

20. S3. #s 1, 4, 7, 9, 10, 12, 17, 23

- ① Assume radius r & Area A are diff'l funcs of t . Write eq's relating $\frac{dA}{dt}$ & $\frac{dr}{dt}$:

$$A = \pi r^2 \Rightarrow$$

$$\boxed{\frac{dA}{dt} = 2\pi r \frac{dr}{dt}}$$

- ④ If $y = 5x$ & $\frac{dx}{dt} = 2$. Find $\frac{dy}{dt}$:

$$\frac{dy}{dt} = 5 \frac{dx}{dt} = 5(2) = \boxed{10 = \frac{dy}{dt}}$$

- ⑦ $x^2 + y^2 = 25$ & $\frac{dx}{dt} = -2$, then find

$$\frac{dy}{dt} \quad \left. \begin{array}{l} x=3, \\ y=4 \end{array} \right. \quad 2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0 \Rightarrow$$

$$2(3)(-2) + 2(4)\left(\frac{dy}{dt}\right) = 0 \Rightarrow$$

$$8y' = 12$$
$$y' = \frac{12}{8} = \boxed{\frac{3}{2} = y'}$$

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(10) $r + s^2 + v^3 = 12$, $\frac{dr}{dt} = 4$, $\frac{ds}{dt} = -3$ given.

Find $\frac{dv}{dt}$ $\left. \begin{matrix} r=3 \\ s=1 \end{matrix} \right.$

$$\frac{dr}{dt} + 2s \frac{ds}{dt} + 3v^2 \frac{dv}{dt} = 0, \text{ so } r=3, s=1 \Rightarrow$$

$$4 + 2(1)(-3) + 3v^2 \frac{dv}{dt} = 0 \Rightarrow$$

$$\frac{dv}{dt} = \frac{-2}{3v^2} \quad \text{Need } v:$$

$$3 + 1^2 + v^3 = 12 \Rightarrow$$

$$v^3 = 8 \Rightarrow$$

$$v=2$$

This gives $\frac{dv}{dt} = \frac{-2}{3(2)^2} = \boxed{\frac{-1}{6}} = \frac{dv}{dt} \left. \begin{matrix} r=3 \\ s=1 \end{matrix} \right.$

(12) $\frac{dA}{dt} = 72 \frac{\text{in}^2}{\text{s}}$ = rate of change of area
 of cube. At what rate is cube's Volume
 changing, when $x=3$ in = edge length?

$$A = 6x^2 \Rightarrow \frac{dA}{dt} = 72 = 12x \frac{dx}{dt} \Rightarrow$$

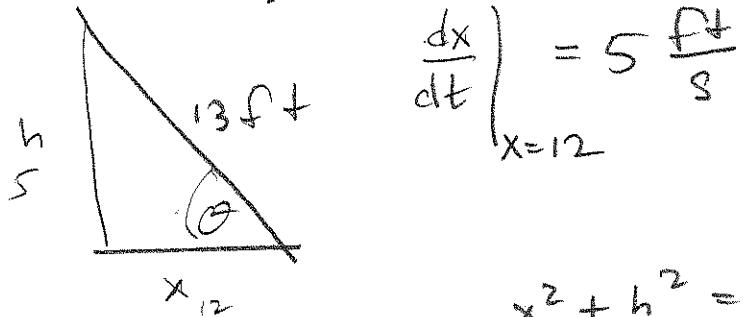
$$\frac{dx}{dt} \Big|_{x=3} = \frac{72}{12(3)} = \frac{72}{36} = 2 \frac{\text{in}}{\text{s}}. \quad V=x^3$$

$$\frac{dV}{dt} \Big|_{x=3} = 3x^2 \frac{dx}{dt} \Big|_{x=3} = 3(9)(2) = 54 \frac{\text{in}^3}{\text{s}}$$

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(17) Done in class, see notes

(23) Sliding Ladder



$$x^2 + h^2 = 13^2$$

$$(a) \left. \frac{dh}{dt} \right|_{x=12} = ?$$

$$2x \cdot x' + 2h \cdot h' = 0$$

$$2(12)(5) + 2(5)h' = 0$$

$$\Rightarrow h' = -\frac{120}{10} = -12 \frac{\text{ft}}{\text{s}} = \left. \frac{dh}{dt} \right|_{x=12}$$

$$x=12 \rightarrow h = \sqrt{69-144} = 5$$

(b) Area of triangle = A .

What's $\left. \frac{dA}{dt} \right|_{x=12}$?

$$A = \frac{1}{2} \times x \times h \Rightarrow$$

$$\left. \frac{dA}{dt} \right|_{x=12} = \frac{1}{2} \left[\frac{dx}{dt} h + x \frac{dh}{dt} \right] = \frac{1}{2} 5 + (12)(-12)$$

$$= \frac{1}{2}[25 - 144] = -\frac{1}{2}[199] = -\frac{199}{2} \frac{\text{ft}^2}{\text{s}} = \left. \frac{dA}{dt} \right|_{x=12}$$

$$-12 = 13 \left(\frac{12}{13} \right) \frac{d\theta}{dt}$$

(c) What's $\left. \frac{d\theta}{dt} \right|_{x=12}$?

$$\frac{h}{13} = \sin \theta \Rightarrow h = 13 \sin \theta \Rightarrow \frac{dh}{dt} = 13 \cos \theta \frac{d\theta}{dt} \Rightarrow \frac{d\theta}{dt} = -\frac{12 \text{ rad}}{5}$$