

Section 11.9 Power Series Representation

Recall the geometric series.

$$S = \sum_{k=1}^{\infty} x^{k-1} = 1 + x + x^2 + \dots + x^{k-1} + \dots$$

It's the limit of the geometric sum

$$S_n = \sum_{k=1}^n x^{k-1} = 1 + x + x^2 + \dots + x^{n-1} = \frac{1 - x^n}{1 - x}$$

$\rightarrow 0$
 in the
 limit
 if $|x| < 1$

And when x is kind enough to satisfy $|x| < 1$, we know that

$$S = \sum_{k=1}^{\infty} x^{k-1} = \frac{1}{1-x}$$

Now, we do something quite naïve, which is nonetheless quite powerful:

We can represent a whole *bunch* of different functions with a power series, now, any time we recognize

$$\frac{1}{1-x}$$

$$\boxed{1} \quad \frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots = \sum_{n=0}^{\infty} x^n \quad |x| < 1$$

$$\frac{1}{1+x^2} = \frac{1}{1-(-x^2)} = 1 + (-x^2) + (-x^2)^2 + (-x^2)^3$$
$$\frac{1}{1-u} = 1+u+u^2+\dots$$
$$= \boxed{1-x^2+x^4-x^6+\dots}$$

Super-cool thing about this is that we can integrate and differentiate these power series, term by term!

And all we need to know is the *power rule for derivatives* to do it!

Recall the anti-derivative of $\frac{1}{1+x^2}$

$$\int \frac{1}{1+x^2} dx = \arctan(x) + C$$

$$\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + \dots$$

$$\int \frac{1}{1+x^2} dx = \int (1 - x^2 + x^4 - x^6 + \dots) dx$$

$$= C + x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \dots = \arctan(x)$$

$$\text{Since } \arctan(0) = 0$$

$$C + 0 - 0 + 0 - 0 = 0 \Rightarrow C = 0, \text{ too!}$$

$$\arctan(x) = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \dots$$

2 Theorem If the power series $\sum c_n(x-a)^n$ has radius of convergence $R > 0$, then the function f defined by

$$f(x) = c_0 + c_1(x-a) + c_2(x-a)^2 + \dots = \sum_{n=0}^{\infty} c_n(x-a)^n$$

is differentiable (and therefore continuous) on the interval $(a-R, a+R)$ and

$$(i) f'(x) = c_1 + 2c_2(x-a) + 3c_3(x-a)^2 + \dots = \sum_{n=1}^{\infty} nc_n(x-a)^{n-1}$$

$$(ii) \int f(x) dx = C + c_0(x-a) + c_1 \frac{(x-a)^2}{2} + c_2 \frac{(x-a)^3}{3} + \dots$$

$$= C + \sum_{n=0}^{\infty} c_n \frac{(x-a)^{n+1}}{n+1}$$

The radii of convergence of the power series in Equations (i) and (ii) are both R .

This is really cool. But you'll still want to check the endpoints, to be sure of the *interval* of convergence.

$$\sin(x) = x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7 + \frac{1}{362880}x^9 - \dots$$

$$= x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \frac{1}{9!}x^9 - \dots \implies$$

$$\frac{d}{dx} [\sin x] = \cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \dots$$

$$7 \left(\frac{1}{7!}\right)x^6$$

$$5 \left(\frac{1}{5!}\right)x^4 = \frac{1}{4!}x^4$$

$$\frac{7}{7!} = \frac{7}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} = \frac{1}{6!}$$

$$J_0(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n}(n!)^2}$$

$$J_0'(x) = \sum_{n=0}^{\infty} \frac{d}{dx} \frac{(-1)^n x^{2n}}{2^{2n}(n!)^2} = \sum_{n=1}^{\infty} \frac{(-1)^n 2n x^{2n-1}}{2^{2n}(n!)^2}$$

1. If the radius of convergence of the power series $\sum_{n=0}^{\infty} c_n x^n$ is 10, what is the radius of convergence of the series $\sum_{n=1}^{\infty} n c_n x^{n-1}$? Why?

