

201 §3.2 #4 Re-do

④ ~~#8~~  $K(z) = \frac{1-z}{2z}$ ,  $K'(1)$ ,  $K'(i)$ ,  $K'(\sqrt{2})$  are to be found.

$$K(z) = \frac{1-z}{2z} = \frac{1}{2z} - \frac{z}{2z} = \frac{1}{2z} - \frac{1}{2} \Rightarrow$$

$$\frac{K(z+h) - K(z)}{h} = \frac{1}{h} [K(z+h) - K(z)]$$

$$= \frac{1}{h} \left[ \frac{1}{2(z+h)} - \frac{1}{2} - \left( \frac{1}{2z} - \frac{1}{2} \right) \right]$$

$$= \frac{1}{h} \left[ \frac{1}{2(z+h)} - \frac{1}{2z} \right] = \frac{1}{h} \left[ \frac{z - (z+h)}{2z(z+h)} \right]$$

$$= \frac{1}{h} \left[ \frac{-h}{2z(z+h)} \right] = \frac{-1}{2z(z+h)} \xrightarrow{h \rightarrow 0} -\frac{1}{2z^2}$$

So

$K'(1) = -\frac{1}{2}$
$K'(i) = -\frac{1}{2}$
$K'(\sqrt{2}) = -\frac{1}{2(2)} = -\frac{1}{4}$