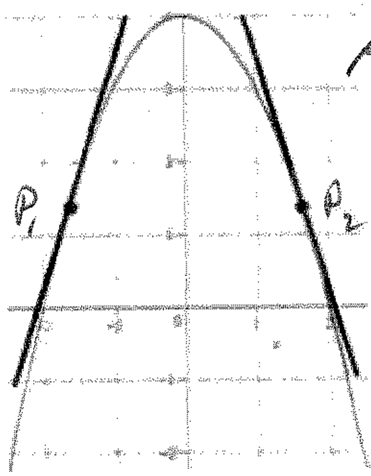
201 §3.1 #s 4, 7, 8, 13, 14, 26, 30, 33, 34

(4)



Approximate the slope of the curve w/ straight edge

@ P_1, $m_{TAN} \approx 2.5$

@ P_2, $m_{TAN} \approx -2.5$

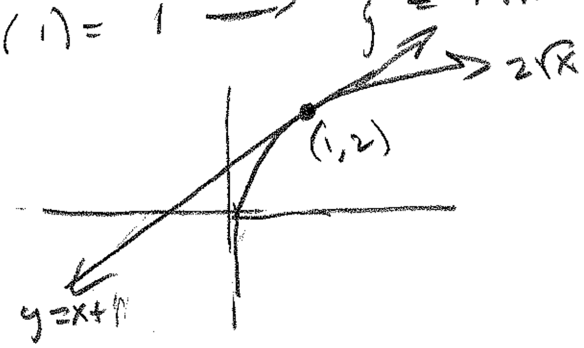
#s 5-10 Find eqn of tangent line @ the given pt. Sketch curve & tangent line together.

(7) $y = 2\sqrt{x}$ @ $(1,2)$

$$\frac{2\sqrt{x+h} - 2\sqrt{x}}{h} \cdot \frac{2\sqrt{x+h} + 2\sqrt{x}}{2\sqrt{x+h} + 2\sqrt{x}} = \frac{4(x+h) - 4(x)}{2h(\sqrt{x+h} + \sqrt{x})}$$

$$= \frac{4x+4h-4x}{2h(\sqrt{x+h} + \sqrt{x})} = \frac{4}{2(\sqrt{x+h} + \sqrt{x})} \xrightarrow{h \to 0} \frac{2}{(2\sqrt{x})} = \frac{1}{\sqrt{x}}$$

$f'(1) = 1 \longrightarrow y = 1(x-1) + 2 = x + 1$



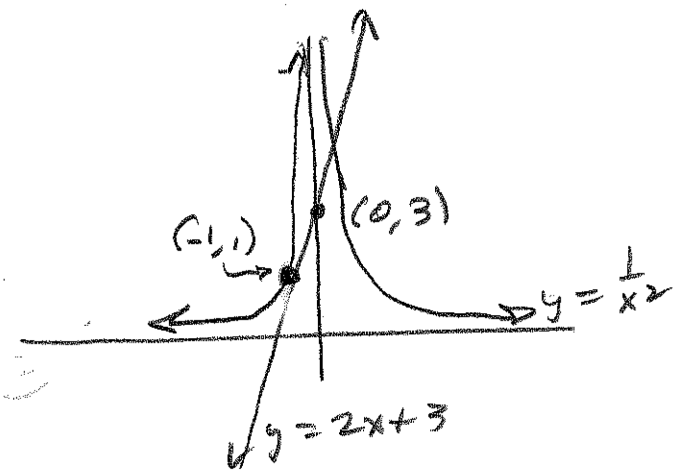
201 §3.1 #s 8, 13, 14, 26, 30, 33, 34

(8) $y = f(x) = \frac{1}{x^2}$ @ $(-1, 1)$

$$\frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} = \frac{1}{h}\left[\frac{x^2 - (x+h)^2}{x^2(x+h)^2}\right] = \frac{1}{h}\left[\frac{x^2 - x^2 - 2xh - h^2}{x^2(x+h)^2}\right]$$

$$= \frac{1}{h}\left[\frac{-2xh - h^2}{x^2(x+h)^2}\right] = \frac{-2x - h}{x^2(x+h)^2} \xrightarrow{h \to 0} \frac{-2x}{x^4} = \boxed{-\frac{2}{x^3} = f'(x)}$$

$$f'(-1) = -\frac{2}{(-1)^3} = 2 \longrightarrow y = 2(x+1) + 1 = \boxed{2x + 3 = y}$$



