201 §3.5 #s 2, 4, 7, 10, 17, 24, 30, 33, 35, 36, 39, 40, 44, 47, 48, 57–61, 65, 66

#s 1–18 Find $\frac{dy}{dx}$

(2) $y = 3x^{-1} + 5\sin x \Longrightarrow \boxed{y' = -3x^{-2} + 5\cos x}$

(4) $y = x^{\frac{1}{2}}\sec x + 3 \longrightarrow \boxed{y' = \frac{1}{2}x^{-\frac{1}{2}}\sec x + x^{\frac{1}{2}}\sec x \tan x}$

(7) $f(x) = \sin x \tan x \longrightarrow \boxed{f'(x) = \cos x \tan x + \sin x \sec^2 x}$

(10) $y = (\sin x + \cos x)\sec x \Longrightarrow$

$f'(x) = y' = (\cos x - \sin x)\sec x + (\sin x + \cos x)\sec x \tan x$

(17) $f(x) = x^3 \sin x \cos x \Longrightarrow$

$\boxed{f'(x) = 3x^2 \sin x \cos x + x^3(\cos^2 x - \sin^2 x)}$

(24) $y = \theta \sin\theta + \cos\theta \Longrightarrow$

$\boxed{\frac{dy}{d\theta} = \sin\theta + \theta\cos\theta - \sin\theta}$

#s 27–34 Find $\frac{dp}{dq}$

(30) $p = \frac{\tan q}{1 + \tan q} \Longrightarrow$

$\boxed{\frac{dp}{dq} = \frac{(\sec^2 q)(1 + \tan q) - \tan q\,(\sec^2 q)}{(1 + \tan q)^2} = \frac{1}{(1 + \tan q)^2}}$

201 §3.5 #s 33, 35, 36, 39, 40, 44, 47, 48, 57–61, 65, 66

(33) Find y'' if

(a) $y = \csc x$

$y' = -\csc x \cot x$

$$\boxed{y'' = (\csc x \cot x)\cot x + (-\csc x)(-\csc^2 x)}$$

$= \csc x \cot^2 x + \csc^3 x$

$= \csc x(\csc^2 x - 1) + \csc^3 x$

$= 2\csc^3 x - \csc x$

(b) $y = \sec x$

$y' = \sec x \tan x$

$$\boxed{y'' = \sec x \tan x \tan x + \sec x(\sec^2 x)}$$

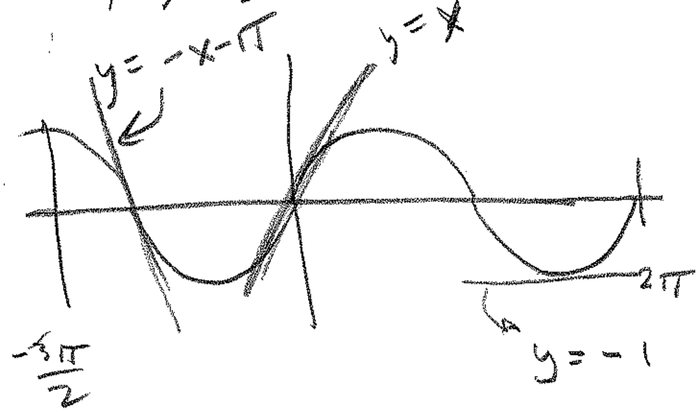
$= \sec x(\sec^2 x - 1) + \sec^3 x$

$= 2\sec^3 x - \sec x$

#s 35–38 Graph over given interval, with tangent lines @ given x-values

(35) $y = \sin x$, $-\frac{3\pi}{2} \le x \le 2\pi$

$x = -\pi, 0, \frac{3\pi}{2}$



@ $x = -\pi$:

$y' = \cos x$

$y'(\pi) = \cos \pi = -1$

$(-\pi, 0) = (x_1, y_1)$

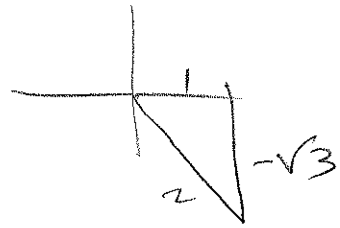
$y = -(x + \pi) + 0$

$= -x - \pi$

201 §3.5 #s 36, 39, 40, 44, 47, 48, 57-61, 65, 66

(36) $y = \tan x \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$

$x = -\frac{\pi}{3}, 0, \frac{\pi}{3}$



$y' = \sec^2 x$

$y'(-\frac{\pi}{3}) = \sec^2(-\frac{\pi}{3}) = \frac{1}{\cos^2(-\frac{\pi}{3})} = \frac{1}{(\frac{1}{2})^2} = 4 = m$

$y(-\frac{\pi}{3}) = \tan(-\frac{\pi}{3}) = -\sqrt{3}$

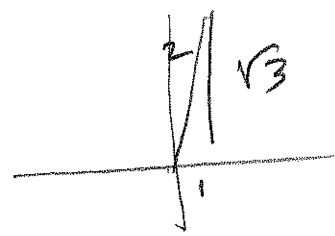
$\rightsquigarrow (-\frac{\pi}{3}, -\sqrt{3})$

$y = 4(x + \frac{\pi}{3}) - \sqrt{3}$ @ $(-\frac{\pi}{3}, -\sqrt{3})$

$y'(0) = \sec^2(0) = 1 = m$

$y(0) = \tan(0) = 0 \rightsquigarrow (0,0), m = 1$

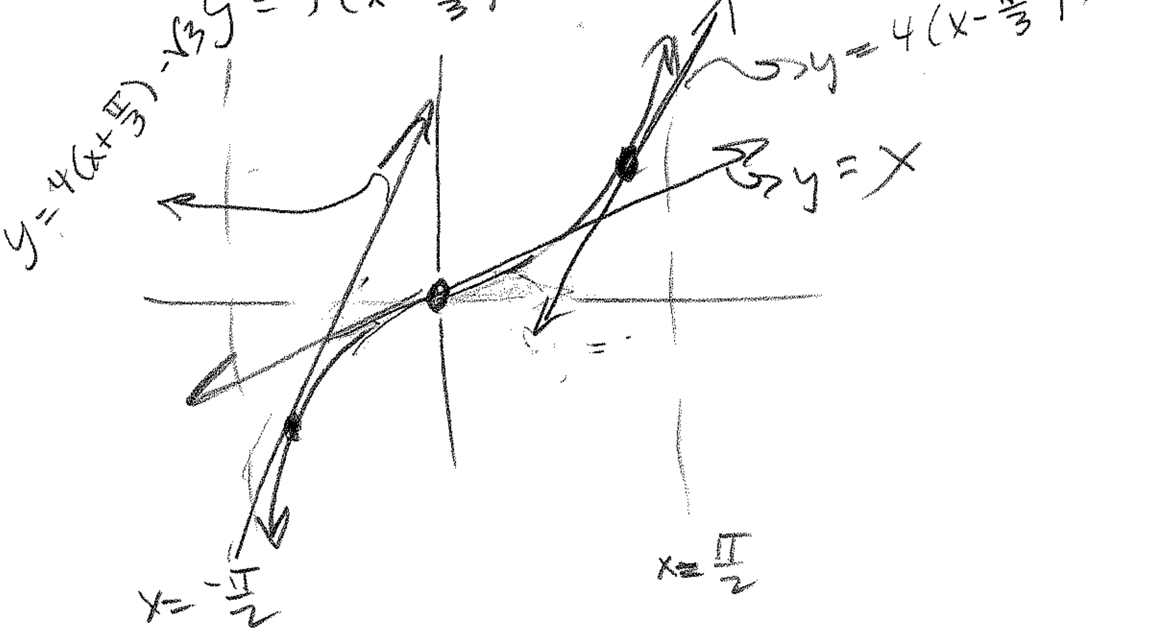
$y = x$



$y'(\frac{\pi}{3}) = \sec^2(\frac{\pi}{3}) = \frac{1}{\cos^2(\frac{\pi}{3})} = \frac{1}{(\frac{1}{2})^2} = 4 = m$

$y(\frac{\pi}{3}) = \tan(\frac{\pi}{3}) = \sqrt{3} \rightsquigarrow (\frac{\pi}{3}, \sqrt{3})$

$y = 4(x - \frac{\pi}{3}) + \sqrt{3}$



201 §3.5 #s 39, 40, 44, 47, 48, 57–61, 65, 66

#s 39–42

Any horizontal tangents in $[0, 2\pi]$? If so where? If not, why not?

