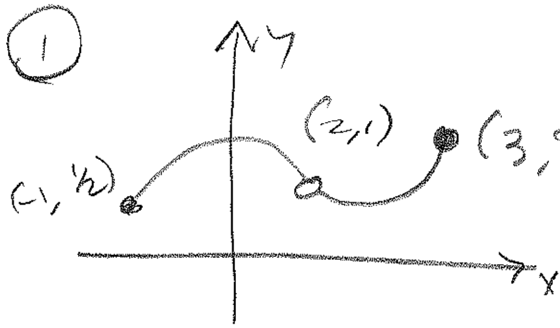
201 §2.5 #s 1-41 odds, 45, 47, 51, 53, 55, 69, 71, 73, 75

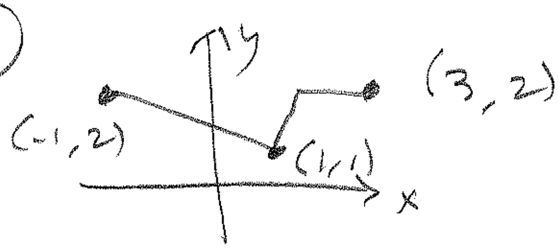
#s 1-4 Is it cnt$^s$ on $[-1,3]$? If not, where does it fail, and why?

(1)




$\boxed{\text{No. } \exists \text{ hole @ } x=2}$

(3)




$\boxed{\text{Yes}}$

#s 5-10 refers to

$$f(x) = \begin{cases} x^2-1 & -1 \le x < 0 \\ 2x & 0 < x < 1 \\ 1 & x = 1 \\ -2x+4 & 1 < x < 2 \\ 0 & 2 < x < 3 \end{cases}$$

(5) (a) $f(-1) = 0 \quad \exists$

(b) $\lim\limits_{x \to -1^+} f(x) = 0 \quad \exists$

(c) $\lim\limits_{x \to -1^+} f(x) = f(-1)$. Yes

(d) $f$ is cnt$^s$ @ $x = -1$

201 §2.5 #s 7–41 odds, 45, 47, 51, 53, 55, 69, 71, 73, 75

7 (a) $f(2) \nexists$

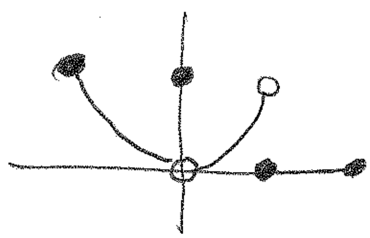
(b) $f$ is <u>NOT</u> $\text{cnt}^s$ @ $x=2$

9 To make $f$ $\text{cnt}^s$ @ $x=2$, define

$$\boxed{f(2) = \lim_{x\to 2} f(x) = 0}$$

11 At what points does §2.4 #1 fail to be $\text{cnt}^s$? Which are / aren't removable? Support your answers

2.4 #1



$x=0$, Removable → Hole.

$x=1$ NOT Removable → Jump Discontinuity

#s 13–30 where are they $\text{cnt}^s$?

13 $y = \dfrac{1}{x-2} - 3x$ : $\boxed{\mathbb{R} \setminus \{2\}}$

15 $y = \dfrac{x+1}{x^2-4x+3} = \dfrac{x+1}{(x-3)(x-1)}$ : $\boxed{\mathbb{R} \setminus \{1,3\}}$

17 $y = |x-1| + \sin x$ : $\boxed{\mathbb{R}}$

19 $y = \dfrac{\cos x}{x}$ : $\boxed{\mathbb{R} \setminus \{0\}}$

201 §2.5 #s 21–41 odds, 45, 47, 51, 53, 55, 69, 71, 73, 75

21 $y = \csc(2x) = \dfrac{1}{\sin(2x)}$ Find where $\sin(2x) = 0$:

$\sin(2x) = 0 \Rightarrow$

$2x = n\pi,\ n \in \mathbb{Z} \Rightarrow$

$x = \dfrac{n\pi}{2},\ n \in \mathbb{Z} \Rightarrow$ cont's $\boxed{\{x \mid x \neq \frac{n\pi}{2}, n \in \mathbb{Z}\}}$

