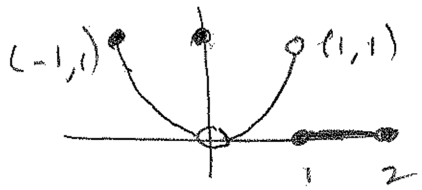
201 §2.4 #5 1-29 odds, 33-39 odds, 47, 52

① Which one true? Which are false?



ⓐ $f(x) \xrightarrow{x \to -1^+} 1$ T

ⓑ $f(x) \xrightarrow{x \to 0^-} 0$ T

ⓒ $f(x) \xrightarrow{x \to 0^-} 1$ F

ⓓ $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$ T

ⓔ $\lim_{x \to 0} f(x)$ exists T

ⓕ $\lim_{x \to 0} f(x) = 0$ T

ⓖ $\lim_{x \to 0} f(x) = 1$ F

ⓗ $\lim_{x \to 1} f(x) = 1$ F

ⓘ $\lim_{x \to 1} f(x) = 0$ F

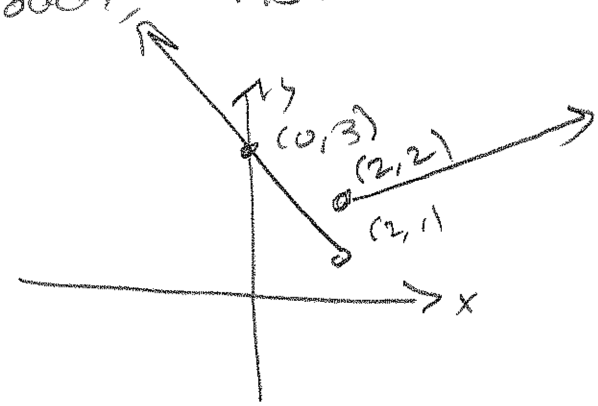
ⓙ $\lim_{x \to 2^-} f(x) = 2$ F

ⓚ $\lim_{x \to -1^-} f(x) \nexists$ T

ⓛ $\lim_{x \to 2^+} f(x) = 0$ F

201 §2.4 #s 3-29 odds, 33-39 odds,

(3) $f(x) = \begin{cases} 3-x, & x<2 \\ \frac{x}{2}+1, & x>2 \end{cases}$



(a) $\lim_{x\to 2^+} f(x) = 2$

$\lim_{x\to 2^-} f(x) = 1$

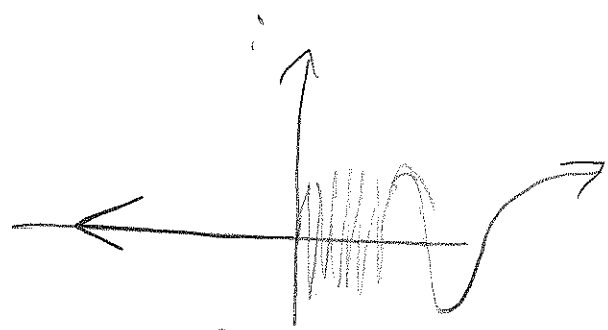
(b) $\lim_{x\to 2} f(x) \nexists$, b/c the 1-sided limits disagree.

(c) $\lim_{x\to 4^-} f(x) = \frac{4}{2}+1 = 3$

$\lim_{x\to 4^+} f(x) = \frac{4}{2}+1 = 3$

(d) $\lim_{x\to 4} f(x) = 3$. Yes

(5) $f(x) = \begin{cases} 0 & x \le 0 \\ \sin\left(\frac{1}{x}\right) & x = 0 \end{cases}$



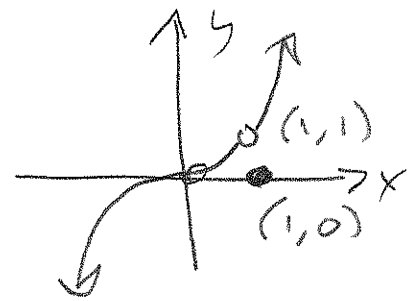
(a) $\lim_{x\to 0^+} f(x) \nexists$, because it jumps from $y=-1$ to $y=1$, on ANY δ-nbhd of 0. So you can't fit it inside an ε-tube for any $\varepsilon < 1$, basically

(b) $\lim_{x\to 0^-} f(x) = 0$

(c) $\lim_{x\to 0} f(x) \nexists$. Left- & Right-sided limits do not agree.

