

Section 11.8: Power Series

A **power series** is a series of the form

$$\boxed{1} \quad \sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots$$

where x is a variable and the c_n 's are constants called the **coefficients** of the series.

For each fixed x , the series (1) is a series of constants that we can test for convergence or divergence.

A power series may converge for some values of x and diverge for other values of x .

Section 11.8 Power Series

11.8 #s 1 - 28 (Quick 'n' dirty. No need to be fancy, other than for future reference.)
I may opt out of some redundant exercises, as I work through them.

The sum of the series is a function

$$f(x) = c_0 + c_1x + c_2x^2 + \dots + c_nx^n + \dots$$

whose domain is the set of all x for which the series converges. Notice that f resembles a polynomial. The only difference is that f has infinitely many terms.

For instance, if we take $c_n = 1$ for all n , the power series becomes the geometric series

$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \dots + x^n + \dots$$

which converges when $-1 < x < 1$ and diverges when $|x| \geq 1$.

Generally, what we'll be doing is applying the ratio test, trying to make

$$\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{c_{n+1}x^{n+1}}{c_nx^n} \right| = \left| \frac{c_{n+1}}{c_n} \right| |x| < 1 \qquad \sum a_n = \sum c_n x^n$$

Hopefully, there's a nice formula for the c_n 's!

$3 |x| < 1$
 $|x| < \frac{1}{3}$
 $-\frac{1}{3} < x < \frac{1}{3}$

$(-\frac{1}{3}, \frac{1}{3}) =$
interval solution.


Since the c_n 's are just numbers, the inequality is a basic absolute-value inequality, we learn to solve in MAT 121.

Solutions to these absolute value inequalities are open intervals. The *radius* of these intervals (all centered at $x = 0$) is called the **Radius of Convergence** R .

The *only* wrinkle new to Calculus II is checking the endpoints of the interval solutions to the inequality. We need to do that, because the Ratio Test is inconclusive, when

$$\left| \frac{a_{n+1}}{a_n} \right| = 1$$

$$\sum_{n=1}^{\infty} \frac{x^n}{n} = \sum_{n=1}^{\infty} \left(\frac{1}{n} x^n \right)$$

$$\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{\frac{x^{n+1}}{n+1}}{\frac{x^n}{n}} \right| = \left| \left(\frac{x^{n+1}}{n+1} \right) \left(\frac{n}{x^n} \right) \right| = \left| \frac{n}{n+1} \right| |x| \quad \text{Want } < 1$$

Want to pass to the limit, here:

$$\frac{n}{n+1} |x| \xrightarrow{n \rightarrow \infty} |x| < 1 \rightarrow R=1$$

$$x < 1 \quad \text{AND} \quad x > -1$$

$$-1 < x < 1$$

Radius of convergence:

$$R=1$$

c So $x \in (-1, 1)$ for sure. Inconclusive (a)

$$x = \pm 1, \text{ where } |x| = 1$$

check endpoints

$$\sum \frac{1}{n} x^n \quad \text{(a) } x=1 \text{ is } \sum \frac{1}{n} \cdot 1^n = \sum \frac{1}{n} \text{ Diverges.}$$

$$\sum \quad \text{(a) } x=-1 \text{ is } \sum \frac{1}{n} (-1)^n =$$

= CONVERGENT ALTERNATING

HARMONIC Series, so

INTERVAL of CONVERGENCE is

$$\boxed{I = [-1, 1)}$$

	Series	Radius of convergence	Interval of convergence
Geometric series	$\sum_{n=0}^{\infty} x^n$	$R = 1$	$I = (-1, 1)$

① $x = 1$

$$\sum_{n=0}^{\infty} 1^n = 1 + 1 + 1 + \dots$$

$$\sum_{n=0}^{\infty} (-1)^n = 1 - 1 + 1 - 1 + \dots$$

Sum oscillates between 1 & 0

$$S_1 = 1$$

$$S_2 = 0$$

$$S_3 = 1$$

$$S_n = \begin{cases} 1 & \text{if } n \text{ is odd } (n=2k+1) \\ 0 & \text{if } n \text{ is even} \end{cases}$$