23 $y = \dfrac{x \tan x}{x^2+1}$. Find where $\cos x = 0$:

$x = \dfrac{(2n+1)}{2}\pi,\ n \in \mathbb{Z}$ So cont's on

$\boxed{\{x \mid x \neq \frac{(2n+1)}{2}\pi, n \in \mathbb{Z}\}}$

25 $y = \sqrt{2x+3}$ Need $2x+3 \geq 0 \Rightarrow$

$2x \geq -3$

$\boxed{\{x \mid x \geq -\frac{3}{2}\}}$

27 $y = (2x-1)^{1/3}$: $\boxed{\mathbb{R}}$

29 $g(x) = \begin{cases} \dfrac{x^2-x-6}{x-3} & x \neq 3 \\ 5 & x = 3 \end{cases}$ $\quad \dfrac{(x-3)(x+2)}{x-3} = x+2 \quad (x \neq 3)$

$\boxed{\mathbb{R}}$

Extends to the "hole"

201 §2.5 #s 31-41 odds, 45, 47, 51, 53, 55, 69, 71

#s 31-36 Find limits. Are they cont @ the points being approached?

(31) $\lim_{x \to \pi} \sin(x - \sin x) = \sin(\pi - \sin \pi) = \sin \pi = \boxed{0}$

$\boxed{\text{Yes}}$

(33) $\lim_{y \to 1} \sec(y \sec^2 y - \tan^2 y - 1)$

$= \boxed{\sec(1 \sec^2 1 - \tan^2 1 - 1) \quad \text{Yes}}$

(35) $\lim_{t \to 0} \dfrac{\pi}{\sqrt{19 - 3\sec(2t)}} = \dfrac{\pi}{\sqrt{19-3}} = \boxed{\dfrac{\pi}{4} \quad \text{Yes}}$

(37) Define $g(3) \ni \dfrac{x^2-9}{x-3}$ is cont's @ $x = 3$

$\dfrac{x^2-9}{x-3} = \dfrac{(x-3)(x+3)}{x-3} = x+3 \xrightarrow{x \to 3} \boxed{6 \equiv g(3)}$

(39) Define $f(1) \ni f(s) = \dfrac{s^3-1}{s^2-1}$ is cont's @ $s = 1$

$\dfrac{s^3-1}{s^2-1} = \dfrac{(s-1)(s^2+s+1)}{(s-1)(s+1)} = \dfrac{s^2+s+1}{s+1} \xrightarrow{s \to 1} \boxed{\dfrac{3}{2} \equiv f(1)}$

201 §2.5 #s 41, 45, 47, 51, 53, 55, 69, 71

(41) For what value of $a$ is

$$f(x)=\begin{cases} x^2-1 & x<3 \\ 2ax & x\geq 3 \end{cases}$$ cnt's?

Want $\lim_{x\to 3^-} f(x) = 3^2-1 = 8$ to be the same as $f(3)$ & $\lim_{x\to 3^+} f(x)$:

Set $2ax = 8$, where $x=3$.

$6a = 8$

$a = \frac{8}{6} = \boxed{\frac{4}{3} = a}$

(45) Find $a$ & $b$ to make

$$f(x)=\begin{cases} -2 & x\leq -1 \\ ax-b & -1<x<1 \\ 3 & x\geq 1 \end{cases}$$ cnt's

Want $\lim_{x\to -1^+} (ax-b) = -2$ $\quad(-1,2)$

& $\lim_{x\to 1^-} (ax-b) = 3$ $\quad(1,3)$

Find eq'n thru $(-1,2)$, $(1,3)$, i.e.

