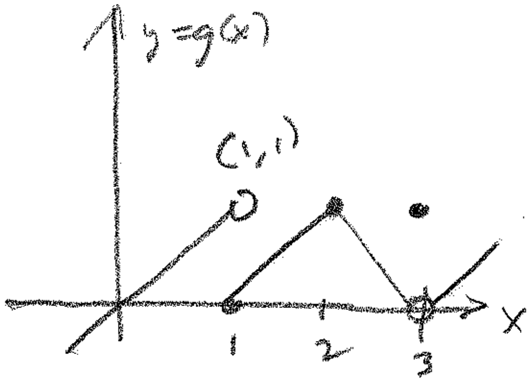
201 §2.2 #s 1, 3, 11, 12, 13, 15, 16, 19, 21–23, 25–53 ODDS, 57, 63, 65, 67, 71, 73, 77, 79

① Find the limits



(a) $\lim_{x \to 1} g(x)\ \nexists$

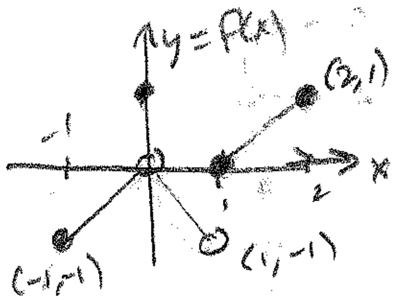
From left, it approaches $y = 1$,
From right, " " $y = 0$.

(b) $\lim_{x \to 2} g(x) = 1$

(c) $\lim_{x \to 3} g(x) = 0$

(d) $\lim_{x \to 2.5} g(x) = 0.5$

③ Which are true & which are false?



(a) $\lim_{x \to 0} f(x)\ \exists$ T

(b) $\lim_{x \to 0} f(x) = 0$ T

(c) $\lim_{x \to 0} f(x) = 1$ F

(d) $\lim_{x \to 1} f(x) = 1$ F

(e) $\lim_{x \to 1} f(x) = 0$ F

(f) $\lim_{x \to x_0} f(x)\ \exists\ \forall x \in (-1, 1)$ T

(g) $\lim_{x \to 1} f(x)\ \nexists$ T

T
T
F
F
F
T
T

#s 11–22 Find the limits.

201 § 2.2 #s 11–13, 15, 16, 19, 21–23, 25–53 odds, 57, 63, 65, 67, 71, 73, 77, 79

(11) $\lim_{x \to -7} (2x+5) = -14+5 = \boxed{-9}$

(12) $\lim_{x \to 2} (-x^2+5x-2) = -4+10-2 = \boxed{4}$

(13) $\lim_{t \to 6} (8)(t-5)(t-7) = 8(1)(-1) = \boxed{-8}$

(15) $\lim_{x \to 2} \frac{x+3}{x+6} = \boxed{\frac{5}{8}}$

(16) $\lim_{s \to \frac{2}{3}} 3s(2s-1) = 3\left(\frac{2}{3}\right)\left(2\left(\frac{2}{3}\right)-1\right)$

$= 2\left(\frac{4}{3}-1\right) = 2\left(\frac{1}{3}\right) = \boxed{\frac{2}{3}}$

(19) $\lim_{y \to -3} (5-y)^{\frac{4}{3}} = 8^{\frac{4}{3}} = 2^4 = \boxed{16}$

(21) $\lim_{h \to 0} \frac{3}{\sqrt{3h+1}+1} = \frac{3}{\sqrt{1}+1} = \boxed{\frac{3}{2}}$

(22) $\lim_{h \to 0} \frac{\sqrt{5h+4}-2}{h} = ?$

$\left(\frac{\sqrt{5h+4}-2}{h}\right)\left(\frac{\sqrt{5h+4}+2}{\sqrt{5h+4}+2}\right) = \frac{5h+4-4}{h(\sqrt{5h+4}+2)} = \frac{5h}{h(\sqrt{5h+4}+2)}$

