Section 10.1 #s 1, 4, 5, 8, 11, 13, 16, 19, 26

V **EXAMPLE 2** What curve is represented by the following parametric equations?

$$x = \cos t \qquad y = \sin t \qquad 0 \leqslant t \leqslant 2\pi$$

SOLUTION If we plot points, it appears that the curve is a circle. We can confirm this impression by eliminating t. Observe that

$$x^2 + y^2 = \cos^2 t + \sin^2 t = 1$$

Thus the point (x, y) moves on the unit circle $x^2 + y^2 = 1$. Notice that in this example the parameter t can be interpreted as the angle (in radians) shown in Figure 4. As t increases from 0 to 2π, the point $(x, y) = (\cos t, \sin t)$ moves once around the circle in the counterclockwise direction starting from the point $(1, 0)$.

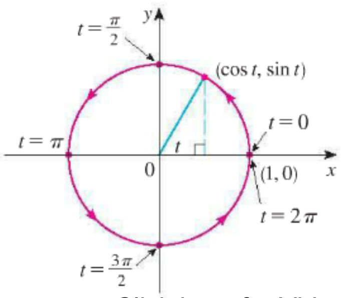


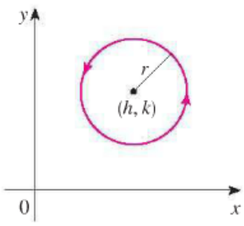
FIGURE 4

Click here for Video from the publisher

http://www.wadsworthmedia.com/math/stewart/solution_videos/6et_10_01_02.html

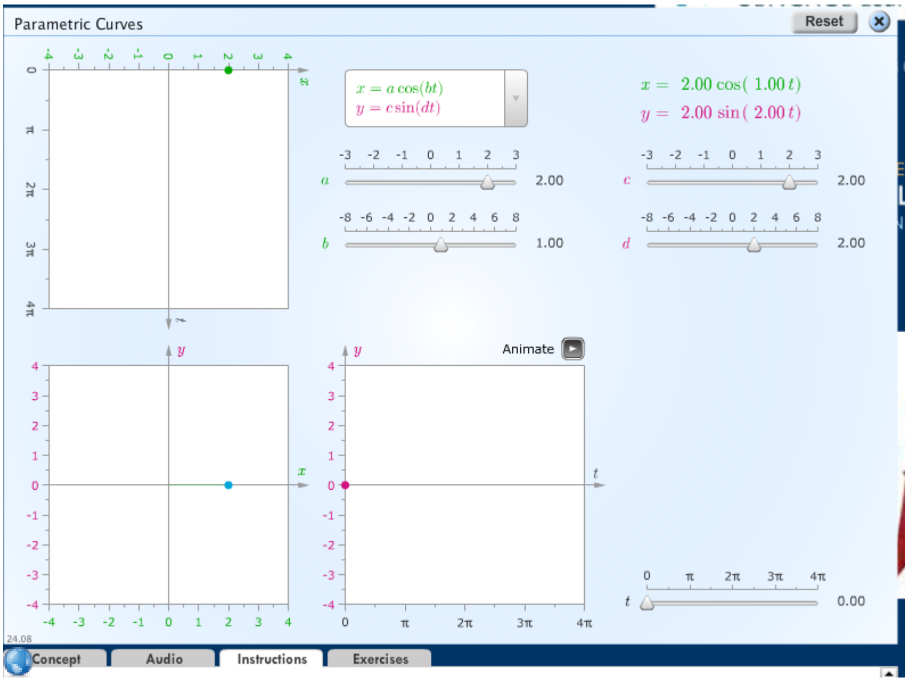
From Example 3... Circle of radius *r,* centered at (*h*, *k*).

$$x = h + r\cos t \qquad y = k + r\sin t \qquad 0 \leqslant t \leqslant 2\pi$$



Pretty Cool Tool for Exploring Parametric Curves. I hope the link works, when you click on the object below. It won't work on video, but it should open from the notes.pdf file.

http://www.cengage.com/math/discipline_content/stewartcalc7/2008/14_cengage_tec/publish/deployments/transcendentals_7e/7e_m10_1a.html



Click here to see a video on Example 6. It uses a TI-84 in Parametric Mode.