Series	Radius of convergence	Interval of convergence
$\sum_{n=0}^{\infty} n! x^n$	$R = 0$	$\{0\}$ = Degenerate interval = single point.

$$0^0 = 1?$$

$$\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{(n+1)! x^{n+1}}{n! x^n} \right|$$

$$= (n+1)|x| \xrightarrow{n \rightarrow \infty} \infty \quad ; \text{if } x \neq 0.$$

Bessel Function

Series	Radius of convergence	Interval of convergence
$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n} (n!)^2}$	$R = \infty$	$(-\infty, \infty)$

$$\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{\frac{x^{2(n+1)}}{2^{2(n+1)} ((n+1)!)^2}}{\frac{x^{2n}}{2^{2n} (n!)^2}} \right| = \frac{x^{2n+2}}{2^{2n+2} ((n+1)!)^2} \cdot \frac{2^{2n} (n!)^2}{x^{2n}}$$

$$\left| \frac{\cancel{(n!)^2} x^2}{2^2 (n+1)^2 \cancel{(n!)^2}} \right| = \frac{x^2}{4(n+1)^2} \xrightarrow{n \rightarrow \infty} 0$$

$$R = \infty$$

$$I = (-\infty, \infty)$$

What value of x will ensure that $0 < 1$?

ANS: ANY DABBONE x !

More generally, a series of the form

$$\boxed{2} \quad \sum_{n=0}^{\infty} c_n(x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \cdots$$

is called a **power series in $(x-a)$** or a **power series centered at a** or a **power series about a** .

Notice that in writing out the term corresponding to $n=0$ in Equations 1 and 2 we have adopted the convention that $(x-a)^0 = 1$ even when $x=a$.

Notice also that when $x=a$ all of the terms are 0 for $n \geq 1$ and so the power series (2) always converges when $x=a$.

Series	Radius of convergence	Interval of convergence
$\sum_{n=1}^{\infty} \frac{(x-3)^n}{n}$	centered @ $x=3$ $R=1$	$[2, 4)$

is Right shift by 3 units of $\sum \frac{x^n}{n}$

$\sum_{n=1}^{\infty} \frac{x^n}{n}$

$R=1$ $[-1, 1)$

centered @ $x=0$

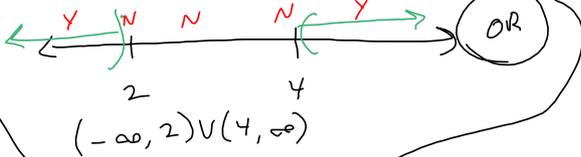
$$\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{(x-3)^{n+1}}{n+1} \cdot \frac{n}{(x-3)^n} \right| = \frac{n}{n+1} |x-3|$$

NOTE

$|x-3| > 1$

$x-3 > 1$ OR $x-3 < -1$

$\{x \mid x > 4 \text{ OR } x < 2\}$



College Algebra review $|A| > B$

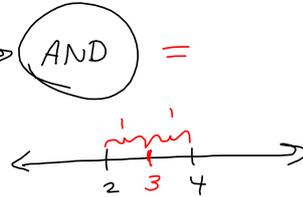
$n \rightarrow \infty \rightarrow |x-3| < 1$

$x-3 < 1$ and $x-3 > -1$

$|A| < B \Rightarrow A < B \text{ AND } A > -B$

$-1 < x-3 < 1$

$\{x \mid x < 4 \text{ and } x > 2\} =$



$x=2:$

$\sum \frac{(2-3)^n}{n} = \sum \frac{(-1)^n}{n}$ conv.

$x=4 = \sum \frac{(4-3)^n}{n} = \sum \frac{1}{n}$ Diverges

$\therefore I = [2, 4)$

3 Theorem For a given power series $\sum_{n=0}^{\infty} c_n(x-a)^n$ there are only three possibilities:

- (i) The series converges only when $x = a$. $\sum n! (x-a)^n$
- (ii) The series converges for all x . Bessel
- (iii) There is a positive number R such that the series converges if $|x-a| < R$ and diverges if $|x-a| > R$. $\sum \frac{x^n}{n}$

1. What is a power series? $\sum_{n=0}^{\infty} c_n x^n = \sum_{n=0}^{\infty} a_n$

2. (a) What is the radius of convergence of a power series? It's the thing that keeps
How do you find it?

