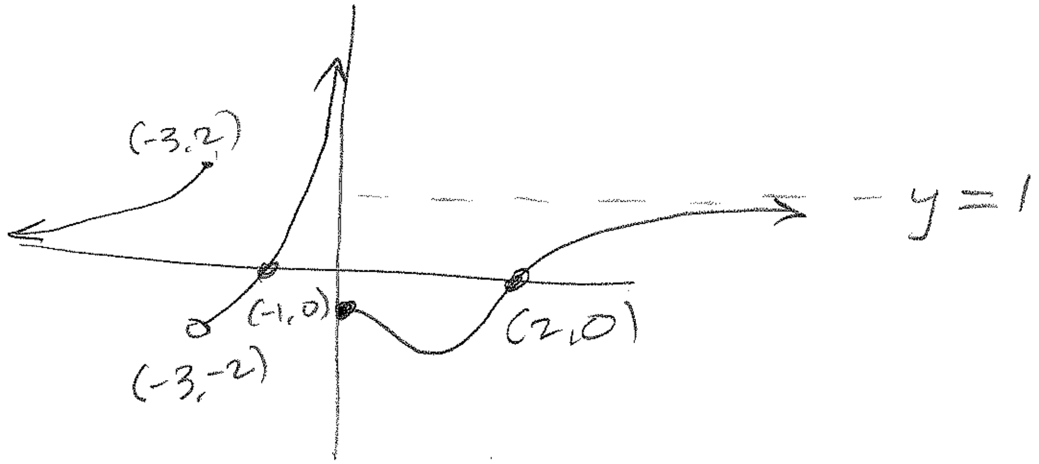
201 §2.6 #s 1-25 odds, 29-59 odds, 81, 99, 107, 109

① Determine the limits from the graph of f(x)



ⓐ $\lim_{x\to 2} f(x) = 0$

ⓑ $\lim_{x\to -3^+} f(x) = -2$

ⓒ $\lim_{x\to -3^-} f(x) = +2$

ⓓ $\lim_{x\to -3} f(x) \nexists$ (by b & c)

ⓔ $\lim_{x\to 0^+} f(x) = -1$

ⓕ $\lim_{x\to 0^-} f(x) = +\infty$

ⓖ $\lim_{x\to 0} f(x) \nexists$ (by e & f)

ⓗ $\lim_{x\to \infty} f(x) = 1$

ⓘ $\lim_{x\to -\infty} f(x) = 0$

201 §2.6 #s 3-25 odds, 29-59 odds, 81, 99, 107, 109

#s 3-8 Find lim ~~as~~ $\overset{(a)}{x \to \infty}$ & $\overset{(b)}{\text{as } x \to -\infty}$

③ $\lim\limits_{x \to \infty} \frac{2}{x} - 3 = -3$

$\lim\limits_{x \to -\infty} \frac{2}{x} - 3 = -3$

⑤ $\lim\limits_{x \to \infty} \frac{1}{2 + \frac{1}{x}} = \frac{1}{2} = \lim\limits_{x \to -\infty} \frac{1}{2 + \frac{1}{x}}$

⑦ $h(x) = \frac{-5 + \frac{7}{x}}{3 - \frac{1}{x^2}} \Rightarrow \lim\limits_{x \to \infty} h(x) = -\frac{5}{3} = \lim\limits_{x \to -\infty} h(x)$

#s 9+12 Find the limits.

⑨ $\lim\limits_{x \to \infty} \frac{\sin(2x)}{x} = 0$

⑪ $\lim\limits_{x \to -\infty} \frac{2 - t + \sin t}{t + \cos t} = -1$

#s 13-22 Find limit of each rational function

(a) as $x \to \infty$ and (b) as $x \to -\infty$.

⑬ $f(x) = \frac{2x+3}{5x+7} \xrightarrow{x \to \infty} \frac{2}{5} = \lim\limits_{x \to -\infty} f(x)$.