R = 10, Derivatives have same R

2. Suppose you know that the series $\sum_{n=0}^{\infty} b_n x^n$ converges for $|x| < 2$. What can you say about the following series? Why?

$$\sum_{n=0}^{\infty} \frac{b_n}{n+1} x^{n+1}$$

*Converges for $|x| < 2$
Not sure about $x = \pm 2$
Antiderivs have same R*

1-10 Find a power series representation for the function and determine the interval of convergence.

(I.e., "How much mileage can they get out of this geometric series thing?" It turns out, quite a *lot*, when you see some of what we can get away with!)

$$3. f(x) = \frac{1}{1+x} = \frac{1}{1-(-x)}$$

$$= 1 - x + x^2 - x^3 + \dots$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$$

$$4. f(x) = \frac{5}{1-4x^2} = 5 \left[\frac{1}{1-4x^2} \right] = 5 \left[1 + 4x^2 + (4x^2)^2 + (4x^2)^3 + \dots \right]$$

$$= 5 \left[1 + 4x^2 + 16x^4 + 64x^6 + \dots \right]$$

Let's make sure

$$\text{convert} \left(\frac{1}{1-4x^2}, \text{parfrac} \right) = \frac{1}{2(2x+1)} - \frac{1}{2(2x-1)} = \frac{1}{2} \left[\frac{1}{1+2x} + \frac{1}{1-2x} \right]$$

$$\frac{1}{2} \left[\frac{1}{1+2x} \right] = \frac{1}{2} \left[\frac{1}{1-(-2x)} \right]$$

$$= \frac{1}{2} \left[1 - (2x) + (2x)^2 - (2x)^3 + (2x)^4 - (2x)^5 + (2x)^6 + \dots \right]$$

$$= \frac{1}{2} \left[1 - 2x + 4x^2 - 8x^3 + 16x^4 - 32x^5 + 64x^6 + \dots \right]$$

$$\frac{1}{2} \left[\frac{1}{1-2x} \right] = \frac{1}{2} \left[1 + (2x) + (2x)^2 + (2x)^3 + (2x)^4 + (2x)^5 + (2x)^6 + \dots \right]$$

$$= \frac{1}{2} \left[1 + 2x + 4x^2 + 8x^3 + 16x^4 + 32x^5 + 64x^6 + \dots \right]$$

Add them:

$$\frac{1}{2} \left[1 - 2x + 4x^2 - 8x^3 + 16x^4 - 32x^5 + 64x^6 + \dots \right]$$

$$+ \frac{1}{2} \left[1 + 2x + 4x^2 + 8x^3 + 16x^4 + 32x^5 + 64x^6 + \dots \right]$$

$$\frac{1}{2} \left[2 \quad + 8x^2 \quad + 32x^4 \quad + 128x^6 + \dots \right]$$

$$= 1 \quad + 4x^2 \quad + 16x^4 \quad + 64x^6 + \dots$$

Previous work

$$\left[1 + 4x^2 + 16x^4 + 64x^6 + \dots \right]$$

Sweet! I CAN play that game.

$$\begin{aligned}
 \text{5. } f(x) = \frac{2}{3-x} &= \frac{2}{3(1-\frac{x}{3})} = \frac{2}{3} \left[\frac{1}{1-\frac{x}{3}} \right] = \frac{2}{3} \left[1 + \frac{1}{3}x + \left(\frac{1}{3}\right)^2 x^2 + \left(\frac{1}{3}\right)^3 x^3 + \dots \right] \\
 &= \frac{2}{3} \left[1 + \frac{1}{3}x + \frac{1}{9}x^2 + \frac{1}{27}x^3 + \dots \right]
 \end{aligned}$$