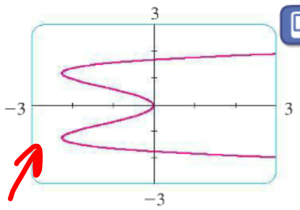


FIGURE 9

EXAMPLE 6 Use a graphing device to graph the curve $x = y^4 - 3y^2$.

SOLUTION If we let the parameter be $t = y$, then we have the equations

$$x = t^4 - 3t^2 \qquad y = t$$

Using these parametric equations to graph the curve, we obtain Figure 9. It would be possible to solve the given equation ($x = y^4 - 3y^2$) for y as four functions of x and graph them individually, but the parametric equations provide a much easier method.

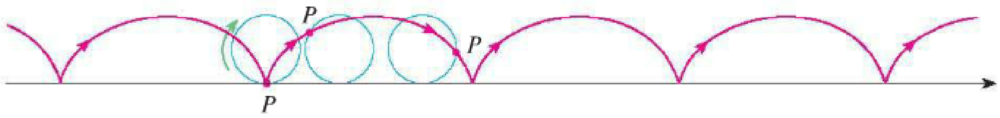
http://college.cengage.com/mathematics/blackboard/shared/content/video_explanations/v00604a.html

$$\frac{1}{2}\sqrt{6+2\sqrt{9+4x}},\ -\frac{1}{2}\sqrt{6+2\sqrt{9+4x}},\ \frac{1}{2}\sqrt{6-2\sqrt{9+4x}},\ -\frac{1}{2}\sqrt{6-2\sqrt{9+4x}}$$

http://www.cengage.com/math/discipline_content/stewartcalc7/2008/14_cengage_tec/publish/deployments/transcendentals_7e/7e_m10_1a.html#

The Cycloid

EXAMPLE 7 The curve traced out by a point P on the circumference of a circle as the circle rolls along a straight line is called a **cycloid** (see Figure 13). If the circle has radius r and rolls along the x-axis and if one position of P is the origin, find parametric equations for the cycloid.



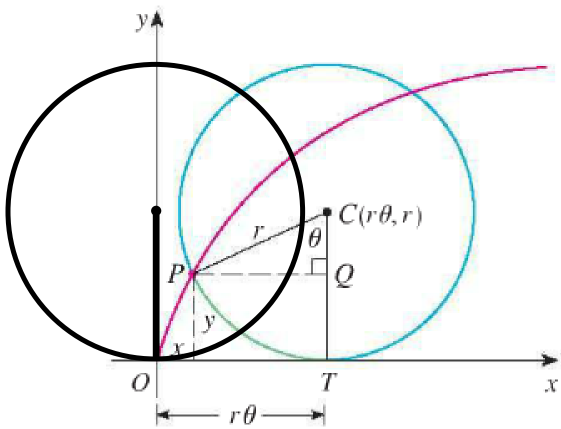


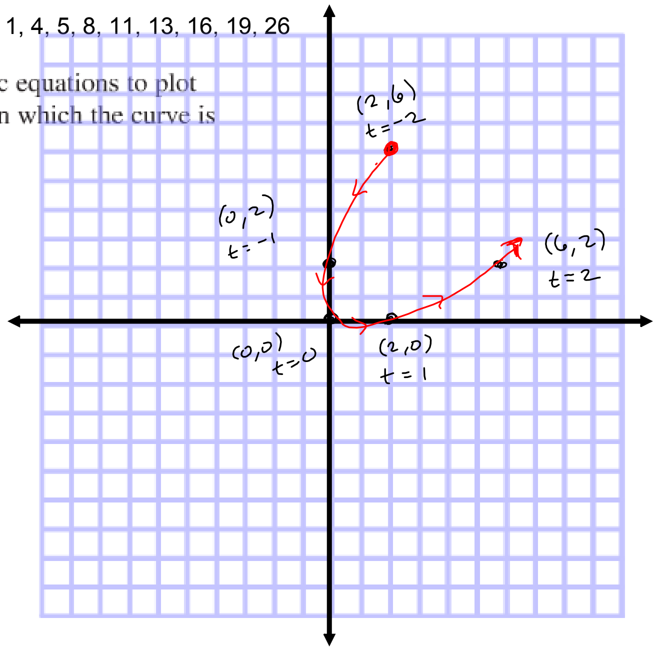
FIGURE 14

Section 10.1 #s 1, 4, 5, 8, 11, 13, 16, 19, 26

1–4 Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as t increases.