$= \frac{5}{\sqrt{5h+4}+2} \xrightarrow{h \to 0} \frac{5}{\sqrt{4}+2} = \frac{5}{2+2} = \boxed{\frac{5}{4}}$

201 §2.2 #s 23, 25-53 ODDS, 57, 63, 65, 67, 71, 73, 77, 79

#23-42 - Find Limits

23 $\lim_{x \to 5} \frac{x-5}{x^2-25} = \lim_{x \to 5} \frac{x-5}{(x-5)(x+5)} = \lim_{x \to 5} \frac{1}{x+5} = \boxed{\frac{1}{10}}$

25 $\lim_{x \to -5} \frac{x^2+3x-10}{x+5} = \lim_{x \to -5} \frac{(x+5)(x-2)}{x+5} = \lim_{x \to -5} (x-2) = \boxed{-7}$

27 $\lim_{t \to 1} \frac{t^2+t-2}{t^2-1} = \lim_{t \to 1} \frac{(t+2)(t-1)}{(t+1)(t-1)} = \lim_{t \to 1} \frac{t+2}{t+1} = \boxed{\frac{3}{2}}$

29 $\lim_{x \to -2} \frac{-2x-4}{x^3+2x^2} = \lim_{x \to -2} \frac{-2(x+2)}{x^2(x+2)} = \lim_{x \to -2} \frac{-2}{x^2} = -\frac{2}{4} = \boxed{-\frac{1}{4}}$

31 $\lim_{x \to 1} \frac{\frac{1}{x}-1}{x-1} = \lim_{x \to 1} \frac{\frac{x-1}{x}}{x-1} = \lim_{x \to 1} \frac{x-1}{x(x-1)} = \lim_{x \to 1} \frac{1}{x} = \boxed{1}$

33 $\lim_{u \to 1} \frac{u^4-1}{u^3-1} = \lim_{u \to 1} \frac{(u^2-1)(u^2+1)}{(u-1)(u^2+u+1)} = \lim_{u \to 1} \frac{(u-1)(u+1)(u^2+1)}{(u-1)(u^2+u+1)}$

$= \lim_{u \to 1} \frac{(u+1)(u^2+1)}{u^2+u+1} = \frac{(2)(2)}{3} = \boxed{\frac{4}{3}}$

35 $\lim_{x \to 9} \frac{\sqrt{x}-3}{x-9} = \lim_{x \to 9} \frac{(\sqrt{x}-3)(\sqrt{x}+3)}{(x-9)(\sqrt{x}+3)} =$

$= \lim_{x \to 9} \frac{x-9}{(x-9)(\sqrt{x}+3)} = \lim_{x \to 9} \frac{1}{\sqrt{x}+3} = \boxed{\frac{1}{6}}$

201 §2.2 #s 37–53 ODDS, 57, 63, 65, 67, 71, 73, 77, 79

(37) $\lim_{x \to 1} \frac{x-1}{\sqrt{x+3}-2} = ?$

$$\left(\frac{x-1}{\sqrt{x+3}-2}\right)\left(\frac{\sqrt{x+3}+2}{\sqrt{x+3}+2}\right) = \frac{(x-1)(\sqrt{x+3}+2)}{x+3-4} = \frac{(x-1)(\sqrt{x+3}+2)}{x-1}$$

$$= \sqrt{x+3}+2 \xrightarrow{x \to 1} \boxed{4}$$

(39) $$\frac{\sqrt{x^2+12}-4}{x-2} = \frac{(\sqrt{x^2+12}-4)(\sqrt{x^2+12}+4)}{(x-2)(\sqrt{x^2+12}+4)}$$

$$= \frac{x^2+12-16}{(x-2)(\sqrt{x^2+12}+4)} = \frac{x^2-4}{(x-2)(\sqrt{x^2+12}+4)} = \frac{(x-2)(x+2)}{(x-2)(\sqrt{x^2+12}+4)}$$

$$= \frac{x+2}{\sqrt{x^2+12}+4} \xrightarrow{x \to 2} \frac{4}{\sqrt{16}+4} = \frac{4}{8} = \boxed{\frac{1}{2}}$$