$$\begin{aligned}
 \text{6. } f(x) = \frac{1}{x+10} &= \frac{1}{10+x} = \frac{1}{10} \left[\frac{1}{1+\frac{x}{10}} \right] = \frac{1}{10} \left[\frac{1}{1-\left(-\frac{x}{10}\right)} \right] \\
 &= \frac{1}{10} \left[1 - \frac{1}{10}x + \frac{1}{100}x^2 - \frac{1}{1000}x^3 + \frac{1}{10^4}x^4 + \dots \right]
 \end{aligned}$$

$$\begin{aligned}
 7. f(x) &= \frac{x}{9+x^2} = \frac{x}{9\left(1+\frac{x^2}{9}\right)} = \frac{x}{9} \left[\frac{1}{1+\frac{x^2}{9}} \right] = \frac{x}{9} \left[\frac{1}{1-\left(-\frac{x^2}{9}\right)} \right] \\
 &= \frac{x}{9} \left[1 - \frac{x^2}{9} + \left(\frac{x^2}{9}\right)^2 - \left(\frac{x^2}{9}\right)^3 + \left(\frac{x^2}{9}\right)^4 + \dots \right] \\
 &= \frac{1}{9}x - \frac{1}{9^2}x^3 + \frac{x^5}{9^3} - \frac{x^7}{9^4} + \frac{x^9}{9^5} - \dots
 \end{aligned}$$

$$\begin{aligned}
 8. f(x) &= \frac{x}{2x^2+1} = \frac{x}{1-(-2x^2)} = x \left[\frac{1}{1-(-2x^2)} \right] = x \left[1 - 2x^2 + (2x^2)^2 - (2x^2)^3 + (2x^2)^4 - \dots \right] \\
 &= x - 2x^3 + 2^2x^5 - 2^3x^7 + 2^4x^9 - \dots
 \end{aligned}$$

$$\begin{aligned}
 \underline{9. f(x) = \frac{1+x}{1-x}} &= \frac{1}{1-x} + \frac{x}{1-x} \\
 &= 1 + x + x^2 + x^3 + x^4 + \dots \\
 &+ \quad x + x^2 + x^3 + x^4 + \dots \\
 \hline
 &1 + 2x + 2x^2 + 2x^3 + 2x^4 + \dots
 \end{aligned}$$

$$\begin{aligned}
 \underline{10. f(x) = \frac{x^2}{a^3 - x^3}} &= \frac{x^2}{a^3} \left[\frac{1}{1 - \frac{x^3}{a^3}} \right] = \frac{x^2}{a^3} \left[1 + \frac{x^3}{a^3} + \left(\frac{x^3}{a^3} \right)^2 + \left(\frac{x^3}{a^3} \right)^3 + \dots \right] \\
 &= \frac{x^2}{a^3} + \frac{x^5}{a^6} + \frac{x^8}{a^9} + \frac{x^{11}}{a^{12}} + \frac{x^{14}}{a^{15}} + \dots
 \end{aligned}$$

11-12 Express the function as the sum of a power series by first using partial fractions. Find the interval of convergence. $\frac{1}{1-x}$

11. $f(x) = \frac{3}{x^2 - x - 2}$ *convert* $\left(\frac{3}{x^2 - x - 2}, \text{parfrac}\right) = \frac{1}{x-2} - \frac{1}{x+1}$

you do this one the rest of the way, if you need more practice.

12. $f(x) = \frac{x+2}{2x^2 - x - 1} = \text{convert}\left(\frac{(x+2)}{2x^2 - x - 1}, \text{parfrac}\right) = \frac{1}{x-1} - \frac{1}{2x+1}$

$= -\frac{1}{1-x} - \frac{1}{1-(2x)}$

$= -\left[1 + x + x^2 + x^3 + \dots\right]$

$+ -\left[1 - (2x) + (2x)^2 - (2x)^3 + \dots\right]$

$= -\left[2 - x + 5x^2 - 7x^3 + \dots\right]$

$-2 + x - 5x^2 + 7x^3 + \dots$

13. (a) Use differentiation to find a power series representation for

$$f(x) = \frac{1}{(1+x)^2} = (1+x)^{-2}$$

What is the radius of convergence?