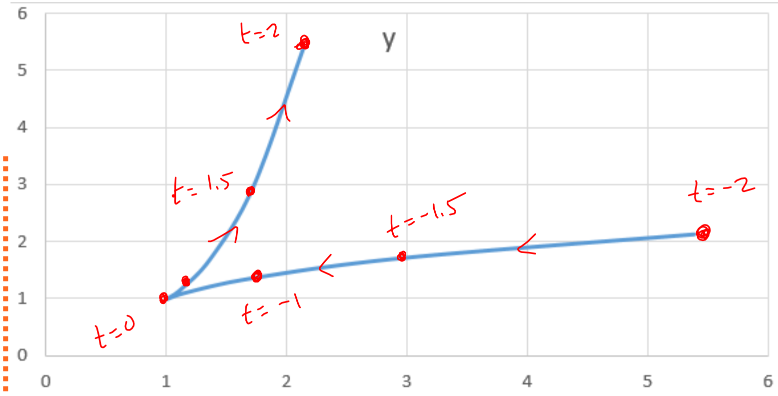
1. $x = t^2 + t, \quad y = t^2 - t, \quad -2 \leqslant t \leqslant 2$

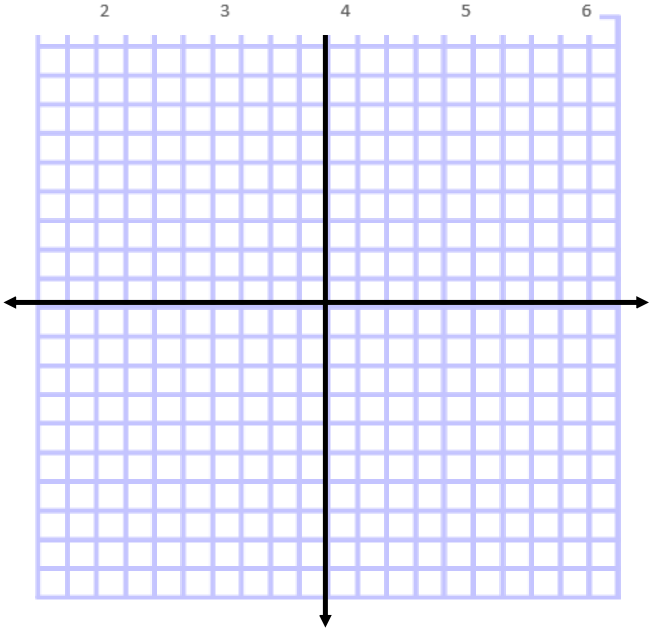
t	x	y
-2	2	6
-1	0	2
0	0	0
1	2	0
2	6	2



4. $x = e^{-t} + t, \quad y = e^{t} - t, \quad -2 \leqslant t \leqslant 2$

t	x	y
-2	5.389056	2.135335
-1.5	2.981689	1.72313
-1	1.718282	1.367879
-0.5	1.148721	1.106531
0	1	1
0.5	1.106531	1.148721
1	1.367879	1.718282
1.5	1.72313	2.981689
2	2.135335	5.389056





5–10

(a) Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as t increases.

(b) Eliminate the parameter to find a Cartesian equation of the curve.

5. $x = 3 - 4t, \quad y = 2 - 3t$

t	x	y
-2	11	8
-1.5	9	6.5
-1	7	5
-0.5	5	3.5
0	3	2
0.5	1	0.5
1	-1	-1
1.5	-3	-2.5
2	-5	-4

5–10

(a) Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as t increases.

(b) Eliminate the parameter to find a Cartesian equation of the curve.

