201 § 3.3 #s 1, 4, 7, 10, 14, 17, 20, 30, 33, 42abc
43, 44, 45, 51, 56, 58

#s 1–12 Find y' & y''

1. $y = -x^2 + 3$
$y' = -2x$ &
$y'' = -2$

4. $w = 3z^7 - 7z^3 + 21z^2$
$w' = 21z^6 - 21z^2 + 42z$
$w'' = 126z^5 - 42z + 42$

7. $w = 3z^{-2} - \frac{1}{z} = 3z^{-2} - z^{-1}$
$w' = -6z^{-3} + z^{-2}$
$w'' = 18z^{-4} - 2z^{-3}$

10. $y = 4 - 2x - x^{-3}$
$y' = -2 + 3x^{-4}$
$y'' = -12x^{-5}$

#s 13–16 Find y' (a) by product rule
(b) multiply & differentiate

14. $\boxed{y = (2x+3)(5x^2 - 4x)}$
(a) $\boxed{y' = (2)(5x^2 - 4x) + (2x+3)(10x - 4)} = 10x^3 + 7x^2 - 12x \rightarrow \boxed{y' = 30x^2 + 14x - 12}$
(b) $y = 10x^3 - 8x^2 + 15x^2 - 12x$

201 §3.3 #s17, 20, 30, 33, 42abc, 43-45, 51, 56, 58

#s17-28 Find the derivative.

17 $y = \dfrac{2x+5}{3x-2} \longrightarrow$

$$\boxed{y' = \frac{2(3x-2)-(2x+5)(3)}{(3x-2)^2}} = \frac{6x-4-6x-15}{(3x-2)^2}$$

$$= \frac{-19}{(3x-2)^2}$$

20 $f(t) = \dfrac{t^2-1}{t^2+t-2} \longrightarrow$

$$\boxed{f'(t) = \frac{2t(t^2+t-2)-(t^2-1)(2t+1)}{(t^2+t-2)^2}}$$

$$= \frac{2t^3+2t^2-4t-[2t^3+t^2-2t-1]}{(t^2+t-2)^2}$$

$$= \frac{2t^3+2t^2-4t-2t^3-t^2+2t+1}{(\quad)^2} = \frac{t^2-2t+1}{(t^2+t-2)^2}$$

$$= \frac{(t-1)^2}{((t-1)(t+2))^2} = \frac{1}{(t-1)(t+2)^2}$$

201 §3.3 #s 30, 33, 42abc, 43-45, 51, 56, 58

#s 29-32 Find derivatives of all orders

30 $y = \frac{x^5}{120}$, $y' = \frac{\cancel{5}x^4}{\cancel{120}_{24}} = \frac{x^4}{24} \Rightarrow$

$y'' = \frac{4x^3}{24} = \frac{x^3}{6} \Rightarrow y''' = \frac{3x^2}{6} = \frac{x^2}{2}$

$\Rightarrow y^{(4)} = \frac{2x}{2} = x \Rightarrow y^{(5)} = 1 \Rightarrow$

$y^{(6)} = y^{(7)} = \ldots = 0.$

#s 33-40 Find 1st & 2nd derivs

33 $y = \frac{x^3+7}{x} = x^2 + 7x^{-1} \Rightarrow$

$y' = 2x - 7x^{-2} \Rightarrow$

$y'' = 2 + 14x^{-3}$

42 If u & v are diff'l funcs of x &

$u(1) = 2$, $u'(1) = 0$, $v(1) = 5$, $v'(1) = -1$

(a) - (d) Find the following @ $x = 1$:

(a) $\frac{d}{dx}[uv]\Big|_{x=1} = [u'v + uv']_{x=1} = (0)(5) + (2)(-1) = -2$

(b) $\frac{d}{dx}\left[\frac{u}{v}\right]\Big|_{x=1} = \left[\frac{u'v - uv'}{v^2}\right]_{x=1} = \frac{0(5) - (2)(-1)}{5^2} = \frac{2}{25}$

201 §3.3 #s 43-45, 51, 56, 58

43 (a) Line $\perp$ to $y = x^3 - 4x + 1$ @ $(2, 1)$:

$y' = 3x^2 - 4 \implies m_{tan} = 3(2)^2 - 4 = 8 \implies$

$\boxed{y = -\frac{1}{8}(x-2) + 1}$ is $\perp$ to curve @ $(2, 1)$

(b) Smallest Slope : minimize y' :

$y' = 3x^2 - 4$ is minimized $\boxed{@\ x = 0}$

@ $x = 0$, slope is $y'\big|_{x=0} = \boxed{-4 = m_{min}}$

(c) Find eq'ns for tangents to curve, where slope is $m_{tan} = 8$.