⑮ $f(x) = \frac{x+1}{x^2+3} \xrightarrow{x \to \infty} 0 = \lim\limits_{x \to -\infty} f(x)$

201 §2.6 #s 17-25 odds, 29-59 odds, 81, 99, 107, 109

⑰ $h(x) = \frac{7x^3}{x^3-3x^2+6x} \xrightarrow{x\to\infty} 7 = \lim_{x\to-\infty} h(x)$

⑲ $g(x) = \frac{10x^5+x^4+31}{x^6} \xrightarrow{x\to\infty} 0 = \lim_{x\to-\infty} g(x)$

㉑ $h(x) = \frac{-2x^3-2x+3}{3x^3+3x^2-5x} \xrightarrow{x\to\infty} -\frac{2}{3} = \lim_{x\to-\infty} h(x)$

#s 23-36 Find the limits

㉓ $\lim_{x\to\infty}\sqrt{\frac{8x^2-3}{2x^2+x}} = \lim_{x\to\infty}\sqrt{\frac{x^2(8-\frac{3}{x^2})}{x^2(2+\frac{1}{x})}}$

$= \lim_{x\to\infty}\sqrt{\frac{8-\frac{3}{x^2}}{2+\frac{1}{x}}} = \sqrt{4} = \boxed{2}$

㉕ $\lim_{x\to-\infty}\left(\frac{1-x^3}{x^2+7x}\right)^5 = \boxed{\infty}$

$-x^3$ is dominant. Nothing as big in the denominator, so I just managed signs.

㉙ $\lim_{x\to-\infty}\frac{\sqrt[3]{x}-\sqrt[5]{x}}{\sqrt[3]{x}+\sqrt[5]{x}} = \lim_{x\to-\infty}\frac{x^{\frac{1}{3}}-x^{\frac{1}{5}}}{x^{\frac{1}{3}}+x^{\frac{1}{5}}} = \boxed{1}$

$x^{\frac{1}{3}}$ dominates.

201 § 2.6 #s 31-59 odds , 81, 99, 107, 109

(31) $\lim_{x\to\infty} \frac{2x^{\frac{5}{3}} - x^{\frac{1}{3}} + 7}{x^{\frac{8}{5}} + 3x + \sqrt{x}} = \boxed{\infty}$

$\frac{5}{3} = \frac{5}{3} \cdot \frac{5}{5} = \frac{25}{15}$

$\frac{8}{5} = \frac{8}{5} \cdot \frac{3}{3} = \frac{24}{15}$

$x^{\frac{5}{3}}$ dominates.

(33) $\lim_{x\to\infty} \frac{\sqrt{x^2+1}}{x+1} = \lim_{x\to\infty} \frac{\sqrt{x^2(1+\frac{1}{x^2})}}{x(1+\frac{1}{x})} = \lim_{x\to\infty} \frac{x\sqrt{1+\frac{1}{x^2}}}{x(1+\frac{1}{x})}$

$(|x| = x$ when $x \longrightarrow$ BIG)

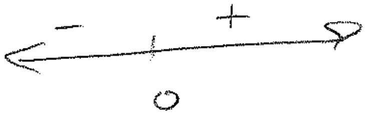
$= \boxed{1}$

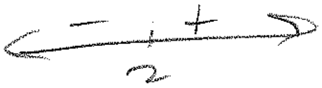
(35) $\lim_{x\to\infty} \frac{x-3}{\sqrt{4x^2+25}} = \lim_{x\to\infty} \frac{x(1-\frac{3}{x})}{\sqrt{x^2(4+\frac{25}{x^2})}} = \lim_{x\to\infty} \frac{x(1-\frac{3}{x})}{x\sqrt{4+\frac{25}{x^2}}}$

$= \lim_{x\to\infty} \frac{1-\frac{3}{x}}{\sqrt{4+\frac{25}{x^2}}} = \boxed{\frac{1}{2}}$