$m = \frac{2-3}{-1-1} = \frac{-1}{-2} = \frac{1}{2}$

$y = \frac{1}{2}(x+1)+2$

$y = \frac{1}{2}x + \frac{1}{2} + 2$

$y = \frac{1}{2}x + \frac{5}{2}$

$$\boxed{\begin{array}{l} a = \frac{1}{2} \\ b = -5/2 \end{array}}$$

201 §2.5 #s 47, 51, 53, 55, 69, 71

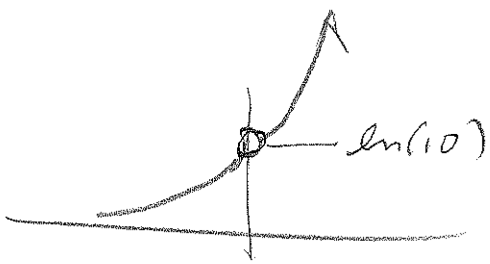
#s 47–50 Graph f to see if it has cnt's extension to $x=0$. If not, can it be so from left or right?

47

$$y = \frac{10^x - 1}{x}$$

L'Hôpital says: $\dfrac{(\ln 10)10^x}{1} = (\ln 10)10^x \xrightarrow{x \to 0} \ln 10$

$\approx 2.302585093 \approx$ $\boxed{2.3026 \text{ to extend to } f(0)}$




51 If $f(x) < 0$ @ $x=0$ &

$f(1) > 0$, and $f$ is cnt's,

then $\exists\, c \in (0,1) \ni f(c) = 0$ by IVT.

53 Show that $x^3 - 15x + 1 = 0$ has 3 sol'ns in $[-4, 4]$:

$$\begin{array}{r|rrrr} -4 & 1 & 0 & -15 & 1 \\ & & -4 & 16 & -4 \\ \hline & 1 & -4 & 1 & -3 \end{array}$$

$f(-4) = -3 < 0$

$\boxed{① \; c \in (-4,-3)}$

$$\begin{array}{r|rrrr} -3 & 1 & 0 & -15 & 1 \\ & & -3 & 9 & 18 \\ \hline & 1 & -3 & -6 & 19 \end{array}$$

$f(-3) = 19 > 0$

201 §2.5 #5 53, 55, 69, 71

53 cntd

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -15 & 1 \\ & & -2 & 4 & 22 \\ \hline & 1 & -2 & -11 & 23 \end{array}$$

$f(-2) = 23 > 0$

$$\begin{array}{r|rrrr} -1 & 1 & 0 & -15 & 1 \\ & & -1 & 1 & 16 \\ \hline & 1 & -1 & -16 & 16 \end{array}$$

$f(-1) = 16 > 0$

$f(0) = 1 > 0$

① 

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -15 & 1 \\ & & 1 & 1 & -14 \\ \hline & 1 & 1 & -14 & -13 \end{array}$$

$f(1) = -13 < 0$

$\boxed{② \ c \in (-3, 1)}$

$$\begin{array}{r|rrrr} 4 & 1 & 0 & -15 & 1 \\ & & 4 & 16 & 4 \\ \hline & 1 & 4 & 1 & 5 \end{array}$$

$f(4) = 5 > 0$

$\boxed{③ \ c \in (1, 4)}$

By IVT!

55 $f(x) = x^3 - 8x + 10 \longrightarrow \exists\, c \ni$

(a) $f(c) = \pi$

(b) $f(c) = -\sqrt{3}$

(c) $f(c) = 5000{,}000$

$f(-10) = -1000 + 80 + 10$
$= -910 < \pi, -\sqrt{3}, 5\times10^6$

$f(500{,}000)$
$= 1.25 \times 10^{17} > \pi, -\sqrt{3}, 5\times 10^6$

DONE!

201 §2.5 #s 69, 71

#s 69-76 use IVT to prove $\exists$ sol'n.

Then use a grapher to find it

69 $x^3 - 3x - 1 = 0$

$f(x) = x^3 - 3x - 1 \implies$

$f(0) = -1$ &

$f(2) = 1$, so IVT says $\exists\, c \in (-1, 1) \ni f(c) = 0$

$\boxed{c \approx -.3472964 \text{ works}}$

71 $x(x-1)^2 = 1$

$f(x) = x(x-1)^2 \implies$

$f(0) = 0 < 1$ and

$f(2) = 2 > 1$ so ... $\exists\, c \in (0, 2) \ni f(x) = 1$

$\boxed{c \approx 1.7548777}$