39 $y = x + \sin x$

$y' = 1 + \cos x \overset{SET}{=} 0 \Longrightarrow$

$\cos x = -1 \Longrightarrow$

$\boxed{x = \pi}$

40 $y = 2x + \sin x \Longrightarrow$

$y' = 2 + \cos x \overset{SET}{=} 0 \Longrightarrow$

No sol'n. $|\cos x| \le 1$, so $\cos x = -2$ is impossible. the $y = 2x$ climbs faster than $y = \sin x$ oscillates

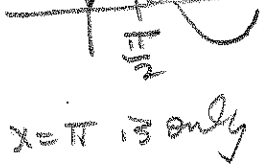
44 Find all pts on $y = \cot x$, $0 < x < \pi$, $\ni$ tangent line is $\parallel$ to $y = -x$. sketch y & tan line.

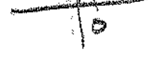
$y' = -\csc^2 x \overset{SET}{=} -1 \Longrightarrow \frac{1}{\sin^2 x} = 1 \Longrightarrow \sin^2 x = 1$

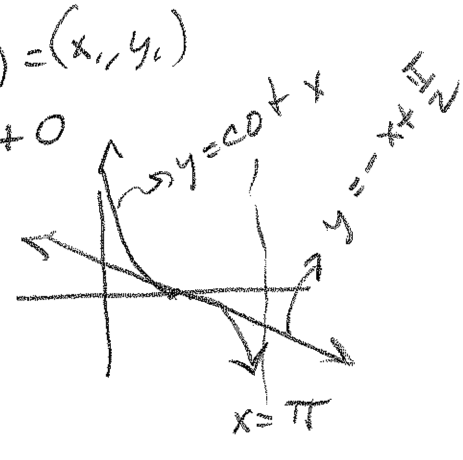
$\Longrightarrow \sin x = \pm 1$

$\cot \frac{\pi}{2} = 0 \leadsto (\frac{\pi}{2}, 0) = (x_1, y_1)$

$y = -1(x - \frac{\pi}{2}) + 0$







$x = \pi$ is only soln

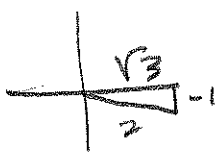
201 § 3.5 #s 47, 48, 57–61, 65, 66

~~#s 47–54~~ Find the limits

(47) $\lim_{x\to 2} \sin\left(\frac{1}{x}-\frac{1}{2}\right) = \sin(0) = \boxed{0}$

(48) $\lim_{x\to -\frac{\pi}{6}} \sqrt{1+\cos(\pi \csc x)}$

$= \sqrt{1+\cos\left(\pi\csc\left(-\frac{\pi}{6}\right)\right)} = \sqrt{1+\cos(-2\pi)} = \sqrt{1+1} = \boxed{\sqrt{2}}$



(57) Is there a value c ∋ f is cnts @ $x = 0$?

$$f(x) = \begin{cases} \dfrac{\sin^2(3x)}{x^2} & x \neq 0 \\ c & x = 0 \end{cases}$$

$$\frac{\sin^2(3x)}{x^2} = \left(\frac{\sin(3x)}{x}\right)^2 = \left(\frac{3\sin(3x)}{3x}\right)^2 \xrightarrow{x\to 0} 3^2 = \boxed{9 = c}$$

We set $f(0) = \lim_{x\to 0} f(x)$ & solved.