(b) What is the interval of convergence of a power series?

How do you find it?

$$= \bar{x} \times \left\{ \sum c_n x^n \text{ converges} \right\}$$

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| < 1 \leftarrow$$

check endpoints of solution to

3-28 Find the radius of convergence and interval of convergence of the series.

3. $\sum_{n=1}^{\infty} (-1)^n n x^n$

$$\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{(n+1) x^{n+1}}{n x^n} \right| = \frac{n+1}{n} |x| \xrightarrow{n \rightarrow \infty} |x| < 1 \Rightarrow \text{Want } R=1$$

$x \in (-1, 1)$

$x = -1 \quad \sum (-1)^n n (-1)^n = \sum n \quad \text{Nope}$

$x = 1 \quad \sum (-1)^n n (1)^n = \sum (-1)^n n \quad \text{Nope}$

$\bar{I} = (-1, 1)$

4. $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{\sqrt[3]{n}}$

$$\left| \frac{a_{n+1}}{a_n} \right| = \frac{x^{n+1}}{\sqrt[3]{n+1}} \cdot \frac{\sqrt[3]{n}}{x^n} = \frac{\sqrt[3]{n}}{\sqrt[3]{n+1}} |x| \xrightarrow{n \rightarrow \infty} |x| < 1 \Rightarrow \text{Want } R=1$$

$x \in (-1, 1)$

$x = -1 \quad \sum \frac{(-1)^n (-1)^n}{\sqrt[3]{n}} = \sum \frac{1}{n^{1/3}} \quad \text{p-test. Nope}$

$x = 1 \quad \sum \frac{(-1)^n}{\sqrt[3]{n}} \quad \text{Yes Alternating Convergent.}$

$\frac{1}{\sqrt[3]{n}} \text{ dec. } \checkmark$

$\frac{1}{\sqrt[3]{n+1}} < \frac{1}{\sqrt[3]{n}} \checkmark \xrightarrow{n \rightarrow \infty} \infty$

$R=1$
 $I = (-1, 1]$

5. $\sum_{n=1}^{\infty} \frac{x^n}{2n-1}$

$$\left| \frac{a_{n+1}}{a_n} \right| = \frac{x^{n+1}}{2(n+1)-1} \cdot \frac{2n-1}{x^n} = \frac{2n-1}{2n+1} |x| \xrightarrow{n \rightarrow \infty} |x| < 1 \Rightarrow R=1$$

$x = -1 : \sum \frac{(-1)^n}{2n-1} \quad \text{Alternating harmonic limit comparison } 2n-1 \quad \text{Converges Yes}$

$x = 1 \quad \sum \frac{1}{2n-1} \text{ Diverges NO}$

$R=1$
 $I = [-1, 1)$

6. $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n^2}$

$R=1$

$I = [-1, 1] = \bar{I}$

$$\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{x^{n+1}}{(n+1)^2} \cdot \frac{n^2}{x^n} \right| = \frac{n^2}{(n+1)^2} |x| \xrightarrow{n \rightarrow \infty} |x| < 1 \Rightarrow R=1$$

$\downarrow n^2 + \text{smaller}$

$x = 1 \quad \sum \frac{(-1)^n}{n^2} \quad \text{Yes}$

$x = -1 \quad \sum \frac{(-1)^n (-1)^n}{n^2} = \sum \frac{1}{n^2} \quad \text{Yes}$

7. $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ $\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{x^{n+1}}{(n+1)!} \cdot \frac{n!}{x^n} \right| = \frac{1}{n+1} |x| \xrightarrow{n \rightarrow \infty} 0 < 1 \forall x!$

$R = \infty!$
 $I = (-\infty, \infty)$

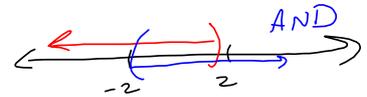
8. $\sum_{n=1}^{\infty} n^n x^n$ $\left| \frac{a_{n+1}}{a_n} \right| = \frac{(n+1)^{n+1} x^{n+1}}{n^n x^n} = \frac{(n+1)^{n+1}}{n^n} |x|$
 $> (n+1) |x| \xrightarrow{n \rightarrow \infty} \infty \forall x \neq 0$
 $R = 0$
 $I = \{0\}$

