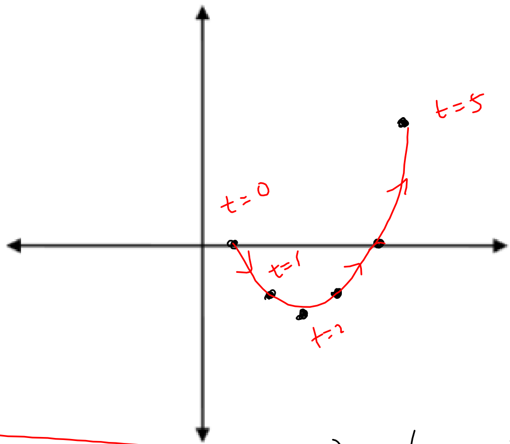
1. Let $x = 1 + t$ and $y = t^2 - 4t$.

   a. Sketch the curve by using the parametric equations to plot points, for $0 \le t \le 5$. Indicate with arrows the direction the curve is traced as t increases.

t	0	1	2	3	4	5
x	1	2	3	4	5	6
y	0	-3	-4	-3	0	5



b. Eliminate the parameter to find a Cartesian equation for the curve.
$x = 1 + t$ and $y = t^2 - 4t$

→ Don't go too far. I went farther to help the explanation

$\Rightarrow x-1 = t \Rightarrow y = (x-1)^2 - 4(x-1)$

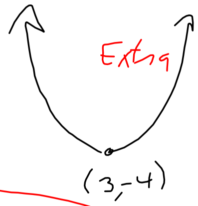
$= x^2 - 2x + 1 - 4x + 4$

$= x^2 - 6x + 5$ Simplified

$= (x^2 - 6x + 3^2) + 5 - 9$

NOT ASKED $= (x-3)^2 - 4 = a(x-h)^2 + k$. Extra

$(h, k) = \text{vertex}$



2. Suppose a curve is described by the parametric equations $x = t^4 + 1, y = t^3 + t$.

a. Find an equation of the tangent to the curve at $t = -1$.

Tangent Line $y = m_{tan}(x - x_1) + y_1 = f'(x_1)(x - x_1) + y_1$

$x_1 = x(-1) = (-1)^4 + 1 = 2 = x_1$

$y_1 = y(-1) = (-1)^3 + (-1) = -1 - 1 = -2 = y_1$

$(x_1, y_1) = (2, -2)$

$f'(x_1) = \left.\frac{dy}{dx}\right|_{t=-1} = \left.\frac{dy}{dx}\right|_{x=2, y=-2}$

$\frac{dx}{dt} = 4t^3, \quad \frac{dy}{dt} = 3t^2 + 1$

$\left.\frac{dy}{dx}\right|_{t=-1} = \left.\frac{3t^2+1}{4t^3}\right|_{t=-1} = \frac{3(-1)^2+1}{4(-1)^3} = \frac{3+1}{-4} = \frac{4}{-4} = -1 = m_{tan}$

$y = -1(x - 2) - 2$

b. Find the point(s) on the curve where the tangent is horizontal, if any.

$\frac{dy}{dx} = \frac{3t^2+1}{4t^3} \overset{SET}{=} 0 \Rightarrow 3t^2 + 1 = 0$ has no real sol'n: $t = \pm \frac{1}{\sqrt{3}} i$

Extra: Vertical tangents:

$x = t^4 + 1, \quad t^3 + t = y$

$4t^3 = 0$

$t = 0$

Check numerator:

$3(0)^2 + 1 \neq 0$ so yeah.

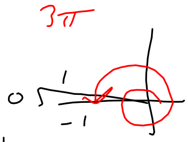
Vertical tangent's @ $t = 0 \Rightarrow$ Vertical. $(x, y) = (1, 0)$

3. Consider the curve given parametrically by
$x = \cos(t) + \cos(2t),\ y = \sin(t) + \sin(2t)$

a. Sketch the curve, using the table, below, as a guide.

t	0	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	2π
x	2	-1	1	0	1	-1	2
y	0	1	0	0	0	-1	0

3π

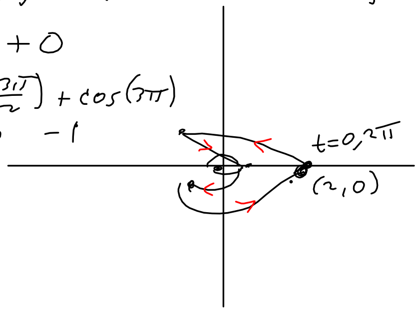


$\sin\left(\frac{3\pi}{2}\right) + \sin(3\pi)$

$= -1 + 0$

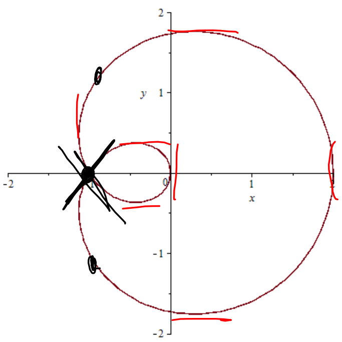
$\cos\left(\frac{3\pi}{2}\right) + \cos(3\pi)$

