201 § 3.2 #s 1, 4, 8, 13, 14, 17, 20, 23, 24, 27–30, 31, 32, 37

#s 1–6 Find $f'(x)$ by the def'n. Then find $f'(a)$ for specified values a.

① $f(x) = 4 - x^2$, $f'(-3), f'(0), f'(1)$

$$\frac{f(x+h) - f(x)}{h} = \frac{4 - (x+h)^2 - (4 - x^2)}{h} =$$

$$= \frac{4 - x^2 - 2xh - h^2 - 4 + x^2}{h} = \frac{-2xh - h^2}{h} = \frac{-2x - h}{1} \xrightarrow{h \to 0} \boxed{-2x = f'(x)}$$

$$\boxed{\begin{aligned} f'(-3) &= -2(-3) = 6 \\ f'(0) &= -2(0) = 0 \\ f'(1) &= -2(1) = 2 \end{aligned}}$$

④ $k(z) = \dfrac{1-z}{2z}$ $\quad k'(-1), k'(1), k'(\sqrt{2})$

$$\frac{k(z+h) - k(z)}{h} = \frac{1}{h}\left[\frac{1 - (z+h)}{2(z+h)} - \frac{1-z}{2z}\right]$$

$$= \frac{2z(1 - z - h) - (1-z)(2)(z+h)}{h(2(z+h))(2z)} = \frac{2z - 2z^2 - 2zh - (2 - 2z)(z+h)}{h(4(z(z+h)))}$$

$$= \frac{2z - 2z^2 - 2zh - (2z + 2h - 2z^2)}{4hz(z+h)}$$

$$= \frac{2z - 2z^2 - 2zh - 2z - 2h + 2z^2}{4hz(z+h)} = \frac{-2zh - 2h}{4hz(z+h)}$$

$$= \frac{-2z - 2}{4z(z+h)} \xrightarrow{h \to 0} \boxed{\frac{-2z - 2}{4z^2} = k'(z)}$$

$$k'(-1) = \frac{2 - 2}{4} = 0, \quad k'(1) = \frac{-4}{4} = -1, \quad k'(\sqrt{2}) = \frac{-2\sqrt{2} - 2}{4(2)} = \frac{-\sqrt{2} - 1}{4}$$

201 §3.2 #s 8, 13, 14, 17, 20, 23, 24, 27–30

#s 7–12 Find the derivative

(8) $\frac{dr}{ds}$ if $r = s^3 - 2s^2 + 3$

$$\frac{(s+h)^3 - 2(s+h)^2 + 3 - (s^3 - 2s^2 + 3)}{h}$$

$$= \frac{s^3 + 3s^2h + 3sh^2 + h^3 - 2(s^2 + 2sh + h^2) + 3 - s^3 + 2s^2 - 3}{h}$$

$$= \frac{s^3 + 3s^2h + 3sh^2 + h^3 - 2s^2 - 4sh - 2h^2 + 3 - s^3 + 2s^2 - 3}{h}$$

$$= \frac{3s^2h + 3sh^2 - 4sh - 2h^2}{h} = 3s^2 + 3sh - 4s - 2h^2$$

$$\xrightarrow{h \to 0} \boxed{3s^2 - 4s = \frac{dr}{ds}}$$

#s 13–16 Differentiate the func. Then find slope of tangent line @ the given value of the independent variable.