5. $x = 3 - 4t, \quad y = 2 - 3t$

$x = 3 - 4t$

$-4t = x - 3$

$t = \frac{x-3}{-4}$

$y = 2 - 3t$

$= 2 - 3\left(\frac{x-3}{-4}\right)$

$$\boxed{\begin{aligned} y &= 2 + \tfrac{3}{4}(x-3) \\ y &= \tfrac{3}{4}(x-3) + 2 \end{aligned}}$$

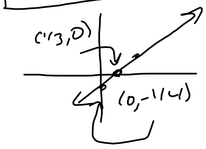
$y = \frac{3}{4}x - \frac{9}{4} + 2$

$= \frac{3}{4}x - \frac{9}{4} + \frac{8}{4}$

$y = \frac{3}{4}x - \frac{1}{4}$

~~$4y = 3x - 1$~~

$y = \frac{3}{4}x - \frac{1}{4}$



$y = 0$

$\frac{3}{4}x - \frac{1}{4} = 0$

$3x - 1 = 0$

$3x = 1$

$x = \frac{1}{3}$

8. $x = t - 1, \quad y = t^3 + 1, \quad -2 \leqslant t \leqslant 2$

$t - 1 = x$

$t = x + 1$

$y = (x+1)^3 + 1$

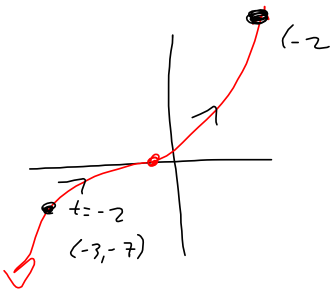
$= x^3 + 3x^2 + 3x + 1 + 1$

$= x^3 + 3x^2 + 3x + 2$

$y' = 3x^2 + 6x + 3$

$= 3(x^2 + 2x + 1)$

$= 3(x+1)^2$



11–18

(a) Eliminate the parameter to find a Cartesian equation of the curve.

(b) Sketch the curve and indicate with an arrow the direction in which the curve is traced as the parameter increases.

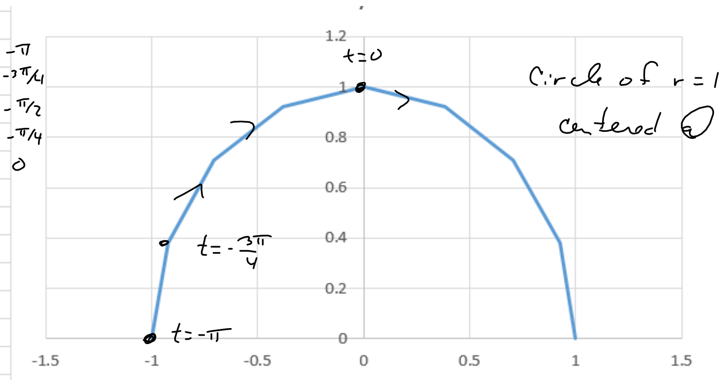
11. $x = \sin \frac{1}{2}\theta, \quad y = \cos \frac{1}{2}\theta, \quad -\pi \leqslant \theta \leqslant \pi$

$\sin^2(\frac{1}{2}\theta) = x^2$

$\cos^2(\frac{1}{2}\theta) = y^2$

$x^2 + y^2 = 1$

t	x	y	
-3.14159	-1	6.13E-17	$-\pi$
-2.35619	-0.92388	0.382683	$-3\pi/4$
-1.5708	-0.70711	0.707107	$-\pi/2$
-0.7854	-0.38268	0.92388	$-\pi/4$
0	0	1	0
0.785398	0.382683	0.92388	
1.570796	0.707107	0.707107	
2.356194	0.92388	0.382683	
3.141593	1	6.13E-17	



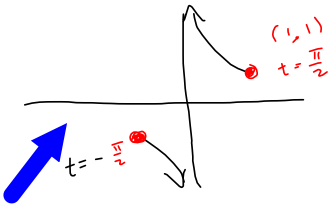
Circle of r=1 centered @ (0,0)

Sorry. Link is password-protected...