$0 \quad -1$

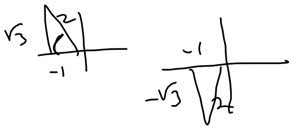


$\sin\left(\frac{2\pi}{3}\right) + \sin\left(\frac{4\pi}{3}\right) = \frac{\sqrt{3}}{2} + -\frac{\sqrt{3}}{2} = 0$

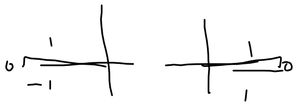
$\sin\left(\frac{4\pi}{3}\right) + \sin\left(\frac{8\pi}{3}\right) = \sin\left(\frac{4\pi}{3}\right) + \sin\frac{2\pi}{3}$



$\cos\left(\frac{2\pi}{3}\right) + \cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2} + -\frac{1}{2} = -1$

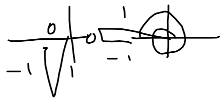


$\cos\pi + \cos 2\pi = -1 + 1 = 0$



$\cos\left(\frac{4\pi}{3}\right) + \cos\left(\frac{8\pi}{3}\right) = \cos\left(\frac{4\pi}{3}\right) + \cos\left(\frac{2\pi}{3}\right) = 0$

$\cos\left(\frac{7\pi}{2}\right) + \cos(3\pi) = 0 - 1 = -1$



b. Find the tangent(s) to the curve at $(x, y) = (-1, 0)$. Add this (these) tangent line(s) to your sketch, above.

$(x, y) = (-1, 0)$

2nd Attempt:

$\cos(t) + \cos(2t)$

$= \cos(t) + 2\cos^2(t) - 1$

$= 2\cos^2(t) + \cos(t) - 1 \overset{SET}{=} -1$

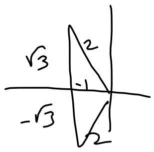
$2\cos^2(t) + \cos(t) = 0$

$\cos(t)\,(2\cos(t) + 1) = 0$

$\cos(t) = 0 \text{ @ } \frac{\pi}{2}, \frac{3\pi}{2}$

$2\cos(t) + 1 = 0$

$\cos(t) = -\frac{1}{2}$



$\frac{2\pi}{3}, \frac{4\pi}{3}$

$\cos(t) + \cos(2t) = -1$

$\cos t + 1 - 2\sin^2(t) = -1$

$\cos t + 1 - 2(1 - \cos^2(t)) = -1$

$\cos t + 1 - 2 + 2\cos^2(t) = -1$

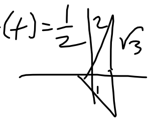
$2\cos^2(t) + \cos(t) - 1 = -1$

$2\cos^2(t) + \cos(t) = 0$

$\cos(t)\,(2\cos(t) + 1) = 0$

$t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{3}, \frac{4\pi}{3}$

$\cos(t) = \frac{1}{2}$



c. Write the integral for finding the arc length from $t = 0$ to $t = \pi$

$x = \cos(t) + \cos(2t),\ y = \sin(t) + \sin(2t)$

$$S = \int_{\alpha}^{\beta} ds = \int_0^{\pi} ds = \int_0^{\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_0^{\pi} \sqrt{5 + 4\cos(t)}\, dt$$

↳ Spent more time simplifying than I would on a time-controlled test.

$$\frac{dx}{dt} = -\sin(t) - 2\sin(2t) \Rightarrow$$

$$\left(\frac{dx}{dt}\right)^2 = \sin^2(t) + 2\cdot 2\sin(t)\sin(2t) + 4\sin^2(2t)$$

$$\frac{dy}{dt} = \cos(t) + 2\cos(2t) \Rightarrow$$

$$\left(\frac{dy}{dt}\right)^2 = \cos^2(t) + 2\cdot 2\cos(t)\cos(2t) + 4\cos^2(2t)$$

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 1 + \underbrace{4\sin(t)\sin(2t) + 4\cos(t)\cos(2t)}_{4\cos(t)} + 4 = 5 + 4\cos(t)$$

$$4\sin(t)\sin(2t) = 4\sin(t)(2\sin(t)\cos(t))$$

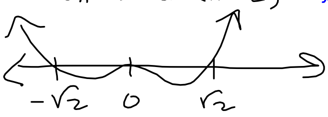
$$4\cos(t)\cos(2t) = 4\cos(t)(1 - 2\sin^2(t))$$

$$= 8\sin^2(t)\cos(t)$$

$$4\cos t - 8\sin^2(t)\cos(t)$$

4. Consider the curve described by the parametric equations $x = \sqrt{t}, y = t^2 - 2t$.
Write the integral for finding the area bounded between the curve and the x-axis.