(13) $f(x) = x + \frac{9}{x}$ @ $x = -3$

$$\frac{f(x+h) - f(x)}{h} = \frac{1}{h}\left[x + h + \frac{9}{x+h} - \left(x + \frac{9}{x}\right)\right] =$$

$$\frac{1}{h}\left[x + h - x + \frac{9}{x+h} - \frac{9}{x}\right] = \frac{1}{h}[h] + \frac{1}{h}\left[\frac{9x - 9(x+h)}{x(x+h)}\right]$$

$$= 1 + \frac{1}{h}\left[\frac{9x - 9x - 9h}{x(x+h)}\right] = 1 + \frac{-9}{x(x+h)} \xrightarrow{h \to 0} \boxed{1 - \frac{9}{x^2} = f'(x)}$$

$$f'(-3) = 1 - \frac{9}{(-3)^2} = \boxed{0 = f'(-3)}$$

801 §3.2 #s 14, 17, 20, 23, 24, 27–30

14 $k(x) = \frac{1}{2+x}$, $x = 2$

$$\frac{k(x+h)-k(x)}{h} = \frac{1}{h}\left[\frac{1}{2+(x+h)} - \frac{1}{2+x}\right] = \frac{1}{h}\left[\frac{x+2-(x+h+2)}{(x+2)(x+h+2)}\right]$$

$$= \frac{1}{h}\left[\frac{x+2-x-h-2}{(x+2)(x+h+2)}\right] = \frac{-h}{h\left[(x+2)(x+h+2)\right]} = \frac{-1}{(x+2)(x+h+2)} \xrightarrow{h\to 0} \frac{-1}{(x+2)^2}$$

$= k'(x)$

$$\Rightarrow k'(2) = \frac{-1}{(2+2)^2} = \boxed{-\frac{1}{16} = k'(2)}$$

#s 17, 18 Differentiate Then find eq'n of tangent line @ the indicated point on the graph

17 $f(x) = \frac{8}{\sqrt{x-2}}$, $(x,y) = (6,4)$

$$\frac{f(x+h)-f(x)}{h} = \frac{1}{h}\left[\frac{8}{\sqrt{x+h-2}} - \frac{8}{\sqrt{x-2}}\right]$$

conjugate trick

$$= \frac{1}{h}\left[\frac{8\sqrt{x-2} - 8\sqrt{x+h-2}}{\sqrt{x-2}\sqrt{x+h-2}}\right] = \frac{8}{h}\left[\frac{x-2-(x+h-2)}{\sqrt{x-2}\sqrt{x+h-2}\left(\sqrt{x-2}+\sqrt{x+h-2}\right)}\right]$$

$$= \frac{8}{h}\left[\frac{-h}{\sqrt{x-2}\sqrt{x+h-2}\left(\sqrt{x-2}+\sqrt{x+h-2}\right)}\right]$$

$$= \frac{8}{\sqrt{x-2}\sqrt{x+h-2}\left(\sqrt{x-2}+\sqrt{x+h-2}\right)} \xrightarrow{h\to 0} \frac{8}{(x-2)\left(2\sqrt{x-2}\right)}$$

$$\xrightarrow{h\to 0} \boxed{\frac{4}{(x-2)\sqrt{x-2}} = f'(x)} \Rightarrow f'(6) = \frac{4}{4(2)} = \frac{1}{2}$$

$$\boxed{y = \tfrac{1}{2}(x-6)+4}$$

$= \frac{1}{2}x + 1$

201 §3.2 #s 20, 23, 24, 27–30

#s 19–22 Find the values of the derivatives

20 $\left.\frac{dy}{dx}\right|_{x=\sqrt{3}}$ if $y = 1 - \frac{1}{x} = f(x)$

$$\frac{f(x+h)-f(x)}{h} = \frac{1}{h}\left[1 - \frac{1}{x+h} - \left(1 - \frac{1}{x}\right)\right] = \frac{1}{h}\left[1 - 1 - \frac{1}{x+h} + \frac{1}{x}\right]$$

$$= \frac{1}{h}\left[\frac{x+h-x}{x(x+h)}\right] = \frac{1}{x(x+h)} \xrightarrow{h \to 0} \boxed{\frac{1}{x^2} = \frac{dy}{dx}} \longrightarrow$$

$$\boxed{\left.\frac{dy}{dx}\right|_{x=\sqrt{3}} = \frac{1}{(\sqrt{3})^2} = \frac{1}{3}}$$