9. $\sum_{n=1}^{\infty} (-1)^n \frac{n^2 x^n}{2^n}$ $\left| \frac{a_{n+1}}{a_n} \right| = \frac{(n+1)^2 x^{n+1}}{2^{n+1}} \cdot \frac{2^n}{n^2 x^n} = \frac{(n+1)^2}{2 n^2} |x| \xrightarrow{n \rightarrow \infty} \frac{1}{2} |x| < 1$ *want*

$(-2)^n = ((-1)(2))^n = (-1)^n (2)^n$

centered @ $x=0$
 $R=2$
 $I = (-2, 2)$ the endpoints?
 $\Rightarrow |x| < 2$
 $\{x | x < 2 \text{ and } x > -2\} = (-2, 2)$

$x = -2: \sum (-1)^n \frac{n^2 (-2)^n}{2^n} = \sum \frac{n^2 \cdot 2^n}{2^n} = \sum n^2$ Nope.



$x = 2: \sum (-1)^n \frac{n^2 (2)^n}{2^n} = \sum (-1)^n n^2$ Nope

10. $\sum_{n=1}^{\infty} \frac{10^n x^n}{n^3}$

$\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{10^{n+1} x^{n+1}}{(n+1)^3} \cdot \frac{n^3}{10^n x^n} \right| = \frac{10^3 (10)}{(n+1)^3} |x| \xrightarrow{n \rightarrow \infty} 10 |x| < 1$ *want*

$\left(\frac{1}{10}\right)^n (10^n) = \left(\frac{1}{10} \cdot 10\right)^n = 1$

$x = -\frac{1}{10}:$

$\sum \frac{10^n (-\frac{1}{10})^n}{n^3}$

$= \sum \frac{(-1)^n}{n^3}$ Yes $p=3 > 1$

$x = \frac{1}{10}: \sum \frac{1}{n^3}$ Yes

$|x| < \frac{1}{10} \Rightarrow \left(-\frac{1}{10}, \frac{1}{10}\right)$
 $R = \frac{1}{10}$
 $I = \left[-\frac{1}{10}, \frac{1}{10}\right]$

11. $\sum_{n=1}^{\infty} \frac{(-3)^n}{n\sqrt{n}} x^n$

$$\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{3^{n+1} x^{n+1}}{(n+1)\sqrt{n+1}} \cdot \frac{n\sqrt{n}}{3^n x^n} \right| = \left| 3 \times \frac{n\sqrt{n}}{(n+1)\sqrt{n+1}} \right| \xrightarrow{n \rightarrow \infty} |3x|$$

want $|3x| < 1$
 $|x| < \frac{1}{3} = R$
 $(-\frac{1}{3}, \frac{1}{3})$ pending endpoints
 $x = -\frac{1}{3} \quad \sum \frac{(-3)^n (-\frac{1}{3})^n}{n\sqrt{n}} = \sum \frac{1}{n^{3/2}}$ (yes) $p = \frac{3}{2}$ - series $\frac{3}{2} > 1$
 $x = \frac{1}{3} \quad \sum \frac{(-1)^n}{n^{3/2}}$ (yes)

12. $\sum_{n=1}^{\infty} \frac{x^n}{n3^n}$

$$\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{x^{n+1}}{(n+1)3^{n+1}} \cdot \frac{n3^n}{x^n} \right| \quad I = \left[-\frac{1}{3}, \frac{1}{3}\right]$$

$$= \frac{n}{n+1} \cdot \frac{1}{3} |x| \xrightarrow{n \rightarrow \infty} \frac{1}{3} |x|$$

want $\frac{1}{3} |x| < 1$
 $|x| < 3 = R$
 $-3 < x < 3$
 $x = -3 \quad \sum \frac{(-3)^n}{n3^n} = \sum \frac{(-1)^n}{n}$ Yes Alt. Series Test
 But $x = 3 \quad \sum \frac{1}{n}$ No $p = 1$ - series.