Maybe I got this one wrong, but I think this calls for ~~anti~~-differentiation.

$$\frac{d}{dx} \left[\frac{1}{1+x} \right] = \frac{d}{dx} \left[(1+x)^{-1} \right] = -1(1+x)^{-2} = -f(x)$$

$$\frac{1}{1+x} = \frac{1}{1-(-x)} = 1 - x + x^2 - x^3 + x^4 - \dots$$

$$\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{(n+1)x^n}{n x^{n-1}} \right|$$

$$n \rightarrow \infty \Rightarrow |x| < 1 = R$$

$$-\frac{d}{dx} \left[\frac{1}{1+x} \right] = - \left[-1 + 2x - 3x^2 + 4x^3 - \dots \right]$$

$$= \boxed{1 - 2x + 3x^2 - 4x^3 + \dots} = \sum_{n=1}^{\infty} (-1)^{n-1} n x^{n-1}$$

Book:

$$\sum_{n=0}^{\infty} (-1)^n (n+1) x^n$$

Same

$$\frac{d}{dx} \left[(1+x)^{-2} \right] = -2(1+x)^{-3} = -2 \left[\frac{1}{(1+x)^3} \right]$$

(b) Use part (a) to find a power series for

$$f(x) = \frac{1}{(1+x)^3}$$

Book:

$$\sum_{n=0}^{\infty} (-1)^n (n+1)(n+2) x^n$$

(c) Use part (b) to find a power series for

$$f(x) = \frac{x^2}{(1+x)^3}$$

$$= \frac{1}{2} \sum_{n=1}^{\infty} (-1)^{n-1} n(n+1) x^{n-1}$$

$$\Rightarrow \frac{1}{(1+x)^3} = -\frac{1}{2} \frac{d}{dx} \left[\frac{1}{(1+x)^2} \right]$$

$$= -\frac{1}{2} \frac{d}{dx} \left[\sum_{n=1}^{\infty} (-1)^{n-1} n x^{n-1} \right] = -\frac{1}{2} \sum_{n=1}^{\infty} (-1)^{n-1} n(n-1) x^{n-2}$$

1st term is zero
n=1: (1-1)x

$$= \sum_{n=1}^{\infty} (-1)^n \frac{1}{2} n(n-1) x^{n-2}$$

$$= \frac{1}{2} \sum_{n=2}^{\infty} (-1)^n n(n-1) x^{n-2}$$

(b) ANS

(c) $\frac{x^2}{(1+x)^3} = \frac{1}{2} \sum_{n=1}^{\infty} (-1)^{n-1} n(n+1) x^{n+1}$

$$\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{(n+1)(n+2) x^{n+2}}{n(n+1) x^{n+1}} \right|$$

$$n \rightarrow \infty \Rightarrow |x| < 1 = R$$

$$\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{(n+1)(n+2) x^n}{n(n+1) x^{n-1}} \right|$$

$$n \rightarrow \infty \Rightarrow |x| < 1 = R$$



14. (a) Use Equation 1 to find a power series representation for $f(x) = \ln(1-x)$. What is the radius of convergence?
 (b) Use part (a) to find a power series for $f(x) = x \ln(1-x)$.
 (c) By putting $x = \frac{1}{2}$ in your result from part (a), express $\ln 2$ as the sum of an infinite series.

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$$

$$\frac{d}{dx} [\ln(1-x)] = \frac{-1}{1-x}$$

$$\ln(1-x) = - \int \frac{1}{1-x} dx = - \int (1 + x + x^2 + x^3 + \dots) dx$$

$$= - \left[x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \dots \right] + C$$

$$\ln(1-0) = \ln(1) = 0 = - [0 + 0 + \dots] + C \Rightarrow C = 0$$

$$\Rightarrow \boxed{\ln(1-x) = -x - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \frac{1}{4}x^4 - \dots}$$