13. $x = \sin t, \quad y = \csc t, \quad 0 < t < \pi/2$

http://college.cengage.com/coursemate/mathematics/stewart_9781111297947/student/ebook/data/homework_helper/videos/scalcet6_10_01_013a.html

$x = \sin t \qquad y = \csc t = \frac{1}{\sin t} = \frac{1}{x}$



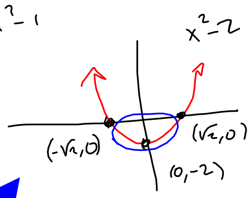
16. $y = \sqrt{t+1}, \quad y = \sqrt{t-1}$ $\quad t \geq 1 \implies x = \sqrt{t+1} \geq \sqrt{1+1} = \sqrt{2}$

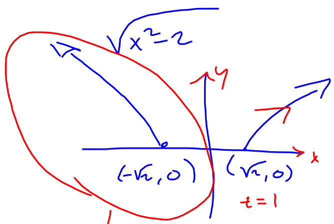
x

$\sqrt{t+1} = x \qquad y = \sqrt{(x^2-1)-1}$

$t+1 = x^2 \qquad = \sqrt{x^2-2}$

$t = x^2 - 1$





Not part of picture.

Sorry, the link is password-protected. I can bring it up, and share it with you, if you want.

19–22 Describe the motion of a particle with position (x, y) as t varies in the given interval.

19. $x = 3 + 2\cos t, \quad y = 1 + 2\sin t, \quad \pi/2 \leqslant t \leqslant 3\pi/2$

$t=\frac{\pi}{2}$	3	3
$t=\pi$	1	1
$t=\frac{3\pi}{2}$	3,	−1

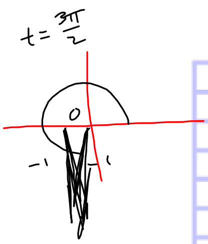
$x-3 = 2\cos t \qquad y-1 = 2\sin t$

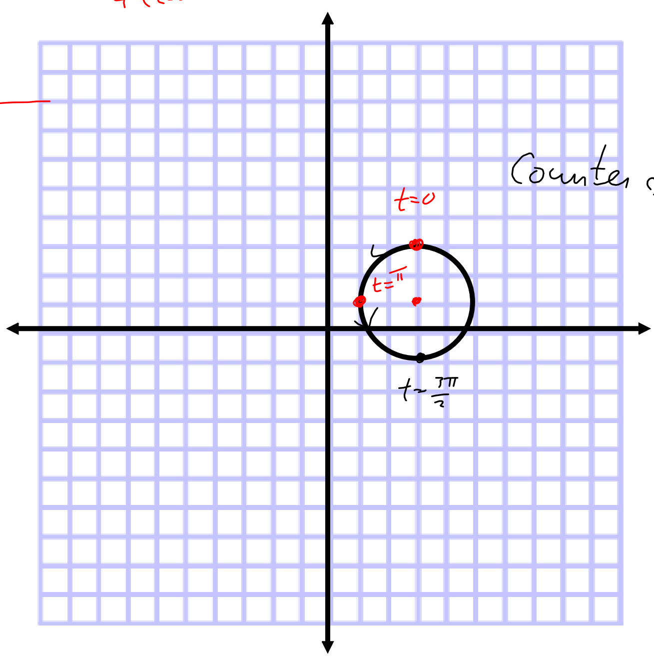
$(x-3)^2 = 4\cos^2 t \qquad (y-1)^2 = 4\sin^2 t$

$4\cos^2 t + 4\sin^2 t = (x-3)^2 + (y-1)^2 = 4$

$4(\cos^2 t + \sin^2 t)$

$r = 2, \quad (h,k) = (3,1)$





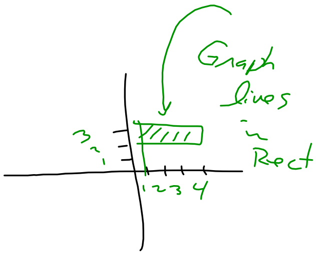
Counter clockwise

23. Suppose a curve is given by the parametric equations $x = f(t)$, $y = g(t)$, where the range of f is $[1, 4]$ and the range of g is $[2, 3]$. What can you say about the curve?