13. $\sum_{n=2}^{\infty} (-1)^n \frac{x^n}{4^n \ln n}$

$$\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{x^{n+1}}{4^{n+1} \ln(n+1)} \cdot \frac{4^n \ln n}{x^n} \right| = \frac{\ln n}{\ln(n+1)} |x| = \frac{1}{4} |x|$$

want $|x| < 4 = R$
 u check endpoints.
 L'H says $\frac{\frac{1}{n}}{\frac{1}{n+1}} = \frac{n+1}{n} \xrightarrow{n \rightarrow \infty} 1$

14. $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$

$$\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{x^{2(n+1)+1}}{(2(n+1)+1)!} \cdot \frac{(2n+1)!}{x^{2n+1}} \right| = \frac{(2n+1)!}{(2n+3)(2n+2)(2n+1)!} |x|^2$$

$$= \frac{|x|^2}{(2n+3)(2n+2)} \xrightarrow{n \rightarrow \infty} 0 \quad \forall x \quad \text{So}$$

$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = 0 < 1 \quad \forall x$, So

$$\boxed{R = \infty}$$

$$\boxed{I = (-\infty, \infty)}$$

15. $\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^2+1}$
centered @ $a=2$

$$\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{(x-2)^{n+1}}{(n+1)^2+1} \cdot \frac{(n^2+1)}{(x-2)^n} \right| = \left(\frac{n^2+1}{(n+1)^2+1} \right) |x-2|$$

$$\xrightarrow{n \rightarrow \infty} |x-2| < \boxed{1 = R}$$

$$-1 < x-2 < 1$$

$$1 < x < 3$$

(1,3) check edpts

$x=1: \sum \frac{(-1)^n}{n^2+1}$ Yes

$x=3: \sum \frac{1}{n^2+1}$ Yes

$I = [1, 3]$

16. $\sum_{n=0}^{\infty} (-1)^n \frac{(x-3)^n}{2n+1}$

$$\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{(x-3)^{n+1}}{2n+3} \cdot \frac{2n+1}{(x-3)^n} \right| = \frac{2n+1}{2n+3} |x-3| \xrightarrow{n \rightarrow \infty} |x-3| < 1$$

$I = (2, 4)$

$x=2: \sum (-1)^n \frac{(-1)^n}{2n+1} = \sum \frac{1}{2n+1}$ No, $-1 < x-3 < 1$
 $2 < x < 4$

$x=4: \sum (-1)^n \frac{1}{2n+1} = \sum (-1)^n b_n$ Yes

$b_n = \frac{1}{2n+1} > \frac{1}{2(n+1)+1}$ decr.
 $\frac{1}{2n+1} \xrightarrow{n \rightarrow \infty} 0$
 $\frac{1}{2n+1} > 0$

17. $\sum_{n=1}^{\infty} \frac{3^n(x+4)^n}{\sqrt{n}}$

$$\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{3^{n+1}(x+4)^{n+1}}{\sqrt{n+1}} \cdot \frac{\sqrt{n}}{3^n(x+4)^n} \right|$$

$x+4 = x - (-4)$
centered @ $x = -4$

$$= \left(3(x+4) \left(\frac{\sqrt{n}}{\sqrt{n+1}} \right) \right) \xrightarrow{n \rightarrow \infty} 3|x+4| < 1$$

$|x+4| < \frac{1}{3} = R$
 $-\frac{1}{3} < x+4 < \frac{1}{3}$
 $-\frac{13}{3} < x < -\frac{11}{3}$

$x = -\frac{13}{3}$

$\sum \frac{3^n \left(-\frac{13}{3} + \frac{12}{3}\right)^n}{\sqrt{n}}$
 $= \sum \frac{\left(-\frac{1}{3}\right)^n (3^n)}{\sqrt{n}}$

$= \sum \frac{(-1)^n}{\sqrt{n}}$ Yes

$x = -\frac{11}{3}$ No

$I = \left[-\frac{13}{3}, -\frac{11}{3}\right)$

18. $\sum_{n=1}^{\infty} \frac{n}{4^n} (x+1)^n$

$$\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{(n+1)(x+1)^{n+1}}{4^{n+1}} \cdot \frac{4^n}{n(x+1)^n} \right| = \frac{n+1}{n} \left| \frac{x+1}{4} \right| \xrightarrow{n \rightarrow \infty} \left| \frac{x+1}{4} \right| < 1$$