(41) $$\frac{2-\sqrt{x^2-5}}{x+3} = \left(\frac{2-\sqrt{x^2-5}}{x+3}\right)\left(\frac{2+\sqrt{x^2-5}}{2+\sqrt{x^2-5}}\right)$$

$$= \frac{4-(x^2-5)}{(x+3)(2+\sqrt{x^2-5})} = \frac{9-x^2}{(x+3)(2+\sqrt{x^2-5})} = \frac{(3-x)(3+x)}{(x+3)(2+\sqrt{x^2-5})}$$

$$= \frac{3-x}{2+\sqrt{x^2-5}} \xrightarrow{x \to -3} \frac{6}{2+\sqrt{4}} = \frac{6}{4} = \boxed{\frac{3}{2}}$$

201 §2.2 #s 43–53 ODDS, 57, 63, 65, 67, 71, 73, 77, 79

(43) $\lim_{x\to 0} (2\sin x - 1) = \boxed{-1}$

(45) $\lim_{x\to 0} \sec x = \boxed{1}$

(47) $\lim_{x\to 0} \dfrac{1+x+\sin x}{3\cos x} = \boxed{\dfrac{1}{3}}$

(49) $\lim_{x\to -\pi} \sqrt{x+4}\,\cos(x+\pi) = \sqrt{4-\pi}\,\cos(0) = \boxed{\sqrt{4-\pi}}$

(51) (a) $\lim\left(\dfrac{p}{q}\right) = \dfrac{\lim p}{\lim q}$

(b) $\lim(p+q) = \lim p + \lim q$ and

$\lim p^n = (\lim p)^n$

(c) $\lim(p+q) = \lim p + \lim q$

$\lim_{x\to c} f(x) = f(c)$, when things go well.

(53) Given $\lim_{x\to c} f(x) = 5$, $\lim_{x\to c} g(x) = -2$. Then

(a) $\lim_{x\to c} f(x)g(x) = (5)(-2) = \boxed{-10}$

(b) $\lim_{x\to c} 2f(x)g(x) = 2(5)(-2) = \boxed{-20}$

(c) $\lim_{x\to c} (f(x)+3g(x)) = 5+3(-2) = \boxed{-1}$

(d) $\lim_{x\to c} \dfrac{f(x)}{f(x)-g(x)} = \dfrac{5}{5-(-2)} = \boxed{\dfrac{5}{7}}$

201 §2.2 #s 57, 63, 65, 67, 71, 73, 77, 79

#s 57-62 Evaluate $\lim\limits_{h\to 0} \frac{f(x+h)-f(x)}{h}$

57 $f(x) = x^2$, $x=1$ $\Longrightarrow$

$$\frac{f(1+h)-f(1)}{h} = \frac{(1+h)^2 - 1^2}{h} = \frac{1+2h+h^2-1}{h} = \frac{2h+h^2}{h}$$

$$= \frac{h(2+h)}{h} = 2+h \xrightarrow{h\to 0} \boxed{2}$$

63 $\wp$ $\sqrt{5-2x^2} \le f(x) \le \sqrt{5-x^2}$ $\forall\ x \in [-1, 1]$

Then $\boxed{\lim\limits_{x\to 0} f(x) = \sqrt{5}}$ by Sandwich.

65 $\wp$ $1 - \frac{x^2}{6} \le \frac{x \sin x}{2-2\cos x} \le 1$ $\forall\ x$ close to $x=0$.

Then $\lim\limits_{x\to 0}\left(1-\frac{x^2}{6}\right) \le \lim\limits_{x\to 0} \frac{x\sin x}{2-2\cos x} \le 1$

$\Longrightarrow$ $\boxed{\lim\limits_{x\to 0} \frac{x \sin x}{2-2\cos x} = 1}$ by Sandwich.