$$(b) \Rightarrow x \ln(1-x) = -x^2 - \frac{1}{2}x^3 - \frac{1}{3}x^4 - \frac{1}{4}x^5 - \dots$$

$$(c) \quad x = \frac{1}{2} \Rightarrow \ln(1-x) = \ln\left(1 - \frac{1}{2}\right) = \ln\left(\frac{1}{2}\right) = -\ln 2$$

$$\ln(2) = - \left[-\frac{1}{2} - \frac{1}{2} \left(\frac{1}{2}\right)^2 - \frac{1}{3} \left(\frac{1}{2}\right)^3 - \frac{1}{4} \left(\frac{1}{2}\right)^4 - \dots \right]$$

$$= \frac{1}{2} + \frac{1}{2} \left(\frac{1}{2}\right)^2 + \frac{1}{3} \left(\frac{1}{2}\right)^3 + \frac{1}{4} \left(\frac{1}{2}\right)^4 + \dots$$

$$= \sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{1}{2}\right)^n$$

15-20 Find a power series representation for the function and determine the radius of convergence.

15. $f(x) = \ln(5-x) = \ln\left(5\left(1-\frac{x}{5}\right)\right) = \ln(5) + \ln\left(1-\frac{x}{5}\right)$

$$\ln(1-x) = -x - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \frac{1}{4}x^4 - \dots$$

$$\ln\left(1-\frac{x}{5}\right) = -\frac{x}{5} - \frac{1}{2}\left(\frac{x}{5}\right)^2 - \frac{1}{3}\left(\frac{x}{5}\right)^3 - \frac{1}{4}\left(\frac{x}{5}\right)^4 - \dots$$

$$\Rightarrow \ln\left(1-\frac{x}{5}\right) = \ln(5) - \frac{x}{5} - \frac{1}{2}\left(\frac{x}{5}\right)^2 - \frac{1}{3}\left(\frac{x}{5}\right)^3 - \frac{1}{4}\left(\frac{x}{5}\right)^4 - \dots = \ln(5) - \sum_{n=1}^{\infty} \frac{1}{n} \frac{x^n}{5^n}$$

Radius of Convergence:

$$\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{\frac{x^{n+1}}{(n+1)5^{n+1}} \cdot \frac{n \cdot 5^n}{x^n}}{\frac{x^n}{n \cdot 5^n}} \right| = \left| \frac{n \cdot x}{(n+1) \cdot 5} \right| \xrightarrow{n \rightarrow \infty} \frac{|x|}{5} < 1$$

$$\Rightarrow \boxed{|x| < 5 = R}$$

16. $f(x) = x^2 \tan^{-1}(x^3)$

$$\arctan(x) = 1 - x^2 + x^4 - x^6 + \dots = \sum_{n=0}^{\infty} (-1)^n x^{2n}$$

$$x^2 \tan^{-1}(x^3) = x^2 \sum_{n=0}^{\infty} (-1)^n (x^3)^{2n} = x^2 \sum_{n=0}^{\infty} (-1)^n x^{6n}$$

$$= \sum_{n=0}^{\infty} (-1)^n x^{6n+2}$$

$$\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{x^{6(n+1)+2}}{x^{6n+2}} \right| = \left| \frac{x^{6n+8}}{x^{6n+2}} \right| = |x^6| = x^6 < 1 \Rightarrow \boxed{|x| < 1 = R}$$

$$\begin{aligned}
 17. f(x) &= \frac{x}{(1+4x)^2} & \frac{1}{(1+x)^2} &= 1 - 2x + 3x^2 - 4x^3 + \dots = \sum_{n=1}^{\infty} (-1)^{n-1} n x^{n-1} \\
 & & & \text{by \#13} & & = \sum_{n=0}^{\infty} (-1)^n (n+1) x^n \\
 &= x \sum_{n=1}^{\infty} (-1)^{n+1} n (4x)^{n-1} & & & & \\
 &= \boxed{\sum_{n=1}^{\infty} (-1)^{n+1} n \cdot 4^{n-1} x^n} & \left| \frac{a_{n+1}}{a_n} \right| &= \left| \frac{(n+1)4^n x^{n+1}}{n 4^{n-1} x^n} \right| \\
 & & & = \frac{n+1}{n} |4x| \xrightarrow{n \rightarrow \infty} 4|x| < 1 \\
 & & & & & \boxed{|x| < \frac{1}{4} = R}
 \end{aligned}$$