$|x+1| < 4 = R$
 $-4 < x+1 < 4$
 $-5 < x < 3$

$I = (-5, 3)$

$x = -5: \sum \frac{n}{4^n} (-5+1)^n$

$= \sum \frac{n(-4)^n}{4^n} = \sum n(-1)^n$

No. Diverge

$x = 3: \sum \frac{n}{4^n} (4)^n = \sum n$ No

19. $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^n}$ woo-hoo! Hugo! Root Test!

$$\sqrt[n]{|a_n|} = \sqrt[n]{\frac{|x-2|^n}{n^n}} = \sqrt[n]{\left(\frac{|x-2|}{n}\right)^n} = \frac{|x-2|}{n} \xrightarrow{n \rightarrow \infty} 0 < 1$$

want < 1
if have < 1, regardless of x, so:
 $R = \infty, I = (-\infty, \infty)$

20. $\sum_{n=1}^{\infty} \frac{(2x-1)^n}{5^n \sqrt{n}}$ Root or Ratio, here, if you're good w/ $n^{\frac{1}{n}}$ on $n^{\frac{1}{2k}}$

$$\sqrt[n]{|a_n|} = \left(\frac{(2x-1)^n}{5^n n^{\frac{1}{2}}}\right)^{\frac{1}{n}} = \frac{|2x-1|}{5 n^{\frac{1}{2n}}}$$

$$\left(n^{\frac{1}{2}}\right)^{\frac{1}{n}} = n^{\frac{1}{2} \cdot \frac{1}{n}} = n^{\frac{1}{2n}}$$

$$n^{\frac{1}{2n}} \xrightarrow{n \rightarrow \infty} 1^{\frac{1}{2}} = 1$$

$\xrightarrow{n \rightarrow \infty} \frac{|2x-1|}{5} < 1$ want < 1

$$|2x-1| < 5 \rightarrow -5 < 2x-1 < 5$$

$$-4 < 2x < 6 \rightarrow -2 < x < 3$$

$\boxed{I = [-2, 3]}$

$\textcircled{q} x = -2$

$$\sum \frac{(2x-1)^n}{5^n \sqrt{n}} = \sum \frac{(-5)^n}{5^n \sqrt{n}} = \sum \frac{(-1)^n}{\sqrt{n}} \text{ Yes}$$

$x = 3$

$$\sum \frac{5^n}{5^n \sqrt{n}} = \sum \frac{1}{\sqrt{n}} \text{ No}$$

21. $\sum_{n=1}^{\infty} \frac{n}{b^n} (x-a)^n, b > 0$

$$\sqrt[n]{|a_n|} = \frac{n^{\frac{1}{n}}}{b} |x-a| \xrightarrow{n \rightarrow \infty} \frac{1}{b} |x-a|$$

want < 1
 $x = -b + a$

$\boxed{I = (a-b, a+b)}$ $|x-a| < b = R$

$$-b < x-a < b \rightarrow -b+a < x < b+a$$

$$\sum \frac{n}{b^n} (a-b-a)^n = \sum \frac{n}{b^n} (-b)^n = \sum \frac{n}{b^n} (-1)^n$$

No
Same for $x = b+a$

22. $\sum_{n=2}^{\infty} \frac{b^n}{\ln n} (x-a)^n, b > 0$

$$= \left(\frac{b^{n+1}}{\ln(n+1)} (x-a)^{n+1} \cdot \frac{\ln(n)}{b^n (x-a)^n} \right)$$

want $b|x-a| \xrightarrow{n \rightarrow \infty} b|x-a| < 1, \text{ etc.}$

23. $\sum_{n=1}^{\infty} n!(2x-1)^n$ Ratio

24. $\sum_{n=1}^{\infty} \frac{n^2 x^n}{2 \cdot 4 \cdot 6 \cdot \dots \cdot (2n)}$ $\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{(n+1)^2 x^{n+1}}{2 \cdot 4 \cdot \dots \cdot (2n) (2n+2)} \cdot \frac{2 \cdot 4 \cdot 6 \cdot \dots \cdot (2n)}{n^2 x^n} \right|$

\uparrow
 $2(n+1) = 2n+2$

$\left| \frac{(n+1)^2 x}{n^2 2n+2} \right| \xrightarrow{n \rightarrow \infty} 0 \quad \forall x$ $2^n \cdot n! =$