201 §3.5 58-61, 65, 66

(58) Does $\exists\ b \ni g$ is cnt^s @ $x=0$?
Difbl @ $x=0$?

$$g(x)=\begin{cases} x+b & x<0 \\ \cos x & x \geq 0 \end{cases}$$

$$\lim_{x\to 0^-} g(x) = b \overset{\text{SET}}{=} \cos(0) = \lim_{x\to 0^+} g(x) = 1$$

So, $\boxed{b=1 \text{ makes it cnt}^s.}$

Now for difbl, we need

$$\lim_{h\to 0^-} \frac{g(0+h)-g(0)}{h} = \lim_{h\to 0^+} \frac{g(0+h)-g(0)}{h} \Longrightarrow$$

$$= -\sin(0) \not\longrightarrow$$

$$1$$

$\boxed{\text{Impossible to make it difbl @ } x=0}$

(59) $\frac{d^{999}}{dx^{999}}(\cos x)$ we need to find 999 mod 4, since $\frac{d^n}{dx^n}\cos x$ repeats after 4 derivatives

$$\frac{999}{4} = 249 + \frac{3}{4} \Longrightarrow 999 = 3 \text{ mod } 4 \text{ so}$$

find 3rd derivative

$y^{(0)} = \cos x$

$y^{(1)} = -\sin x$

$y^{(2)} = -\cos x$

$$\boxed{y^{(3)} = \sin x = \frac{d^{999}}{dx^{999}}(\cos x)}$$

201 §3.5 #s 60, 61, 65, 66

(60) Derive the formulas

a. $\frac{d}{dx}[\sec x] = \frac{d}{dx}\left[\frac{1}{\cos x}\right] = \frac{0 \cdot \cos x - 1 \cdot (-\sin x)}{\cos^2 x}$

$= \frac{\sin x}{\cos^2 x} = \boxed{\sec x \tan x}$

(b) $\frac{d}{dx}[\csc x] = \frac{d}{dx}\left[\frac{1}{\sin x}\right] = \frac{0 \cdot \sin x - 1 \cdot \cos x}{\sin^2 x}$

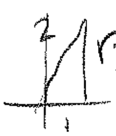
$= \frac{-\cos x}{\sin^2 x} = \boxed{-\csc x \cot x}$

(c) $\frac{d}{dx}[\cot x] = \frac{d}{dx}\left[\frac{\cos x}{\sin x}\right] = \frac{-\sin^2 x - (\cos^2 x)}{\sin^2 x}$

$= \frac{-(1)}{\sin^2 x} = \boxed{-\csc^2 x}$

(61) $x = 10\cos t$ describes weight on a spring.

(a) Find $x(0)$, $x(\frac{\pi}{3})$, $x(\frac{3\pi}{4})$



$x(0) = 10$ cm

$x(\frac{\pi}{3}) = 10\cos\frac{\pi}{3} = 5$ cm

$x(\frac{3\pi}{4}) = 10\cos\frac{3\pi}{4} = -\frac{10}{\sqrt{2}}$ cm

(b) Same for $x' = -10\sin t$

$x'(0) = \boxed{0}$ cm/s

$x'(\frac{\pi}{3}) = -10\sin\frac{\pi}{3} = \boxed{-5\sqrt{3}}$ cm/s

$x'(\frac{3\pi}{4}) = 10\sin\frac{3\pi}{4} = \boxed{\frac{10}{\sqrt{2}}}$ cm/s

201 §3.5 #s 65, 66

(65) Centered difference quotient.

$$\frac{f(x+h) - f(x-h)}{2h}$$

(See #64)

(a) $\frac{\sin(x+h) - \sin(x-h)}{2h}$ compared to $y = \cos x = y'$ for $\sin x$