#s 11-18 Find eq'n of tangent line @ the given point

(13) $g(x) = \frac{x}{x-2}$ @ $(3, 3)$

$$\frac{1}{h}\left[\frac{x+h}{(x+h)-2} - \frac{x}{x-2}\right] = \frac{1}{h}\left[\frac{(x+h)(x-2) - x(x+h-2)}{(x+h-2)(x-2)}\right]$$

$$= \frac{1}{h}\left[\frac{x^2 - 2x + hx - 2h - x^2 - xh + 2x}{(x+h-2)(x-2)}\right] = \frac{1}{h}\left[\frac{hx - 2h - xh}{(x+h-2)(x-2)}\right]$$

$$= \frac{-2}{(x+h-2)(x-2)} \xrightarrow{h \to 0} \frac{-2}{(x-2)^2} = g'(x) \Rightarrow g'(3) = -2$$

$$\Rightarrow \boxed{y = -2(x-3) + 3}$$

$$= -2x + 9$$

201 §3.1 #s 14, 26, 30, 33, 34

(14) $g(x) = \frac{8}{x^2}$ @ $(2,2)$

$$\frac{1}{h}\left[\frac{8}{(x+h)^2} - \frac{8}{x^2}\right] = \frac{8x^2 - 8(x^2+2xh+h^2)}{h(x^2(x+h)^2)}$$

$$= \frac{8x^2 - 8x^2 - 16xh - 8h^2}{h(x^2)(x+h)^2} = \frac{-16xh - 8h^2}{h(x^2)(x+h)^2}$$

$$= \frac{-16x - 8h}{x^2(x+h)^2} \xrightarrow{h \to 0} \frac{-16x}{x^4} = -\frac{16}{x^3} \Longrightarrow$$

$$g'(2) = -\frac{16}{2^3} = -2 \Longrightarrow \boxed{y = -2(x-2)+2}$$

$$= -2x + 6$$

(26) Find eq'n of line with slope $m = \frac{1}{4}$ thats tangent to $y = \sqrt{x}$.

By previous work, $y' = \frac{1}{2\sqrt{x}}$ SET $\frac{1}{4}$ $\Longrightarrow$

$2\sqrt{x} = 4 \Longrightarrow \sqrt{x} = 2 \Longrightarrow x = 4$. @

$x = 4$, $y = \sqrt{4} = 2 \rightsquigarrow (4,2)$, so

$\boxed{y = \frac{1}{4}(x-4)+2}$ does it.

$= \frac{1}{4}x + 1$

201 §3.1 #s 30, 33, 34

(30) What's the rate of change of the volume V of a ball wrt radius r when the radius is $r = 3$?

$V = \frac{4}{3}\pi r^3$

$$\frac{dV}{dr} = 4\pi r^2$$

$$\left.\frac{dV}{dr}\right|_{r=3} = 4\pi(3)^2 = 36\pi \ \frac{\text{units}^3}{\text{time unit}}$$

$V = V(r) = \frac{4}{3}\pi r^3$

$V'(r) = 4\pi r^2$

$V'(3) = 4\pi(3)^2 = 36\pi$

(33) The graph of $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$

Does have a tangent @ the origin.

PF $\frac{1}{h}\left[(x+h)^2 \sin\left(\frac{1}{x+h}\right) - x^2 \sin\left(\frac{1}{x}\right)\right]$ @ $x \neq 0$

and $\frac{1}{h}\left[h^2 \sin\left(\frac{1}{h}\right) - 0\right] = h \sin\left(\frac{1}{h}\right)$ @ $x = 0$,

and $\xrightarrow{h \to 0}$ gives $f'(0) = 0$!

This limit exists!

(34) Same thing only $f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$

It's cnt@ $x = 0$ (Damped) but

$\frac{1}{h}\left[h \sin\left(\frac{1}{h}\right) - 0\right] = \sin\left(\frac{1}{h}\right)$ has no limit as $h \to 0$.