

201 §2.1 #5 18, 20

18

$$F(x) = \frac{x+2}{x-2}, \quad x = 1.2, 1.1, 1.01, 1.001, 1.0001, 1$$

(a) Find  $m_{AVG}$  for  $F$  on  $[1, x]$ , using a table

$x$	$F(x)$	$m_{AVG}$ on $[1, x]$
1	-3	<del>3</del>
1.2	-4	-5
1.1	-3.444	-4.444
1.01	-3.04	-4.04
1.001	-3.004	-4.004
1.0001	-3	-4
1.00001	-3.0004	-4.0004

(b) By this work, we guess that the rate of change at  $x=1$  is  $f'(1) = -4$

20 Let  $f(t) = \frac{1}{t}$  for  $t \neq 0$

(a) Find AVG rate of change of  $f$  w.r.t.  $t$

over (i)  $[2, 3]$ :  $\frac{\frac{1}{3} - \frac{1}{2}}{3-2} = \frac{2-3}{6} = \boxed{-\frac{1}{6} = m_{AVG}}$

(ii)  $[2, T]$ :  $\frac{\frac{1}{T} - \frac{1}{2}}{T-2} = \frac{2-T}{2T(T-2)}$

$= \frac{-(T-2)}{2T(T-2)} = \boxed{-\frac{1}{2T} = m_{AVG}}$

20)  $S^2, 1 \#s 2, 6, 8, 14, 18, 20$

#s 1-6 Find average rate of change over the given interval.

(2)  $g(x) = x^2$

(a)  $[a, b] = [-1, 1] \Rightarrow \frac{g(b) - g(a)}{b - a} = \frac{g(1) - g(-1)}{1 - (-1)}$

$= \frac{1^2 - (-1)^2}{2} = \frac{0}{2} = \boxed{0} = m_{avg}$

(b)  $[a, b] = [-2, 0] \Rightarrow \frac{g(b) - g(a)}{b - a} = \frac{0^2 - (-2)^2}{0 - (-2)} = \frac{-4}{2} = \boxed{-2} = m_{avg}$

(6)  $f(\theta) = \theta^3 - 4\theta^2 + 5\theta$ ;  $[1, 2] \Rightarrow$

$\frac{f(2) - f(1)}{2 - 1} = \frac{2^3 - 4(2)^2 + 5(2) - (1^3 - 4(1)^2 + 5(1))}{1}$

$= 8 - 16 + 10 - (1 - 4 + 5) = 2 - (2) = \boxed{0} = m_{avg}$

#s 7-14 Use method of Example 3 to find

(a) the slope of the curve @ P and

(b) " eq'n of the tangent line @ P.

Sketch  $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = m$ , then

$y = m(x - a) + f(a)$

201 \$2.1 \neq 8, 14, 18, 20\$

8  $y = 5 - x^2 = f(x), P(1, 4)$

#8 (2)  $\frac{f(1+h) - f(1)}{h} = \frac{5 - (1+h)^2 - (5 - 1^2)}{h}$

$$= \frac{5 - (1 + 2h + h^2) - (5 - 1)}{h} = \frac{5 - 1 - 2h - h^2 - 4}{h}$$

$$= \frac{-2h - h^2}{h} = -2 - h \xrightarrow{h \rightarrow 0} \boxed{-2 = m_{\text{tan}}}$$

This is actually  $f'(1)$

(b)  $y = m(x - x_1) + f(x_1)$

$$y = -2(x - 1) + 4 = -2x + 2 + 4 = -2x + 6$$

In genl, for these:

$$L(x) = \text{Linearization} = f'(x_1)(x - x_1) + f(x_1)$$

is the tangent line equation.

14  $y = x^3 - 3x^2 + 4; P(2, 0)$

(a)  $\frac{f(2+h) - f(2)}{h} = \frac{(2+h)^3 - 3(2+h)^2 + 4 - (2^3 - 3(2)^2 + 4)}{h}$

$$= \frac{2^3 + 3 \cdot 2^2 \cdot h + 3 \cdot 2 \cdot h^2 + h^3 - 3(2^2 + 2 \cdot 2h + h^2) + 4 - (8 - 12 + 4)}{h}$$

$$= \frac{8 + 12h + 6h^2 + h^3 - 12 - 12h - 3h^2 + 4 - 0}{h}$$

$$= \frac{12h + 6h^2 + h^3 - 12h - 3h^2}{h} = \frac{h^3 + 3h^2}{h} = h^2 + 3h \xrightarrow{h \rightarrow 0} \boxed{0 = m}$$

(b)  $y = 0(x - 2) + 0 = 0; \text{i.e. } \boxed{y = 0}$