Same ? Find Limits #s 37-48

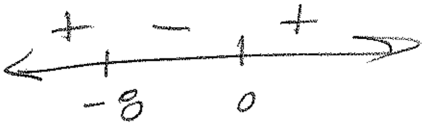
(37) $\lim_{x\to 0^-} \frac{1}{3x} = \boxed{-\infty}$





(39) $\lim_{x\to 2^-} \frac{3}{x-2} = \boxed{-\infty}$

(41) $\lim_{x\to -8^+} \frac{2x}{x+8} = \boxed{-\infty}$

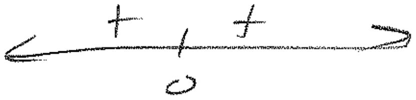


(43) $\lim_{x\to 7} \frac{4}{(x-7)^2} = \boxed{\infty}$

(45) ⓐ $\lim_{x\to 0^+} \frac{2}{3x^{\frac{1}{3}}} = \boxed{\infty}$ ⓑ $\lim_{x\to 0^-} \frac{2}{3x^{1/3}} = \boxed{-\infty}$

201 §2.6 #s 47-59 odds, 81, 99, 107, 109

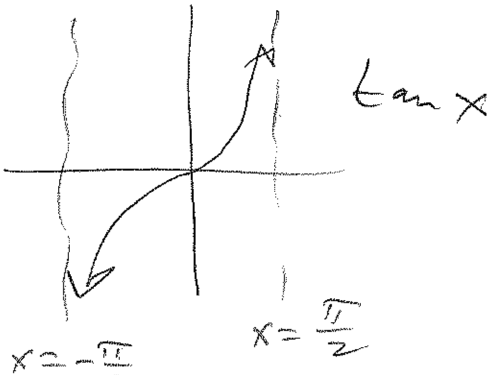
(47) $\lim_{x\to 0}\frac{4}{x^{2/5}} = \infty$



$x^{2/5} = (x^2)^{1/5} = (x^{\frac{1}{5}})^2 \ge 0$

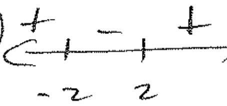
49-58 Find the limits

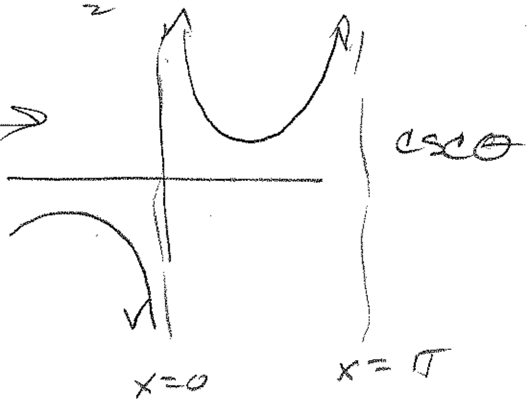
(49) $\lim_{x\to \frac{\pi}{2}^-}\tan x = \boxed{\infty}$



(51) $\lim_{\theta\to 0^-}(1+\csc\theta) = \boxed{-\infty}$

(53) ⓐ $\lim_{x\to 2^+}\frac{1}{x^2-4} = \boxed{\infty}$





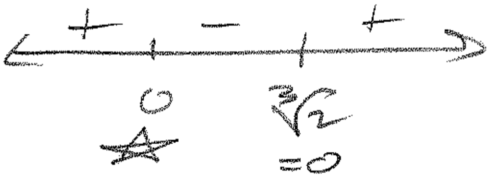
ⓑ $\lim_{x\to 2^-}\frac{1}{x^2-4} = \boxed{-\infty}$

