201 §3.6 #s 1-4, 9-12, 19-26, 37-44, 58-60, 65, 67, (73) optional

#s 1-8 $y = f(u)$ & $u = g(x)$, Find $\frac{dy}{dx} = f'(g(x))\,g'(x)$

(1) $y = 6u - 9$, $u = \frac{1}{2}x^4$ ⟹

$$\frac{dy}{dx} = (6)(2x^3) = 12x^3$$

(2) $y = 2u^3$, $u = 8x - 1$ ⟹

$$\frac{dy}{dx} = (6u^2)(8) = 6(8x-1)^2(8)$$

(3) $y = \sin u$, $u = 3x + 1$ ⟹

$$\frac{dy}{dx} = (\cos u)(3) = \cos(3x+1)(3) = 3\cos(3x+1)$$

(4) $y = \cos u$, $u = -\frac{x}{3}$ ⟹

$$\frac{dy}{dx} = (-\sin u)(-\tfrac{1}{3}) = \tfrac{1}{3}\sin(-\tfrac{x}{3})$$

#s 9-18 write as $y = f(u)$ & $u = g(x)$. Then find $\frac{dy}{dx}$ as func. of x.

(9) $y = (2x+1)^5$ ⟹ $y = u^5$ & $u = 2x+1$ ⟹

$$\frac{dy}{dx} = (5u^4)(2) = 5(2x+1)^4(2)$$

201 § 3.6 #s 10-12, 19-26, 37-44, 58-60, 65, 67, 73

(10) $y = (4-3x)^9 \Rightarrow y = u^9,\ u = 4-3x \Rightarrow$

$$\frac{dy}{dx} = (9u^8)(-3) = 9(4-3x)^8(-3)$$

(11) $y = \left(1-\frac{x}{7}\right)^{-7} \Rightarrow y = u^{-7},\ u = 1-\frac{x}{7} \Rightarrow$

$$\frac{dy}{dx} = -7\left(1-\frac{x}{7}\right)^{-8}\left(-\frac{1}{7}\right)$$

(12) $y = \left(\frac{x}{2}-1\right)^{-10} \Rightarrow y = u^{-10},\ u = \frac{x}{2}-1 \Rightarrow$

$$\frac{dy}{dx} = (-10u^{-11})\left(\frac{1}{2}\right) = -10\left(\frac{x}{2}-1\right)^{-11}\left(\frac{1}{2}\right)$$

#s 19-40 Find the derivatives

(19) $p = \sqrt{3-t} = (3-t)^{\frac{1}{2}} \Rightarrow$

$$\frac{dp}{dt} = \frac{1}{2}(3-t)^{-\frac{1}{2}}(-1)$$

(20) $q = \sqrt[3]{2r-r^2} = (2r-r^2)^{\frac{1}{3}} \rightarrow$

$$\frac{dq}{dr} = \frac{1}{3}(2r-r^2)^{-2/3}(2-2r)$$

201 § 3.6 #s 21-26, 37-44, 58-60, 65, 67, 73

(21) $s = \frac{4}{3\pi}\sin(3t) + \frac{4}{5\pi}\cos(5t) \Rightarrow$

$$\frac{ds}{dt} = \frac{4}{\pi}\cos(3t) - \frac{1}{\pi}\sin(5t)$$

(22) $s = \sin\left(\frac{3\pi t}{2}\right) + \cos\left(\frac{3\pi t}{2}\right) \Rightarrow$

$$\frac{ds}{dt} = \frac{3\pi}{2}\cos\left(\frac{3\pi t}{2}\right) - \frac{3\pi}{2}\sin\left(\frac{3\pi t}{2}\right)$$

(23) $r = (\csc\theta + \cot\theta)^{-1} \Rightarrow$

$$\frac{dr}{d\theta} = -(\csc\theta + \cot\theta)^{-2}(-\csc\theta\cot\theta - \csc^2\theta)$$

(24) $r = 6(\sec\theta - \tan\theta)^{3/2} \Rightarrow$

$$\frac{dr}{d\theta} = 9(\sec\theta - \tan\theta)^{\frac{1}{2}}(\sec\theta\tan\theta - \sec^2\theta)$$

(25) $y = x^2\sin^4 x + x\cos^{-2}x = x^2(\sin x)^4 + x(\cos x)^{-2} \Rightarrow$

$$\frac{dy}{dx} = 2x\sin^4 x + x^2(4\sin^3 x)(\cos x) + (\cos x)^{-2} + x(-2\cos^{-3}x) \cdot (-\sin x)$$

(26) $y = x^{-1}\sin^{-5}x - \frac{1}{3}x\cos^3 x \Rightarrow$

$$\frac{dy}{dx} = -x^{-2}\sin^{-5}x + x^{-1}(-5\sin^{-6}x)\cos x - \frac{1}{3}\cos^3 x$$
$$-\frac{1}{3}x(3\cos^2 x)(-\sin x)$$

201 §3.6 #s 37-44, 58-60, 65, 67, 73

(37) $r = \sin(\theta^2)\cos(2\theta)$ ⟶

$$\frac{dr}{d\theta} = (\cos(\theta^2))(2\theta)\cos(2\theta) + \sin(\theta^2)(-2\sin(2\theta))$$

(38) $r = \sec(\theta^{\frac{1}{2}})\tan(\theta^{-1})$ ⟶

$$\frac{dr}{d\theta} = (\sec(\theta^{\frac{1}{2}})\tan(\theta^{\frac{1}{2}}))(\tfrac{1}{2}\theta^{-\frac{1}{2}})\tan(\theta^{-1}) + \sec(\theta^{\frac{1}{2}})(\sec^2(\theta^{-1}))(-1\theta^{-2})$$