$\mathcal{R}(f(t)) = \mathcal{D}$ of graph x-values

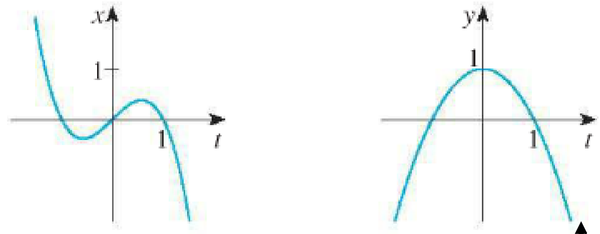
$\mathcal{R}(g(t)) = \mathcal{R}$ of grap y-values

A window with $x \in [1,4]$
and $y \in [2,3]$
would contain the whole graph

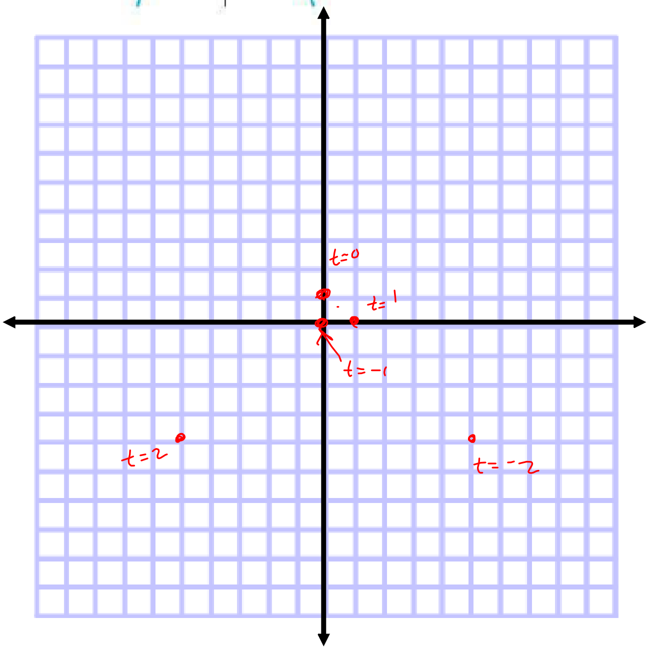


25–27 Use the graphs of $x = f(t)$ and $y = g(t)$ to sketch the parametric curve $x = f(t)$, $y = g(t)$. Indicate with arrows the direction in which the curve is traced as t increases.

26.



t	x	y
0	0	1
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{2}{3}$?
1	1	0
2	-5?	-4?
-1	0	0
-2	5?	-4?



This one is not assigned, but it might be a good one for visualiziing. Maybe this is one y'all want to work, together, on the whiteboard.

I think the way they're arranged could work on the whiteboard. Just run the screen up.

28. Match the parametric equations with the graphs labeled I-VI. Give reasons for your choices. (Do not use a graphing device.)

(a) $x = t^4 - t + 1, \quad y = t^2$

(b) $x = t^2 - 2t, \quad y = \sqrt{t}$

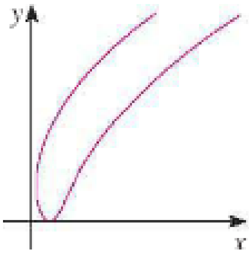
(c) $x = \sin 2t, \quad y = \sin(t + \sin 2t)$

(d) $x = \cos 5t, \quad y = \sin 2t$

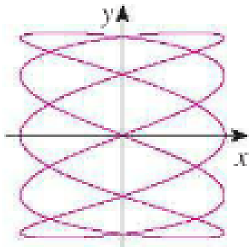
(e) $x = t + \sin 4t, \quad y = t^2 + \cos 3t$

(f) $x = \dfrac{\sin 2t}{4 + t^2}, \quad y = \dfrac{\cos 2t}{4 + t^2}$

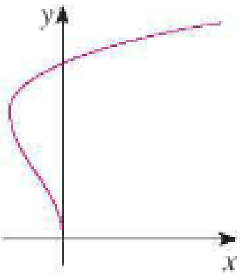
V



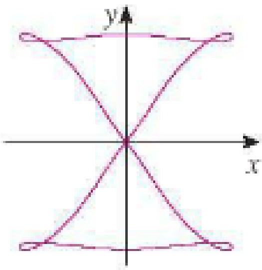
VI



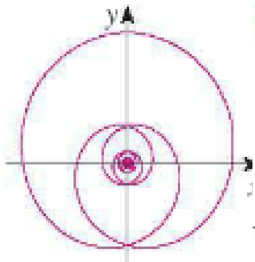
I



II



III



IV