left(\frac{1}{2}D\right)^2 h$

$= \frac{\pi}{4} D^2 h \Longrightarrow$

$V = \frac{10\pi}{4} D^2 = \frac{5\pi}{2} D^2 \Longrightarrow$

$dV = 5\pi D \, dD \approx \Delta V$ want

$|\Delta V| \approx 5\pi D \, dD \leq .01 V = .01 \left(\frac{5\pi}{2} D^2\right)$

$= \frac{\pi D^2}{40} \Longrightarrow$

$5\cancel{\pi} D \, dD \leq \frac{\cancel{\pi} D^2}{40} \Longrightarrow$

$\frac{dD}{D} \leq \frac{1}{200} = 0.005 = \boxed{0.5\%}$

That's as specific as we can be w/o knowing D.

201 §3.9 #48, 54

48 (b) How accurately must you measure D to Estimate amt of paint needed w/in 5%, where D = exterior diameter?

Again, this <u>looks</u> like a dV thing, but the BOOK is looking for surface area:

$$S = 2\pi r h = \pi D h$$

$$\frac{dS}{dD} = \pi h \Longrightarrow dS = \pi h \, dD \overset{\text{want}}{\leq} .05\, \pi D h$$

$$\Longrightarrow \boxed{\frac{dD}{D} \leq .05 = 5\%.}$$

54 (a) $Q(x) = b_0 + b_1 (x-a) + b_2 (x-a)^2$ be a quadratic approximation to $f(x)$ @ $x = a$ with three properties:

(1) $Q(a) = f(a)$

(2) $Q'(a) = f'(a)$

(3) $Q''(a) = f''(a)$

<u>Determine</u> b_0, b_1, b_2: Differentiate Q!

$$\boxed{\begin{aligned} Q(a) &= b_0 = f(a) \\ Q'(a) &= b_1 = f'(a) \\ Q''(a) &= b_2 = f''(a) \end{aligned}}$$

201 §3.9 #54b

(b) Find the quadratic approximation to $f(x) = \frac{1}{1-x}$ @ $x=0$

$f(x) = (1-x)^{-1}$

$\boxed{f(0) = 1}$

$f'(x) = -(1-x)^{-2}(-1) = \frac{1}{(1-x)^2}$

$\boxed{f'(0) = 1}$

$f''(x) = 2(1-x)^{-3}(-1) = -\frac{2}{(1-x)^3}$

$\boxed{f''(0) = -2}$

$f(x) \approx Q_0(x) = 1 + 1(x-0) - 2(x-0)^2$

$= \boxed{1 + x - 2x^2 = Q_0(x)}$

This is our first glimpse of Taylor's Series, which takes up a good chunk of Calc II. We basically view each term as being generated by the respective derivative. Make the poly agree with $f'(0)$, $f(0)$, $f''(0)$, $f'''(0)$. What do you suppose a CUBIC approximation would look like?