$3x^2 - 4 \overset{SET}{=} 8 \implies 3x^2 = 12$

$\implies x^2 = 4$

$\implies x = \pm 2$

$y(2) = 2^3 - 4(2) + 1$
$= 8 - 8 + 1 = 1$
$y(-2) = (-2)^3 - 4(-2) + 1$
$= -8 + 8 + 1 = 1$

So, $\boxed{\begin{aligned} y &= 8(x-2) + 1 \\ y &= 8(x+2) + 1 \end{aligned}}$ & are the two tan lines

201 §3.3 ~~#5~~ 44, 45, 51, 56, 58

44a) Find eq'ns for horizontal tangents to $y = x^3 - 3x - 2$. Also find eq'ns for lines $\perp$ @ those pts

$y' = 3x^2 - 3 \overset{\text{SET}}{=} 0 \Rightarrow$

$x^2 = 1 \Rightarrow$

$x = \pm 1$

$y(1) = 1^3 - 3(1) - 2$

$= -4 \rightarrow (1, -4)$

$y(-1) = (-1)^3 - 3(-1) - 2$

$= -1 + 3 - 2$

$= 0 \rightarrow (-1, 0)$

Tan lines:	$\perp$ lines:
$y = 0$	$x = -1$
$y = -4$	$x = 1$

44b) Smallest slope?

Minimize $y' = 3x^2 - 3$: Happens @ $x = 0$

$x = 0$ corresponds to $y = -2$ on the curve.

$\rightarrow (0, -2) = (x_1, y_1)$ Now Slope @ $x = 0$ is

$y'(0) = -3 = m$. Now, slope of $\perp$ line is

$m_\perp = \frac{1}{3}$, so eq'n of $\perp$ line is

$$\boxed{y = \frac{1}{3}(x - 0) - 2}$$

201 § 3.3 # s 45, 51, 56, 58

(45) Find tangents to Newton's Serpentine

(a) (0,0) and (1,2)

$$f(x) = \frac{4x}{x^2+1} \implies$$

$$f'(x) = \frac{4(x^2+1) - (4x)(2x)}{(x^2+1)^2} \implies$$

$$f'(0) = \frac{4(1) - (0)(0)}{1^2} = 4 = m_{tan} \text{ @ } (0,0)$$

$$f'(1) = \frac{4(2) - (4)(2)}{(1+1)^2} = \frac{8-8}{4} = 0 = m_{tan} \text{ @ } (1,2)$$

(51) Find all pts (x,y) on $y = \frac{x}{x-2}$ where tangent is $\perp$ to $y = 2x+3$.

$$y' \overset{SET}{=} m_\perp = -\frac{1}{2} \implies$$

$$\frac{(1)(x-2) - x(1)}{(x-2)^2} = \frac{x-2-x}{(x-2)^2} = -\frac{2}{(x-2)^2} \overset{SET}{=} -\frac{1}{2}$$

$$\implies 4 = (x-2)^2 \implies (x-2)^2 = 4$$

$$x-2 = \pm 2$$

$$x = 2 \pm 2 \quad x=4, \quad x=0$$

$$f(4) = \frac{4}{4-2} = \frac{4}{2} = 2 \rightsquigarrow (4,2)$$

$$f(0) = \frac{0}{0-2} = 0 \rightsquigarrow (0,0)$$

201 §3.3 #s 56, 58

56 Eval by recognizing it as a derivative at a particular x-value.

$$\lim_{x\to -1} \frac{x^{2/9}-1}{x+1} = f'(-1) \text{ for } f(x) = x^{\frac{2}{9}}, \text{ so}$$

$$f'(x) = \frac{2}{9}x^{-\frac{7}{9}} \Longrightarrow f'(-1) = \frac{2}{9}(-1)^{-\frac{7}{9}} = \boxed{-\frac{2}{9}}$$

58 Find a & b ∋ f(x) is dif bl ∀ x.

$$f(x) = \begin{cases} ax+b & x>-1 \\ bx^2-3 & x\le -1 \end{cases} = \begin{cases} 3x-\frac{3}{2} \\ -\frac{3}{2}x^2-3 \end{cases}$$

Need f to be cnt's, so

$$a(-1)+b = b(-1)^2-3 \Longrightarrow$$

$$-a+b = b-3 \Longrightarrow$$

$$-a = -3 \Longrightarrow$$

$$\boxed{a=3}$$

$$f'(x) = \begin{cases} 3 & x>-1 \\ -3x & x\le -1 \end{cases}$$

Need f to be dif bl, so

$$a = 2b(-1) \Longrightarrow$$

$$3 = -2b \Longrightarrow$$

$$\boxed{b = -\frac{3}{2}}$$

Check

$$\lim_{x\to -1^+} f(x) = 3(-1) - \frac{3}{2} = -\frac{9}{2} \checkmark$$

$$\lim_{x\to -1^-} f(x) = -\frac{3}{2}(-1)^2 - 3 = -\frac{9}{2} \checkmark$$

$$\lim_{h\to 0^+} \frac{f(-1+h)-f(-1)}{h} = 3 \checkmark \qquad \lim_{h\to 0^-} \frac{f(-1+h)-f(-1)}{h} = -3(-1) = 3 \checkmark$$