$$18. f(x) = \left(\frac{x}{2-x}\right)^3 = \frac{x^3}{(2-x)^3} = \frac{x^3}{2^3 \left(1 - \frac{x}{2}\right)^3} = \frac{x^3}{2^3} \left(\frac{1}{1 - \frac{x}{2}}\right)^3$$

$$\frac{1}{(1+x)^3} = \frac{1}{2} \sum_{n=1}^{\infty} (-1)^{n-1} (n+1)n x^{n-1} \implies$$

$$\frac{x^3}{2^3} \left(\frac{1}{\left(1 - \frac{x}{2}\right)^3}\right) = \left(\frac{x^3}{2^3}\right) \left(\frac{1}{2}\right) \sum_{n=1}^{\infty} (-1)^{n-1} n(n+1) \left(\frac{x}{2}\right)^{n-1}$$

$$= \frac{x^3}{2^4} \sum_{n=1}^{\infty} (-1)^{n-1} n(n+1) \frac{1}{2^{n-1}} x^{n-1}$$

$$= \boxed{\sum_{n=1}^{\infty} (-1)^{n-1} n(n+1) \frac{1}{2^{n+1}} x^{n+2}}$$

$$\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{(n+1)(n+2) x^{n+3}}{2^{n+2}} \cdot \frac{2^{n+1}}{n(n+1) x^{n+2}} \right|$$

$$\xrightarrow{n \rightarrow \infty} \frac{|x|}{2} < 1 \implies$$

$$|x| < \boxed{2 = R}$$

33. Use the result of Example 7 to compute arctan 0.2 correct to five decimal places.

$$\begin{aligned} \tan^{-1}x &= \int \frac{1}{1+x^2} dx = \int (1 - x^2 + x^4 - x^6 + \dots) dx \\ &= C + x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots \end{aligned}$$

Put $x = 0$ and obtain $C = \tan^{-1}0 = 0$. Therefore

$$\tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$$

$$\tan^{-1}(0.2) = \sum_{n=0}^{\infty} (-1)^n \frac{0.2^{2n+1}}{2n+1}$$

This is an alternating series. I prefer not to have a “specified decimal-place accuracy,” but would prefer to make it an *easy* question, and more *practical*, by specifying a bound on the allowed *error*, because in real life, that’s more likely how it’ll be put to you. “How far off can we afford to be, and make sure your error is smaller than that amount.

These specified decimal-place questions, I’d just as soon open up a spreadsheet and see when the 5th or 4th or whatever decimal place settles down.

See 11-9-spreadsheet.

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	B	C	D	E
n		$\frac{(-1)^k \cdot 2^{2k+1}}{2k+1}$	$\sum_{k=0}^n (-1)^k \frac{0.2^{2k+1}}{2k+1}$	
0		0.2	0.2	
1		-0.002666667	0.197333333	
2		0.000064	0.197397333	
3		-1.82857E-06	0.197395505	
4		5.68889E-08	0.197395562	
5		-1.86182E-09	0.19739556	
6		6.30154E-11	0.19739556	
7		-2.18453E-12	0.19739556	
8		7.71012E-14	0.19739556	
9		-2.75941E-15	0.19739556	
10		9.98644E-17	0.19739556	
11		-3.64722E-18	0.19739556	
12		1.34218E-19	0.19739556	
13		-4.97103E-21	0.19739556	
14		1.85128E-22	0.19739556	
15		-6.92737E-24	0.19739556	
16		2.60301E-25	0.19739556	
17		-9.81707E-27	0.19739556	
18				
19				
20				