ⓒ $\lim_{x\to -2^+}\frac{1}{x^2-4} = \boxed{-\infty}$

ⓓ $\lim_{x\to -2^-}\frac{1}{x^2-4} = \boxed{\infty}$

$\frac{x^2}{2} - \frac{1}{x} = \frac{x^3-2}{2x}$

$= \frac{(x-\sqrt[3]{2})(x^2+\sqrt[3]{2}\,x+2^{2/3})}{2x}$



(55) ⓐ $\lim_{x\to 0^+}\left(\frac{x^2}{2}-\frac{1}{x}\right) = \boxed{-\infty}$

ⓑ $\lim_{x\to 0^-}\left(\frac{x^2}{2}-\frac{1}{x}\right) = \boxed{\infty}$

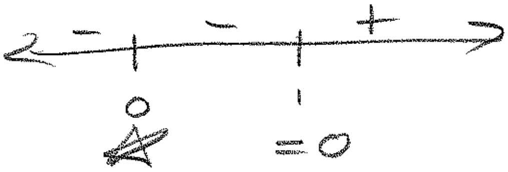
ⓒ $\lim_{x\to \sqrt[3]{2}}\left(\frac{x^2}{2}-\frac{1}{x}\right) = \boxed{0}$

ⓓ $\lim_{x\to -1}\left(\frac{x^2}{2}-\frac{1}{x}\right) = \frac{1}{2}+1 = \boxed{\frac{3}{2}}$

201 § 2.6 #s 57, 59, 81, 99, 107, 109

(57) $f(x) = \frac{x^2-3x+2}{x^3-2x^2} = \frac{(x-2)(x-1)}{x^2(x-2)} = \frac{x-1}{x^2}, x \neq 2$

(a) $\lim_{x\to 0^+} f(x) = \boxed{-\infty}$



(b) $\lim_{x\to 2^+} f(x) = \frac{2-1}{2^2} = \boxed{\frac{1}{4}}$

(c) $\lim_{x\to 2^-} f(x) = \boxed{\frac{1}{4}}$

(d) $\lim_{x\to 2} f(x) = \boxed{\frac{1}{4}}$

(e) $\lim_{x\to 0} f(x) = \boxed{-\infty}$

see sign pattern

#s 59-62 Find limits Same for #81

(59) $f(t) = 2 - \frac{3}{t^{1/3}}$

(a) $\lim_{t\to 0^+} f(t) = \boxed{-\infty}$

(b) $\lim_{t\to 0^-} f(t) = \boxed{+\infty}$

(81) $\lim_{x\to\infty}\left(\sqrt{x^2+25} - \sqrt{x^2-1}\right) = ?$

$$\frac{\sqrt{x^2+25} - \sqrt{x^2-1}}{1} \cdot \frac{\sqrt{x^2+25} + \sqrt{x^2-1}}{\sqrt{x^2+25} + \sqrt{x^2-1}} = \frac{x^2+25-(x^2-1)}{\sqrt{x^2+25} + \sqrt{x^2-1}}$$

$$= \frac{26}{\sqrt{x^2+25} + \sqrt{x^2-1}} \xrightarrow{x\to\infty} \boxed{0}$$

Book DOES :