201 §2.4 #5 7-29 odds, 33-39 odds, 47, 52

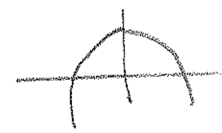
7 (a) Graph $f(x) = \begin{cases} x^3 & x \neq 1 \\ 0 & x = 1 \end{cases}$



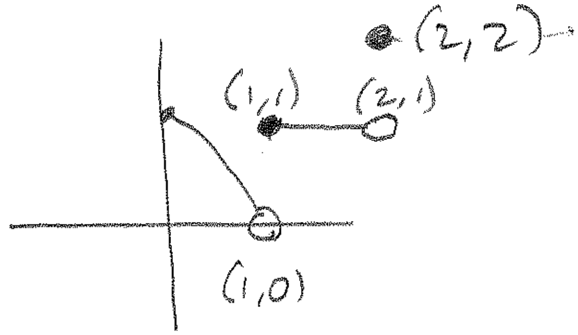
(b) $\lim_{x\to 1^-} f(x) = \lim_{x \to 1^+} f(x) = 1$

(c) ~~See~~ $\lim_{x\to 1} f(x) = 1$

9 $f(x) = \begin{cases} \sqrt{1-x^2} & 0 \le x < 1 \\ 1 & 1 \le x < 2 \\ 2 & x = 2 \end{cases}$



Graph:



(a) $\mathcal{D} = [0,2]$, $\mathcal{R} = (0,1] \cup \{2\}$

(b) For what c does $\lim_{x \to c} f(x)$ $\exists$?

$c \in (0,1) \cup (1,2)$

(c) Where does only left-side limit exist?

$c \in \{1, 2\}$

(d) ONLY Right-sided? $c \in \{0\}$

201 §2.4 #s 11–29 odds, 33–39 odds, 47, 52

#s 11–18 Find one-sided limits

(11) $\lim_{x \to -.5^-} \sqrt{\dfrac{x+2}{x+1}} = \sqrt{\dfrac{1.5}{.5}} = \boxed{\sqrt{3}}$

(13) $\lim_{x \to 2^+} \left(\dfrac{x}{x+1}\right)\left(\dfrac{2x+5}{x^2+x}\right) = \left(\dfrac{2}{3}\right)\left(\dfrac{9}{6}\right) = \boxed{1}$

(15) $\lim_{h \to 0^+} \dfrac{\sqrt{h^2+4h+5} - \sqrt{5}}{h} =$

$\lim_{h \to 0^+} \dfrac{\sqrt{h^2+4h+5} - \sqrt{5}}{h} \cdot \dfrac{\sqrt{h^2+4h+5} + \sqrt{5}}{\sqrt{h^2+4h+5} + \sqrt{5}}$

$= \lim_{h \to 0^+} \dfrac{h^2+4h+5-5}{h\left(\sqrt{h^2+4h+5} + \sqrt{5}\right)} = \lim_{h \to 0^+} \dfrac{h(h+4)}{h\left(\sqrt{h^2+4h+5} + \sqrt{5}\right)}$

$= \dfrac{4}{\sqrt{5}+\sqrt{5}} = \dfrac{4}{2\sqrt{5}} = \boxed{\dfrac{2}{\sqrt{5}}}$ OR $\dfrac{2\sqrt{5}}{5}$

(17) (a) $\lim_{x \to -2^+} (x+3) \dfrac{|x+2|}{x+2}$. When $x > -2$,

$|x+2| = (x+2)$, so it becomes

$\lim_{x \to -2^+} (x+3)(1) = \boxed{1}$

201 §2.4 #s 17–29 odds, 33–39 odds, 47, 52

17 (b) $\lim_{x \to -2^-} (x+3)\frac{|x+2|}{x+2} = \boxed{-1}$ (see part a)

Use graph of greatest integer function $y = \lfloor x \rfloor$ to find limits

19 (a) $\lim_{\theta \to 3^+} \frac{\lfloor \theta \rfloor}{\theta} = \lim_{\theta \to 3^+} \frac{3}{\theta} = \boxed{1}$

(b) $\lim_{\theta \to 3^-} \frac{\lfloor \theta \rfloor}{\theta} = \lim_{\theta \to 3^-} \frac{2}{\theta} = \boxed{\frac{2}{3}}$!

