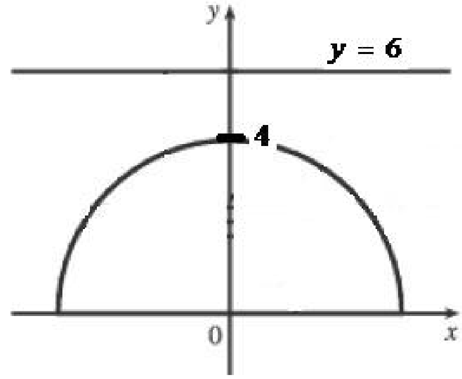
MAT 202 Test 3, Fall, 2011

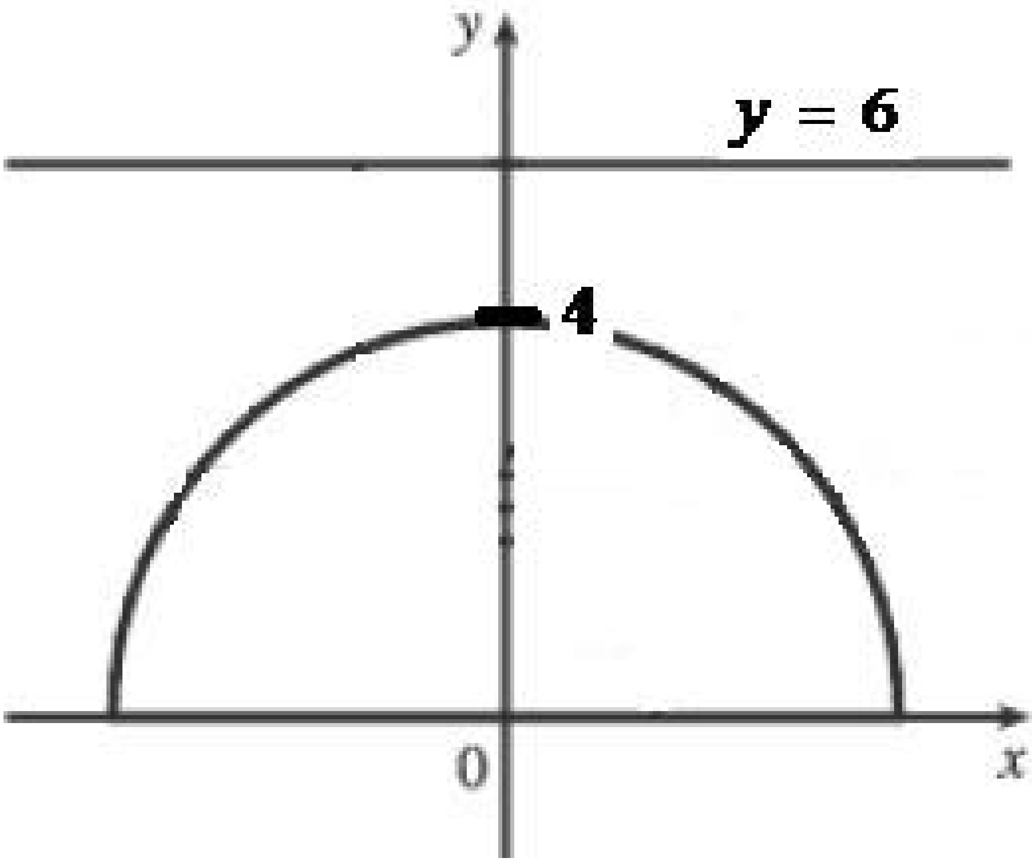
100 Points Name__________________________

1. Find the arc length of the curve $y = 1 + 2x^{\frac{3}{2}}$ between $x = 0$ and $x = 1$.

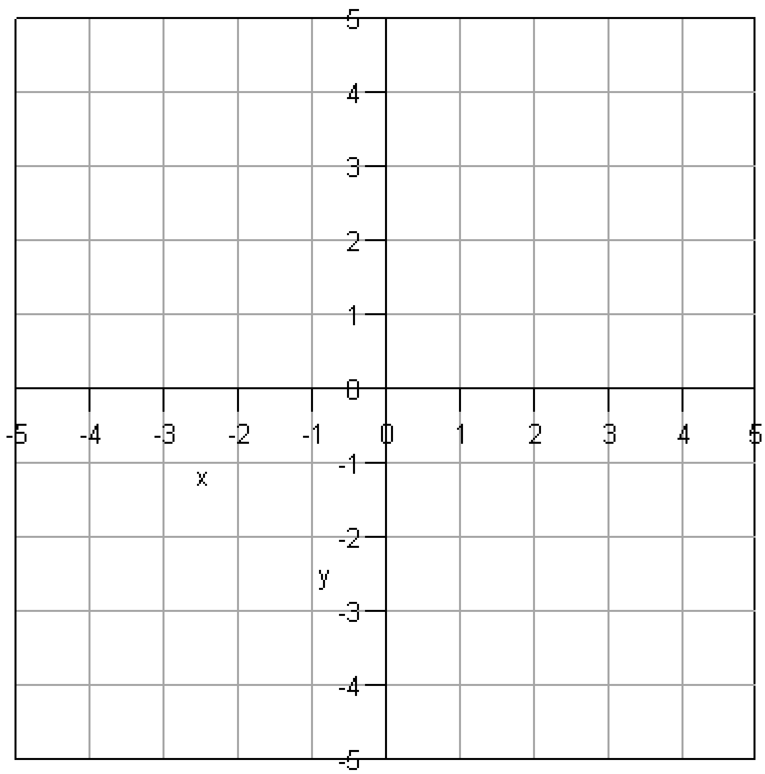
2. Find the surface area of the surface of revolution obtained by revolving $x = y^3$, from $x = 1$ to $x = 8$, about the y-axis.