$$\frac{\frac{26}{x}}{\sqrt{1+\frac{25}{x^2}} + \sqrt{1-\frac{1}{x^2}}}$$

201 §2.6 #s 99, 107, 109

(99) Graph it: $y = \frac{x^2}{x-1} = x+1+\frac{1}{x-1}$

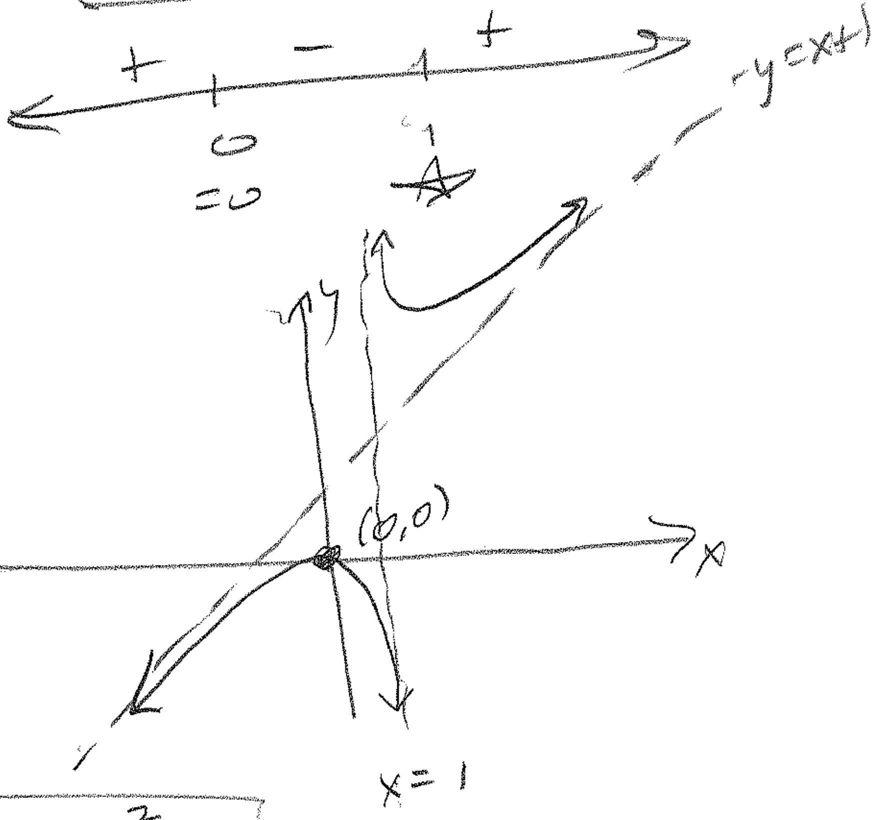
$\boxed{y = x+1 \text{ is O.A.}}$

$$\begin{array}{r|rrr} 1 & 1 & 0 & 0 \\ & & 1 & 1 \\ \hline & 1 & 1 & 1 \end{array}$$

$\boxed{x = 1 \text{ is V.A.}}$

$\boxed{y\text{-int}: (0,0)}$

$\boxed{x\text{-int}: (0,0)}$

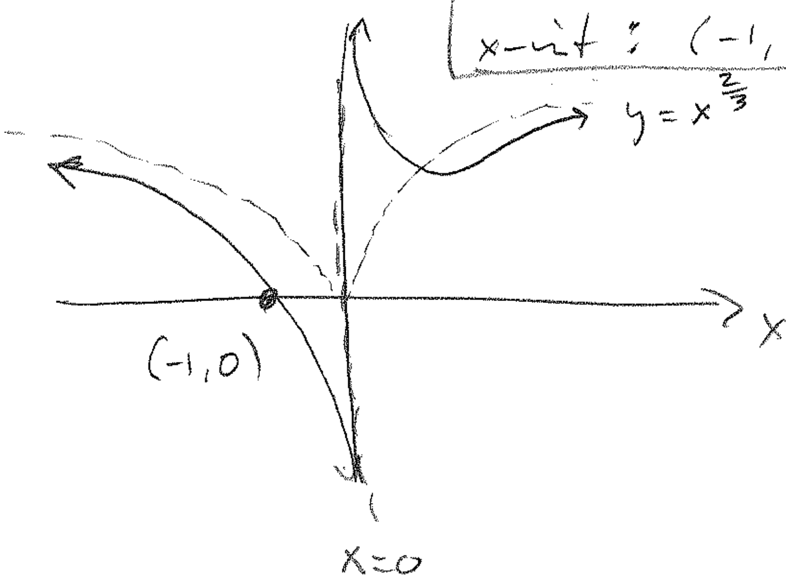


(107) $y = x^{2/3} + \frac{1}{x^{1/3}}$

Graph & relate formula to what you see.

$y = \frac{x+1}{x^{\frac{1}{3}}}$

$\boxed{\begin{array}{l} \text{O.A.}: y = x^{\frac{2}{3}} \\ \text{V.A}: x = 0 \\ x\text{-int}: (-1,0) \end{array}}$



It acts like $\frac{1}{x^{1/3}}$ near $x=0$ & it acts like $x^{2/3}$ as we move far away from $x=0$.