$h = 1, .5, .3$ on $[-\pi, 2\pi]$

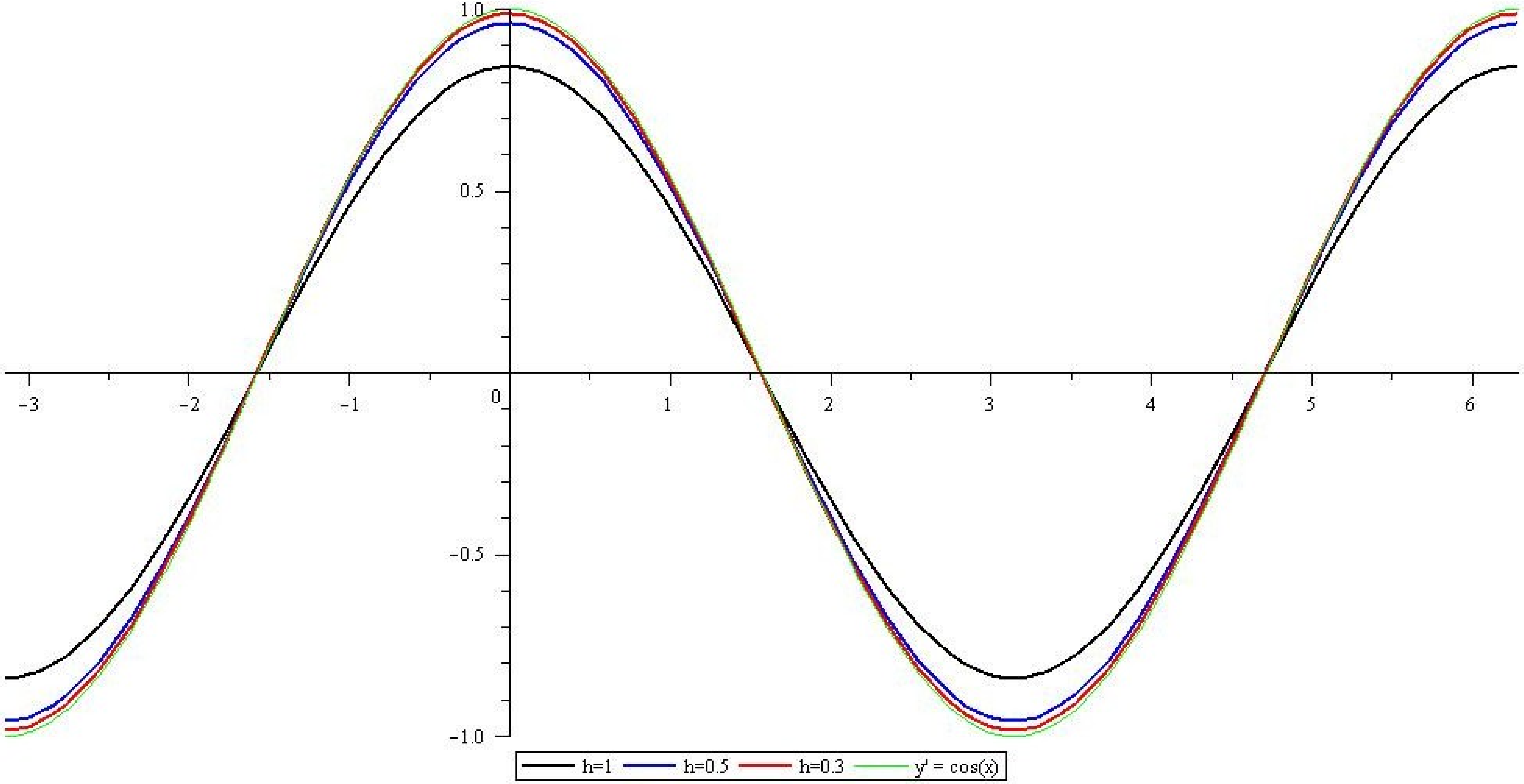
The points:

(1) You need access to a decent grapher.

(2) $h = .5$ & $.3$ are very close to $y' = \cos x$

(3) start early so you can anticipate these graders @ the end and ask about them!

See next page...


1.0
0.5
-0.5
-1.0
-3
-2
-1
0
1
2
3
4
5
6
h=1
h=0.5
h=0.3
y' = cos(x)

201 §3.5 #66

66 Caution! Centered difference CAN lead you astray!

Consider $f(x) = |x|$ @ $x = 0$:

$$\lim_{h \to 0} \frac{|0+h| - |0-h|}{2h} = \lim_{h \to 0} \frac{h-h}{2h} = \lim_{h \to 0} \frac{0}{2h} = 0$$

This limit exists, even though $f(x) = |x|$ is <u>not</u> difb^le @ $x = 0$.

<u>MORAL</u>: Centered difference is great if you want a quick, digital estimate for $f'(x)$, but if $f'(x)$ $\nexists$, your estimate is worthless!