$x^2 = t \Rightarrow y = (x^2)^2 - 2(x^2) = x^4 - 2x^2 = x^2(x^2-2) = x^2(x-\sqrt{2})(x+\sqrt{2})$



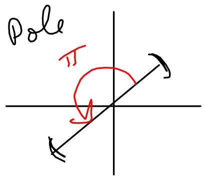
$A = -2\int_0^{\sqrt{2}} (x^4 - 2x^2)\,dx = +\frac{16}{15}\sqrt{2}$

OR $\int |y|\,dx = -2\int_0^2 (t^2-2t)\left(\frac{1}{2}t^{-\frac{1}{2}}\right)dt = +\frac{16}{15}\sqrt{2}$

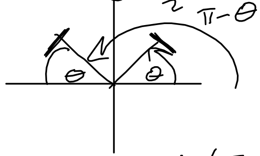
$x = t^{\frac{1}{2}} \Rightarrow dx = \frac{1}{2}t^{-\frac{1}{2}}dt$

5. Check the function (in polar coordinates) $r = 1 + 2\sin\theta$ for symmetry and sketch its graph.

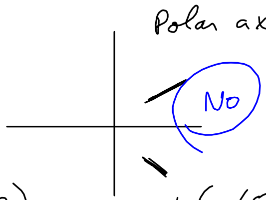
Pole



$\theta = \frac{\pi}{2}$



Polar axis ($\theta = 0$)



$r = 1 + 2\sin(-\theta)$
$= 1 - 2\sin\theta$

$-r = 1 + 2\sin\theta$ — No

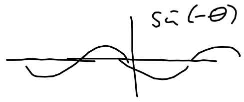
$r = 1 + 2\sin(\pi + \theta)$
$= 1 - 2\sin\theta$

No

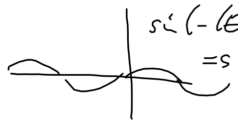
$r = 1 + 2\sin(\pi - \theta) = 1 + 2\sin(-(\theta - \pi)) = 1 + 2\sin(\theta)$

$= 1 + 2$ — Yes

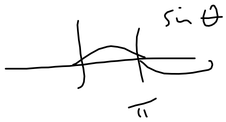
$\sin(-\theta)$



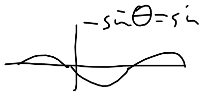
$\sin(-(\theta - \pi)) = \sin\theta$!



$\sin\theta$



$-\sin\theta = \sin(\pi + \theta)$



6. Convert each of the following to Cartesian form, and identify each graph as a common geometric figure.

See the Notes for 3/30/18

(180330-review):

http://harryzaims.com/202/202-spring-18/notes/chapter-10/180330-review.pdf

a. $r = 3\cos\theta$

b. $r = \sec\theta\tan\theta$

(a) $r = 3\cos\theta = 3\,\frac{x}{r}$

$r^2 = 3x \implies$

$\left(\sqrt{x^2+y^2}\right)^2 = 3x$

$x^2+y^2 = 3x$

$x^2+y^2-3x = 0$

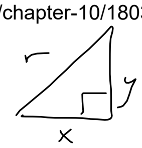
$x^2-3x+\left(\frac{3}{2}\right)^2+y^2 = 0 + \frac{9}{4}$

$\left(x-\frac{3}{2}\right)^2+y^2 = \frac{9}{4} = r^2$

Circle, radius $\frac{3}{2}$

center: $\left(\frac{3}{2}, 0\right)$

$r = \sec\theta\tan\theta$

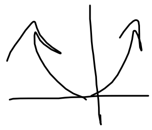


$r = \frac{r}{x}\cdot\frac{y}{x}$

$\frac{y}{x^2} = 1$

$y = x^2$ Parabola

$(h,k) = (0,0)$



7. Find the *exact* length of the curve $r = e^{2\theta}, 0 \le \theta \le \frac{\pi}{2}$. $\Longrightarrow$ $r^2 = (e^{2\theta})^2 = e^{4\theta}$

Also in the same set of notes from live lecture:

http://harryzaims.com/202/202-spring-18/notes/chapter-10/180330-review.pdf

$$ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$$\frac{dr}{d\theta} = 2e^{2\theta}$$

$$\left(\frac{dr}{d\theta}\right)^2 = 4e^{4\theta}$$

$$ds = \sqrt{e^{4\theta} + 4e^{4\theta}}\, d\theta$$

$$S = \int_\alpha^\beta ds = \int_0^{\frac{\pi}{2}} \sqrt{5e^{5\theta}}\, d\theta$$

$$= \sqrt{5}\int_0^{\frac{\pi}{2}} e^{\frac{5}{2}\theta}\, d\theta = \sqrt{5}\cdot\frac{2}{5}\int_{\theta=0}^{\theta=\frac{\pi}{2}} e^{\frac{5}{2}\theta}\cdot\frac{5}{2}\, d\theta = \frac{2\sqrt{5}}{5} e^{\frac{5}{2}\theta}\Big]_{\theta=0}^{\theta=\frac{\pi}{2}} = \boxed{\frac{2\sqrt{5}}{5}\left[e^{\frac{5\pi}{4}} - 1\right]}$$

$u = \frac{5}{2}\theta$

$du = \frac{5}{2}\, d\theta$

=

$$\sqrt{e^{5\theta}} = \left(e^{5\theta}\right)^{\frac{1}{2}} = e^{\frac{5}{2}\theta}$$