3. A vertical plate is submerged in water and it has the shape of a semicircle of radius 4, as shown in the picture. Express the force of the water against the plate as an integral, using the picture to make your choice of coordinate axes, and set things up. Then, *evaluate* the integral. I prefer to see a symbolically precise answer in terms of Pi, fractions, and radicals (if any), rather than calculated decimal approximations. Final decimal answers should be accurate to 4 places. Use $\rho = 62.5 \frac{lb}{ft^3}$ for the (constant) density of water. Use the graph, below, provided for this problem.





4. Sketch the direction field for $y' = y - x$, using the graph paper provided. Show the graph of the solutions corresponding to $y(0) = 2$ and $y(0) = 0$ on your direction field. Then solve this linear differential equation for y (Hint: Re-write in the form $y' - y = -x$.) Your picture should be in agreement with your symbolic solution.



5. Use the characteristic polynomial to find the general solution to $y'' - 16y = 0$. Then find the particular solution corresponding to the boundary conditions $y'(0) = y(0) = 1$.

6. $\int_0^3 \frac{x\,dx}{\sqrt{25 - x^2}}$ Special Instructions:

a. Use trigonometric substitution. 5 points bonus if you make the substitution for the limits of integration, as well, and work it all without a calculator. There will be an $\arcsin\left(\frac{3}{5}\right)$ involved. Call it α, and don't let it bother you.

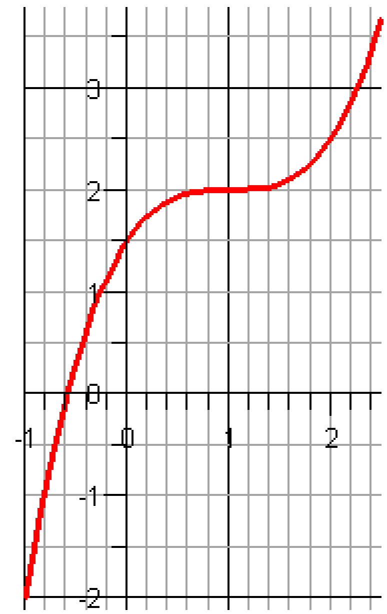
b. Use a *u*-substitution.

7. State whether the statement is true or false. If it is true, explain why. If it is false, explain why or provide a counterexample.

a. (4 pts) If f is one-to-one, with domain **R**, then $f^{-1}(f(6)) = 6$.

b. (4 pts) If f is one-to-one and differentiable, with domain **R**, then $\left(f^{-1}\right)'(6) = \dfrac{1}{f'(6)}$.

8. The graph of g is given.



a. (4 pts) Why is g one-to-one?

b. (4 pts) Estimate the value of $g^{-1}(2)$.

c. (4 pts) Sketch the graph of g^{-1}.

9. Find the exact value of each of the following:

a. (4 pts) $\ln\left(e^{\pi}\right)$

b. (4 pts) $\cos\left(\arctan\sqrt{3}\right)$

10. Solve the following equations for x.

a. (4 pts) $\ln\left(1 + e^{-x}\right) = 3$

b. (4 pts) $\ln(x+1) + \ln(x-1) = 1$

11. Differentiate.

a. (4 pts) $f(t) = t^2 \ln t$

b. (4 pts) $g(x) = 3^{mx}\cos(nx)$

c. (4 pts) $V(t) = \arctan\left(\arcsin\sqrt{t}\right)$

d. (4 pts) $y = \dfrac{\left(x^2+1\right)^4}{(2x+1)^3(3x-1)^5}$ (Use logarithmic differentiation.)

e. (4 pts) $y = (\cos(3x))^{\tan(5x)}$

12. A bacterial culture contains 200 cells initially and grows at a rate proportional to its size. After half an hour, the population has increased to 360 cells.

a. (4 pts) Find the number of bacteria after t hours.

b. (4 pts) Find the growth rate after 5 hours.

c. (4 pts) When will the population reach 10,000?

13. Cobalt-60 has a half-life of 5.24 years.

a. (4 pts) Find the mass that remains from a 100-mg sample after t years.

b. (4 pts) How long would it take for the mass to decay to 1 mg?

14. Evaluate the integral.

a. (4 pts) $\int_0^{\pi/2} \frac{\cos x}{1+\sin^2 x}\,dx$

b. (4 pts) $\int \frac{x}{\sqrt{1-x^4}}\,dx$

c. (4 pts) $\int \ln(\cos x)\tan x\,dx$

15. (4 pts) If $f(x) = x + x^2 + e^x$, find $\left(f^{-1}\right)'(1)$.

16. (4 pts) Find $f'(x)$ for $f(x) = \int_1^{\sqrt{x}} \frac{e^t}{t}\,dt$

17. (4 pts) If $\tanh x = \frac{3}{5}$, find the value of the other 5 hyperbolic trigonometric functions. This should not require a calculator.

18. (Bonus) (4 pts) Use mathematical induction to show that if $f(x) = xe^x$, then $f^{(n)}(x) = (x+n)e^x$.