See Videos here:

https://harryzaims.com/202/videos/chapter-11/11-06-ratio-and-root-absolute-convergence/notes-from-4-13-20/

$\sum_{n=1}^{\infty} \frac{\sin(n)}{3^n}$ converges. The $\sin(n)$ is bdd above & below by 1 & −1, respectively, i.e. $|\sin(n)| \le 1$ and the 3^n in the denominator drags things to zero very rap. Here's the idea:

$|a_n| = \frac{|\sin(n)|}{3^n} \le \frac{1}{3^n}$ & $\sum \left(\frac{1}{3}\right)^n$ is convergent geometric series with $r = \frac{1}{3} < 1$. No ratio test needed! <u>But</u> if you want to go 'round in circles, try what I tried on 4/10/20:

$$\left| \frac{\sin(n+1)}{3^{n+1}} \cdot \frac{3^n}{\sin(n)} \right| = \left| \frac{\sin(n+1)}{3\sin(n)} \right|$$

$$= \left| \frac{\sin(n)\cos(1) + \sin(1)\cos(n)}{3\sin(n)} \right| =$$

$$\left| \frac{\sin(n)\cos(1) + \sin(1)\left(\sin\left(\frac{\pi}{2} - n\right)\right)}{3\sin(n)} \right|$$

$$= \left| \frac{\sin(n)\cos(1) + \sin(1)\left(\sin\frac{\pi}{2}\cos(-n) + \sin(-n)\cos\frac{\pi}{2}\right.}{3\sin(n)} \right|$$

$$= \left| \frac{\sin(n)\cos(1) - \sin(1)\cos(n)}{3\sin(n)} \right|$$

$$= \left| \frac{\sin(n)\cos(1) + \sin(1)\sin(n)}{3\sin(n)} \right| = \left| \frac{\cos(1) + \sin(1)}{3} \right| \le \frac{2}{3} < 1$$

Apr 13-11:16 AM

$\sum \frac{n!}{n^n}$ $\qquad$ $\sum \frac{n^n}{n!} \not\Longrightarrow$

$\frac{(n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{n!} = (n+1)\left(\frac{n^n}{(n+1)^{n+1}}\right)$ $\qquad$ $n \ln\left(\frac{n}{n+1}\right) = \frac{n}{\frac{1}{\ln\left(\frac{n}{n+1}\right)}} = \frac{\infty}{\infty}$

$= \frac{(n+1)n^n}{(n+1)(n+1)^n} = \left(\frac{n}{n+1}\right)^n = y \Longrightarrow$

$\ln y = n \ln\left(\frac{n}{n+1}\right) \xrightarrow{n \to \infty} \infty \cdot \ln(1) = \infty \cdot 0$, so,

L'Hôpital it: $\frac{\ln\left(\frac{n}{n+1}\right)}{\frac{1}{n}}$ is $\frac{0}{0}$ situation.

$\frac{d}{dn} \ln\left(\frac{n}{n+1}\right) = \frac{\frac{(n+1)-n}{(n+1)^2}}{\frac{n}{n+1}} = \frac{1}{(n+1)^2} \cdot \frac{n+1}{n} = \frac{1}{n(n+1)}$

$\frac{d}{dn}\left(\frac{1}{n}\right) = -\frac{1}{n^2}$,

so $\frac{\ln\left(\frac{n}{n+1}\right)}{\frac{1}{n}} \xrightarrow[L'H]{n \to \infty} \frac{\frac{1}{n(n+1)}}{-\frac{1}{n^2}} = \frac{1}{n(n+1)} \cdot \frac{n^2}{1} \cdot (-1)$

$\xrightarrow{n \to \infty} 1 \cdot (-1) = -1$. So,

$\lim_{n \to \infty} \left(\ln\left(\left|\frac{a_{n+1}}{a_n}\right|\right)\right) = -1 \Longrightarrow$

$\lim_{n \to \infty} \left|\frac{a_{n+1}}{a_n}\right| = e^{-1} = \frac{1}{e} < 1$, b/c $e > 1$!

$$\left(\frac{n}{n+1}\right)^n = \left(\left(\frac{n+1}{n}\right)^{-1}\right)^n = \left(\left(1+\frac{1}{n}\right)^{-1}\right)^n$$

$$= \left(\frac{1}{\frac{n+1}{n}}\right)^n$$

$$= \left(1+\frac{1}{n}\right)^{-n}$$

$$= \left(\left(1+\frac{1}{n}\right)^n\right)^{-1} \xrightarrow{n\to\infty}$$

$$e^{-1} = \frac{1}{e} < 1$$

$\Rightarrow$ converges!

For what values of p does $\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^p}$ converge?

$$\int_2^{\infty} \frac{dx}{x(\ln(x))^p}$$

$$\int \frac{dx}{x(\ln(x))^p} = \int (\ln(x))^{-p} \cdot \frac{1}{x}\,dx \qquad \begin{aligned} u &= \ln(x) \\ du &= \frac{1}{x}\,dx \end{aligned}$$

$$= \frac{1}{-p+1}(\ln(x))^{-p+1} + C \qquad \text{Need for } \lim_{x\to\infty} \ln(x) \text{ to } \exists.$$

$$\int_2^{\infty} \frac{dx}{x(\ln(x))^p} = \lim_{t\to\infty} \int_2^{t} \frac{dx}{x(\ln(x))^p} = \lim_{t\to\infty} \left[\frac{1}{-p+1} \ln(x)^{-p+1}\right]_2^t$$

$$= \lim_{t\to\infty} \left(\frac{1}{-p+1}(\ln(t))^{-p+1}\right) - \left(\frac{1}{-p+1}(\ln(2))^{-p+1}\right)$$

Need $-p+1 < 0$

$-p < -1$

$\boxed{p > 1}$