201 §2.2 #s 67, 71, 73, 77, 79

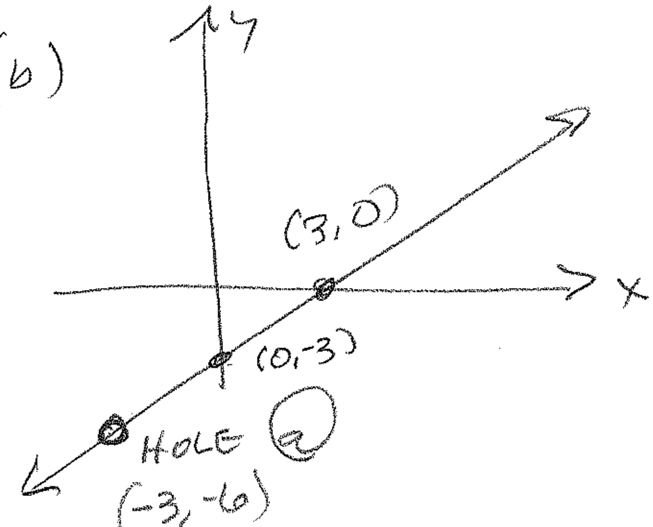
(67) $f(x) = \dfrac{x^2-9}{x+3}$.

(a) We tabulate $f(x)$ @ $x = -3.1, -3.01, -3.001, \ldots$ Then estimate $\lim_{x \to -3} f(x)$. What happens if you go at it from the left, w/ $x = -2.9, -2.99, -2.999$?

x	$f(x)$
-3.1	-6.1
-3.01	-6.01
-3.001	-6.001
-3.0001	-6.0001
-2.9	-5.9
-2.99	-5.99
-2.999	-5.999
-2.9999	-5.9999

$\boxed{\lim_{x \to -3} f(x) = -6}$ from either side.

(b)

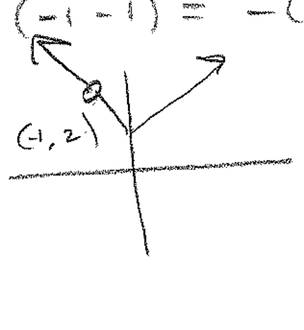


(71) $f(x) = \dfrac{x^2-1}{|x|-1}$ in the nbhd of $x = -1$

is given by $\dfrac{x^2-1}{-x-1} = \dfrac{(x+1)(x-1)}{-(x+1)} = -(x-1)$,

and ditching the busywork, I conclude that

$\lim_{x \to -1} f(x) = -(-1-1) = -(-2) = \boxed{2}$



201 §2.2 #s 73, 77, 79

(73) $g(\theta) = \frac{\sin\theta}{\theta}$ is even, since $\sin\theta$ & θ are odd.

we tabulate near zero

θ	$g(\theta)$
1	.84147
.5	.95885
.25	.98962
.1	.99833
.01	.99998
.001	.9999998333

(77) $\lim_{x\to 4} \frac{f(x)-5}{x-2} = 1$. Then

$$\frac{\lim_{x\to 4}(f(x)-5)}{\lim_{x\to 4}(x-2)} = 1 \Rightarrow \frac{\lim_{x\to 4} f(x)-5}{4-2} = 1$$

$$\Rightarrow \frac{\lim_{x\to 4} f(x) - 5}{2} = 1 \Rightarrow \lim_{x\to 4} f(x) - 5 = 2$$

$$\Rightarrow \boxed{\lim_{x\to 4} f(x) = 7}$$

(79) $\lim_{x\to 2}\left(\frac{f(x)-5}{x-2}\right) = 3 \xrightarrow{(a)} \lim_{x\to 2} f(x) = 5$

and $\lim_{x\to 2} \frac{f(x)-5}{x-2} = 4 \Rightarrow \lim_{x\to 2} f(x) = 5$!