201 §3.7 #s 1, 4, 7, 10, 13, 16, 19, 22, 30, 33, 36, 39, 42, 50

#s 1-14 Find $\frac{dy}{dx} = y'$ "implicitly"

(1) $x^2y + xy^2 = 6$

$2xy + x^2y' + y^2 + 2xyy' = 0$

$x^2y' + 2xyy' = -2xy - y^2$

$\boxed{y' = \frac{-2xy - y^2}{x^2 + 2x}}$

(4) $x^3 - xy + y^3 = 1$

$3x^2 - y - xy' + 3y^2y' = 0$

$-xy' + 3y^2y' = y - 3x^2$

$\boxed{y' = \frac{y - 3x^2}{-x + 3y^2}}$ OR $\frac{3x^2 - y}{3y^2 - x}$

(7) $y^2 = \frac{x-1}{x+1} \Longrightarrow y^2x + y^2 = x - 1$

$2yy'x + y^2 + 2yy' = 1$

$2yy'x + 2yy' = 1 - y^2 \Longrightarrow \boxed{y' = \frac{1 - y^2}{2yx + 2y}}$

201 §3.7 #s 10, 13, 16, 19, 22, 30, 33, 36, 39, 42, 50

10 $xy = \cot(xy)$

$y + xy' = (-\csc^2(xy))(y + xy') = -y\csc^2(xy) - xy'\csc^2(xy)$

$xy' + xy'\csc^2(xy) = -y - y\csc^2(xy)$

$$\boxed{y' = \frac{-y - y\csc^2(xy)}{x + x\csc^2(xy)}} =$$

13 $y \sin\left(\frac{1}{y}\right) = 1 - xy$

$y'\sin\left(\frac{1}{y}\right) + y\cos\left(\frac{1}{y}\right)(-y^{-2}) = -y - xy'$

$y'\left(\sin\left(\frac{1}{y}\right) + x\right) = \frac{\cos\left(\frac{1}{y}\right)}{y} - y$

$$\boxed{y' = \frac{\frac{\cos\left(\frac{1}{y}\right)}{y} - y}{\sin\left(\frac{1}{y}\right) + x}}$$

#s 15-18 Find $\frac{dr}{d\theta}$:

16 $r - 2\theta^{1/2} = (3/2)(\theta^{2/3}) + (4/3)(\theta^{3/4})$

$\frac{dr}{d\theta} - \theta^{-\frac{1}{2}} = \theta^{-1/3} + \theta^{-1/4}$

$$\boxed{\frac{dr}{d\theta} = \theta^{-1/2} + \theta^{-1/3} + \theta^{-1/4}}$$

201 §3.7 #s 19, 22, 30, 33, 36, 39, 42, 50

#s 19–26 Use implicit differentiation to find $\frac{dy}{dx}$ & then $\frac{d^2y}{dx^2}$.

(19) $x^2+y^2=1$

$2x+2yy'=0$

$$\boxed{y'=-\frac{x}{y}}$$

$$y''=-\left(\frac{1y-xy'}{y^2}\right)=\frac{xy'-y}{y^2}=\frac{x\left(-\frac{x}{y}\right)-y}{y^2}$$

$$=\frac{-\frac{1}{y}-y}{y^2}=\frac{\frac{-1-y^2}{y}}{y^2}=\boxed{\frac{-1-y^2}{y^3}=y''}$$

(22) $y^2-2x=1-2y$

$2yy'-2=-2y'$

$2yy'+2y'=2$

$y'(y+1)=1$

$$\boxed{y'=\frac{1}{y+1}}=(y+1)^{-1}$$

$$y''=-(y+1)^{-2}(y')$$

$$=-(y+1)^{-2}(y+1)^{-1}$$

$$=\boxed{-(y+1)^{-3}=y''}$$

201 §3.7 #s 30, 33, 36, 39, 42, 50

#s 29-38 Verify pt is on curve & find lines tangent (a) & normal (b) to the curve there.

30 $x^2+y^2=25$ @ $(3,-4)$

$3^2+4^2=25$ ✓

(a) tan: $\boxed{y=\frac{3}{4}(x-3)-4}$

(b) norm: $\boxed{y=-\frac{4}{3}(x-3)-4}$

$2x+2yy'=0$

$y'=-\frac{x}{y}$

$y'\Big|_{(3,-4)}=-\frac{3}{-4}=\frac{3}{4}=m \Longrightarrow m_\perp=-\frac{4}{3}$

33 $6x^2+3xy+2y^2+17y-6=0$ $(-1,0)$

$6(-1)^2+3(-1)(0)+2(0)^2+17(0)-6=0$ ✓

$12x+3y+3xy'+4yy'+17=0$

$3xy'+4yy'=-12x-17$

$y'=\frac{-12x-17}{3x+4y}$

(a) $y=\frac{5}{3}(x+1)+0$

(b) $y=-\frac{3}{5}(x+1)+0$

$m_{tan}=y'\Big|_{(-1,0)}=\frac{-12(-1)-17}{3(-1)+0}=\frac{-5}{-3}=\frac{5}{3}=m_{tan}$