#s 23–26 Use $\lim_{z \to x} \frac{f(z)-f(x)}{z-x}$ to calculate the derivatives

23 $f(x) = \frac{1}{x+2} \Longrightarrow \frac{f(z)-f(x)}{z-x} =$

$$\frac{\left(\frac{1}{z+2} - \frac{1}{x+2}\right)}{z-x} = \frac{x+2-z-2}{(z+2)(x+2)(z-x)} = \frac{x-z}{(z+2)(x+2)(z-x)}$$

$$= \frac{-1}{(z+2)(x+2)} \xrightarrow{z \to x} \boxed{\frac{-1}{(x+2)^2}}$$

201 § 3.2 #s 24, 27–30

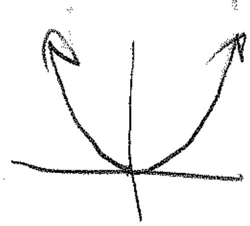
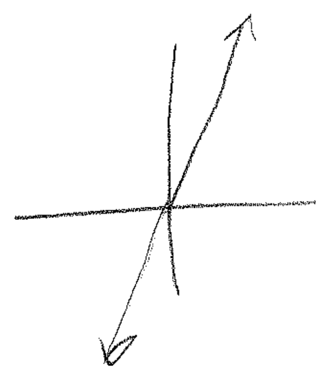
24 $f(x) = x^2 - 3x + 4 \longrightarrow \dfrac{f(z) - f(x)}{z - x} =$

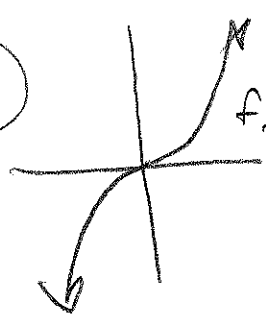
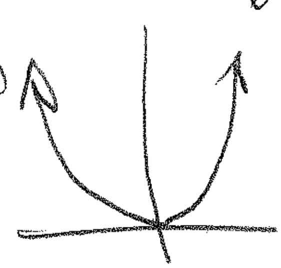
$$\frac{z^2 - 3z + 4 - x^2 + 3x - 4}{z - x} = \frac{z^2 - x^2 - 3z + 3x}{z - x}$$

$$= \frac{(z - x)(z + x) - 3(z - x)}{z - x} = \frac{(z + x - 3)(z - x)}{z - x}$$

$$= z + x - 3 \xrightarrow{z \to x} \boxed{2x - 3 = f'(x)}$$

#s 27–30 Match funcs in 27–30 with derivatives in (a)–(d)

27  $f_1(x) \rightarrow$ (b) 

28  $f_2(x) \rightarrow$ (a) 

201 § 3.2 #s 29, 30

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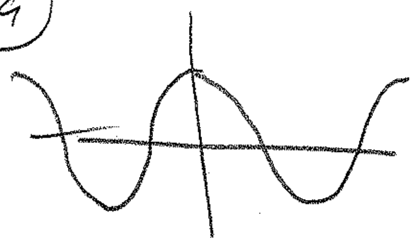
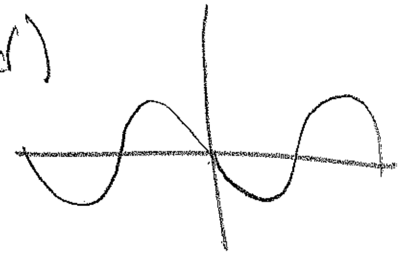
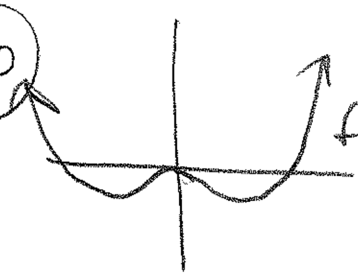


$f_3(x) \longrightarrow$ (d)



30



$f_4(x) \longrightarrow$ (c)