#s 21–42 Find limits

21 $\lim_{\theta \to 0} \frac{\sin\sqrt{2\theta}}{\sqrt{2\theta}} = 1$

23 $\lim_{y \to 0} \frac{\sin(3y)}{4y} = \lim_{y \to 0} \frac{3}{4} \frac{\sin(3y)}{(3y)} = \frac{3}{4}$

25 $\lim_{x \to 0} \frac{\tan(2x)}{x} = \lim_{x \to 0} \left(\frac{\sin(2x)}{x} \cdot \frac{1}{\cos(2x)} \right)$

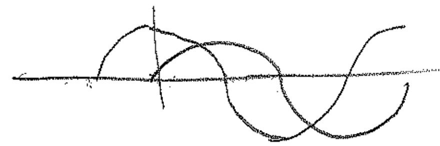
$= \lim_{x \to 0} \left(\frac{2\sin(2x)}{2x} \cdot \frac{1}{\cos(2x)} \right) = 2$

27 $\lim_{x \to 0} \frac{x\csc(2x)}{\cos(5x)} = \lim_{x \to 0} \frac{1}{\cos(5x)} \cdot \frac{2x}{\sin(2x)} \cdot \frac{1}{2} = \frac{1}{2}$

201 §2.4 #s 29–39 odds, 47, 52

(29) $$\frac{x + x\cos x}{\sin x \cos x} = \left(\frac{x}{\sin x}\right)\left(\frac{1+\cos x}{\cos x}\right)$$

$$\xrightarrow{x \to 0} (1)\left(\frac{2}{1}\right) = 2$$



(31) $$\lim_{\theta \to 0}\left(\frac{1-\cos\theta}{\sin(2\theta)}\right) = \lim_{\theta \to 0} \frac{(1-\cos\theta)(1+\cos\theta)}{(\sin\theta)(1+\cos\theta)}$$

$$= \lim_{\theta \to 0} \frac{1-\cos^2\theta}{\sin\theta\,(1+\cos\theta)} = \lim_{\theta \to 0} \frac{\sin^2\theta}{\sin\theta(1+\cos\theta)}$$

$$= \lim_{\theta \to 0} \frac{\sin\theta}{1+\cos\theta} = 0$$

(33) $$\lim_{t \to 0} \frac{\sin(1-\cos t)}{1-\cos t} = 1$$

(35) $$\lim_{\theta \to 0} \frac{\sin\theta}{\sin(2\theta)} = \lim_{\theta \to 0} \frac{\sin\theta}{\theta} \cdot \frac{2\theta}{\sin(2\theta)} \cdot \frac{1}{2} = \frac{1}{2}$$

(37) $$\lim_{\theta \to 0} \theta\cos\theta = 0$$

(39) $$\lim_{x \to 0} \frac{\tan(3x)}{\sin(8x)} = \lim_{x \to 0} \frac{\sin(3x)}{1} \cdot \frac{1}{\cos(3x)} \cdot \frac{1}{\sin(8x)}$$

$$= \lim_{x \to 0} \frac{\sin(3x)}{3x} \cdot \frac{1}{\cos(3x)} \cdot \frac{8x}{\sin(8x)} \cdot \frac{3}{8} = \frac{3}{8}$$

201 §2.4 #s 47, 52

(47) Let $\varepsilon>0$. Find $I=(5,5+\delta)$, $\delta>0$, $\ni$ $x \in I \Longrightarrow \sqrt{x-5} < \varepsilon$. What limit is being verified?

$$\sqrt{x-5} < \varepsilon$$

$$x-5 < \varepsilon^2$$

$$x < 5+\varepsilon^2$$

$\boxed{\text{Let } \delta = \varepsilon^2}$. Then $\boxed{I=(5,5+\varepsilon^2)}$ and we're verifying $\boxed{\lim_{x\to 5^+} \sqrt{x-5} = 0}$

(52) $$f(x)=\begin{cases} x^2\sin\left(\frac{1}{x}\right) & x<0 \\ \sqrt{x} & x>0 \end{cases}$$

(a) $\lim_{x\to 0^+} f(x) = \sqrt{0} = 0$

(b) $\lim_{x\to 0^-} f(x) = 0$

(c) By (a) & (b), we conclude that

$$\boxed{\lim_{x\to 0} f(x) = 0}$$