$\Longrightarrow m_\perp=-\frac{3}{5}$

201 §3.7 #s 39, 42, 50

(39) Find 2 pts where $x^2+xy+y^2=7$ crosses x-axis and show the tangents are parallel there. x-int: $y=0$

$x^2+0+0=7 \longrightarrow$

$x=\pm\sqrt{7} \rightsquigarrow (\sqrt{7},0), (-\sqrt{7},0)$

$2x+y+xy'+2yy'=0$

$xy'+2yy'=-2x-y$

$y'=\dfrac{-2x-y}{x+2y} \longrightarrow$

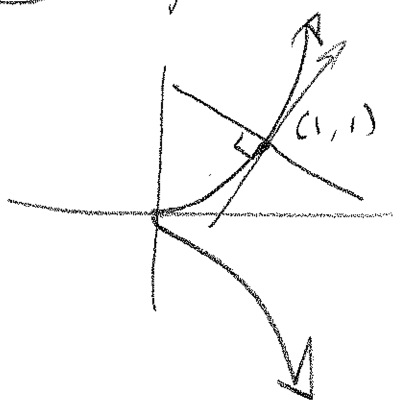
$y'\Big|_{(\sqrt{7},0)}=\dfrac{-2\sqrt{7}}{\sqrt{7}}=-2$

$y'\Big|_{(-\sqrt{7},0)}=\dfrac{-2(-\sqrt{7})}{-\sqrt{7}}=-2$

✓

201 §3.7 #42 - 50

42 $\quad y^2(2-x) = x^3$



Find eq'ns for tan & normal lines to Cissoid of Diocles @ $(1,1) = (x, y)$

$$2yy'(2-x) + y^2(-1) = 3x^2$$

$$2yy' = 3x^2 + y^2$$

$$y' = \frac{3x^2+y^2}{2y}$$

$$y'\Big|_{(1,1)} = \frac{3+1}{2} = \frac{4}{2} = 2 = m_{tan} \longrightarrow$$

$$-\frac{1}{2} = m_{\perp} \longrightarrow$$

$y = 2(x-1) + 1$ is tangent line

$y = -\frac{1}{2}(x-1) + 1$ is normal line

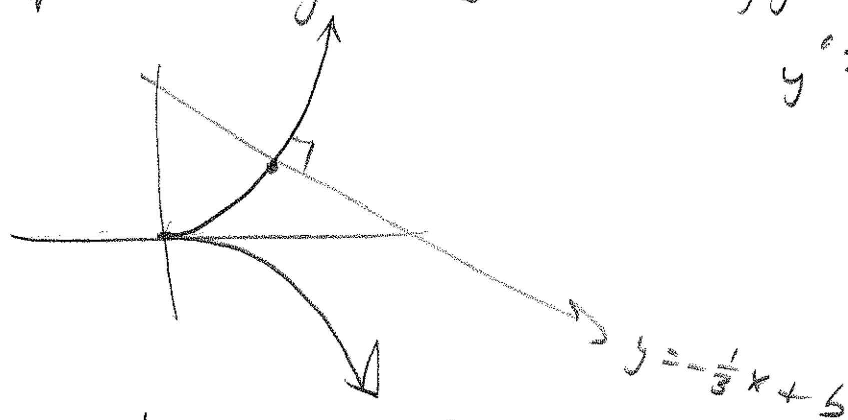
201 §3.7 #50

(50) $y^2 = x^3$ is semicubical parabola

Determine $b \ni y = -\frac{1}{3}x + b$ meets the graph orthogonally.

$2yy' = 3x^2$

$y' = \frac{3x^2}{2y} \overset{SET}{=} 3$



$m_\perp = -\frac{1}{3} \Rightarrow m_{tan} = 3$

$y = \pm x^{3/2}$ $\quad m_\perp = -\frac{1}{3} \Rightarrow$ up on top $\frac{1}{2}$, so

$y = x^{3/2} \Rightarrow y' = \frac{3}{2}x^{\frac{1}{2}} \overset{SET}{=} 3 \Rightarrow$

$x^{\frac{1}{2}} = 2 \Rightarrow x = 4$ when $y' = 3$

so $y = -\frac{1}{3}(x-4) + y(4) \Rightarrow y = -\frac{1}{3}(x-4) + 8$

$y = x^{3/2} \rightarrow y(4) = (4)^{3/2} = 2^3 = 8 = y$

$y = -\frac{1}{3}x + \frac{4}{3} + 8$

$y = -\frac{1}{3}x + \frac{28}{3}$

$\boxed{b = \frac{28}{3}}$