$R = \infty$
 $I = (-\infty, \infty)$ $2 \cdot \dots \cdot 2 \cdot 2 \cdot 1 \cdot 2 \cdot 3 \cdot 4 \cdot \dots \cdot n$

$x = c$
 $x = 2$
 $(5x-4)^n = \left(5\left(x-\frac{4}{5}\right)\right)^n = 5^n \left(x-\frac{4}{5}\right)^n$

25. $\sum_{n=1}^{\infty} \frac{(5x-4)^n}{n^3} = \sum_{n=1}^{\infty} \frac{5^n \left(x-\frac{4}{5}\right)^n}{n^3}$ $\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{5^{n+1} \left(x-\frac{4}{5}\right)^{n+1}}{(n+1)^3} \cdot \frac{n^3}{5^n \left(x-\frac{4}{5}\right)^n} \right|$

$= \frac{n^3}{(n+1)^3} 5 \left|x-\frac{4}{5}\right| \xrightarrow{n \rightarrow \infty} 5 \left|x-\frac{4}{5}\right| < 1$ want

$\left|x-\frac{4}{5}\right| < \frac{1}{5} = R$

$-\frac{1}{5} < x-\frac{4}{5} < \frac{1}{5}$ $x = \frac{3}{5}$ $\sum \frac{(-1)^n}{n^3}$ Yes

$\frac{3}{5} < x < 1$ $x = -\frac{3}{5}$ $\sum \frac{1}{n^3}$ Yes

$I = \left[\frac{3}{5}, 1\right]$

26. $\sum_{n=2}^{\infty} \frac{x^{2n}}{n(\ln n)^2}$ $2(n+1) = 2n+2$

$\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{x^{2n+2}}{(n+1)(\ln(n+1))^2} \cdot \frac{n(\ln n)^2}{x^{2n}} \right|$

$= \frac{n(\ln n)^2}{(n+1)(\ln(n+1))^2} x^2 \xrightarrow{n \rightarrow \infty} x^2 < 1$ want

$\sqrt{x^2} < \sqrt{1}$
 $|x| < 1 = R$
 $-1 < x < 1$

$x = -1$ $\sum \frac{(-1)^{2n}}{n(\ln n)^2}$
 $= \sum \frac{1}{n(\ln n)^2}$ (Same @ $x=+1$)

Converges:
 use Integral Test. Yes!
 Yes!

$\int_2^{\infty} \frac{1}{n} \cdot (\ln n)^{-2} dn$ $I = [-1, 1]$

$u = \ln n$
 $du = \frac{1}{n} dn$

$= \lim_{t \rightarrow \infty} \left[-\ln n^{-1} \right]_2^t = \frac{1}{\ln 2}$ ✓

$$27. \sum_{n=1}^{\infty} \frac{x^n}{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-1)}$$

Should converge $\forall x$.

Intuition: Like $\frac{x^n}{n!}$ deal

$$28. \sum_{n=1}^{\infty} \frac{n! x^n}{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-1)} \quad \left| \frac{z_{n+1}}{z_n} \right| = \frac{(n+1)! x^{n+1}}{1 \cdot 3 \cdot \dots \cdot (2n-1) (2n+1)} = \frac{1 \cdot 3 \cdot \dots \cdot (2n-1)}{n! x^n} \quad \left| \right.$$

$$= \left| \frac{(n+1)x}{2n+1} \right| \xrightarrow{n \rightarrow \infty} \frac{1}{2} |x| < 1 \quad 2(n+1) - 1 = 2n + 2 - 1 = 2n + 1$$

$$|x| < 2 = R$$

$$-2 < x < 2$$

$$x=2 \quad \sum_{n=1}^{\infty} \frac{n! 2^n}{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-1)} = \sum_{n=1}^{\infty} \frac{2 \cdot 4 \cdot 6 \cdot \dots \cdot 2n}{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-1)} \quad \text{Diverges}$$

$$x=-2 \quad \sum_{n=1}^{\infty} \frac{n! (-2)^n}{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-1)} \quad \text{No}$$

$$I = (-2, 2) \quad \text{I'm shaky.}$$