201 §2.6 #s 109

Graph & answer questions

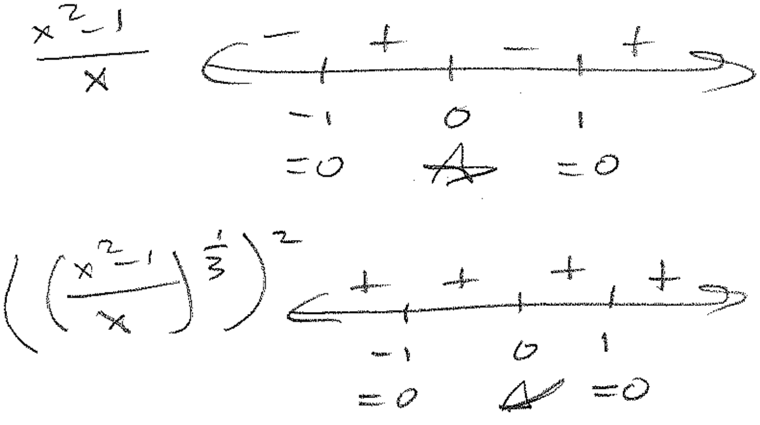
(109) $y = \frac{3}{2}\left(x - \frac{1}{x}\right)^{\frac{2}{3}}$

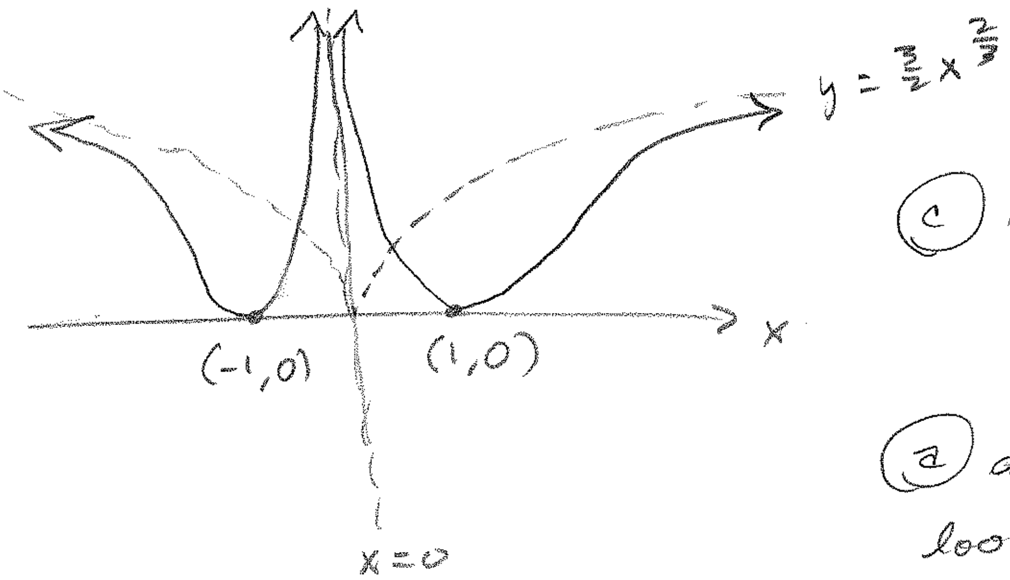
$= \frac{3}{2}\left(\frac{x^2-1}{x}\right)^{\frac{2}{3}}$

O.A.: $y = \frac{3}{2}x^{\frac{2}{3}}$

V.A.: $x = 0$

x-int: $(\pm 1, 0)$





(c) Near $x = \pm 1$: looks like $y = x^2 - 1$

(a) as $x \to 0^+$, it looks like $\left(\frac{1}{x}\right)^{2/3}$

(b) as $x \to \pm\infty$, it looks like $\frac{3}{2}x^{2/3}$