(39) $q = \sin\left(\frac{t}{\sqrt{t+1}}\right) = \sin(t(t+1)^{-\frac{1}{2}})$ ⟶

$$\frac{dq}{dt} = (\cos(t(t+1)^{-\frac{1}{2}}))((t+1)^{-\frac{1}{2}} + t(-\tfrac{1}{2}(t+1)^{-\frac{3}{2}}(1)))$$

(40) $q = \cot\left(\frac{\sin t}{t}\right)$ ⟶

$$\frac{dq}{dt} = (-\csc^2\left(\frac{\sin t}{t}\right))\left(\frac{(\cos t)(t) - (\sin t)(1)}{t^2}\right)$$

#s 41-58 Find $\frac{dy}{dt}$

(41) $y = \sin^2(\pi t - 2)$ ⟶

$$\frac{dy}{dt} = (2\sin(\pi t - 2))(\cos(\pi t - 2))(\pi)$$

201 § 3.6 #s 42-44, 58-60, 65, 67, 73

(42) $y = \sec^2(\pi t) \longrightarrow$

$$\frac{dy}{dt} = (2\sec(\pi t))(\sec(\pi t)\tan(\pi t))(\pi)$$

(43) $y = (1+\cos(2t))^{-4} \longrightarrow$

$$\frac{dy}{dt} = -4(1+\cos(2t))^{-5}(-\sin(2t))(2)$$

(44) $y = \left(1+\cot\left(\frac{t}{2}\right)\right)^{-2} \longrightarrow$

$$\frac{dy}{dt} = -2\left(1+\cot\left(\frac{t}{2}\right)\right)^{-3}\left(-\csc^2\left(\frac{t}{2}\right)\right)\left(\frac{1}{2}\right)$$

(58) $y = \sqrt{3t+\sqrt{2+\sqrt{1-t}}}$

$$= \left(3t+\left(2+(1-t)^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}} \longrightarrow$$

$$\frac{dy}{dt} = \frac{1}{2}\left(3t+\left(2+(1-t)^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{-\frac{1}{2}} \cdot$$

$$\cdot\left(3+\frac{1}{2}\left(2+(1-t)^{\frac{1}{2}}\right)^{-\frac{1}{2}}\left(\frac{1}{2}(1-t)^{-\frac{1}{2}}\right)(-1)\right)$$

work your way in from the outside.

#s 59-64 find y''

(59) $y = (1+x^{-1})^3 \longrightarrow y' = 3(1+x^{-1})^2(-x^{-2}) \longrightarrow$

$$y'' = 6(1+x^{-1})'(-x^{-2})(-x^{-2}) + 3(1+x^{-1})^2(2x^{-3})$$

201 §3.6 #s 60, 65, 67, 73

(60) $y = (1 - x^{\frac{1}{2}})^{-1} \longrightarrow$

$y' = -(1-x^{\frac{1}{2}})^{-2}(-\frac{1}{2}x^{-\frac{1}{2}}) \longrightarrow$

$y'' = 2(1-x^{\frac{1}{2}})^{-3}(-\frac{1}{2}x^{-\frac{1}{2}})(-\frac{1}{2}x^{-\frac{1}{2}})$

$-(1-x^{\frac{1}{2}})^{-2}(\frac{1}{4}x^{-3/2})$

#s 65-70 find $(f \circ g)'$ @ the given x-value.

(65) $f(u) = (u^5 + 1)$, $g(x) = u = \sqrt{x} = x^{\frac{1}{2}}$ @ $x = 1$

$(f \circ g)' = f'(g)\, g'(x)$

$= (5g^4)\frac{1}{2}x^{-\frac{1}{2}} \longrightarrow$ @ $x = 1$, we have

$5(g(1))^4 \cdot \frac{1}{2}(1)^{-\frac{1}{2}} = 5(1)\cdot\frac{1}{2} = \boxed{\frac{5}{2}}$

(67) $f(u) = \cot(\frac{\pi u}{10})$, $g(x) = u = 5x^{\frac{1}{2}}$ @ $x = 1$

$u(1) = 5(1)^{\frac{1}{2}} = 5$

$\frac{df}{du}\cdot\frac{du}{dx} = (-\frac{\pi}{10}\csc^2 u)(\frac{5}{2}x^{-\frac{1}{2}})$

$\longrightarrow$ @ $x = 1$, we have $(-\frac{\pi}{10}\csc^2(5))(\frac{5}{2}(1)^{-\frac{1}{2}})$

$= \boxed{-\frac{\pi}{4}\csc^2(5)}$

201 § 3.6 # 73

x	f(x)	g(x)	f'(x)	g'(x)
2	8	2	$\frac{1}{3}$	-3
3	3	-4	2π	5

(a) $2f(x)$ @ $x=2$ is $2(8) = 16$

(b) $f(x)+g(x)$ @ $x=3$ is $3-4=-1$

(c) $f(x)g(x)$ @ $x=3$ is $(3)(-4)=-12$

(d) $\frac{f(x)}{g(x)}$ @ $x=2$ is $\frac{8}{2}=4$

(e) $f(g(x))$ @ $x=2$ is $f(2)=8$

(f) $\sqrt{f(x)}$ @ $x=2$ is $\sqrt{8}=2\sqrt{2}$

(g) $\frac{1}{g^2(x)}$ @ $x=3$ is $\frac{1}{(-4)^2}=\frac{1}{16}$

(h) $\sqrt{f^2(x)+g^2(x)}$ @ $x=2$ is

$\sqrt{8^2+2^2}=\sqrt{16+4}=\sqrt{20}=2\sqrt{5}$