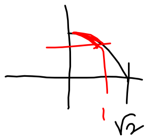
1. Use the arc length formula to find the length of the curve $y=\sqrt{2-x^2}, 0 \le x \le 1$. (You can check by noting this is part of a circle.)

$$S = \int_a^b \sqrt{1+(y')^2}\,dx$$



$x^2+y^2=(\sqrt{2})^2$

$y^2 = 2-x^2$

$y = \pm\sqrt{2-x^2}$

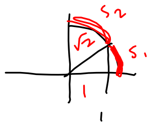
$y=\sqrt{2-x^2}$ is top $\frac{1}{2}$ of circle of radius $\sqrt{2}$ $\Rightarrow$ arc length $=\frac{1}{2}(2\pi r) = \pi r = \sqrt{2}\pi$

$y=(2-x^2)^{\frac{1}{2}} \Rightarrow y' = \frac{1}{2}(2-x^2)^{-\frac{1}{2}}(-2x)$

$= \dfrac{-x}{\sqrt{2-x^2}} \Rightarrow$

$(y')^2 = \dfrac{x^2}{2-x^2}$

sol'ns for this test have it wrong

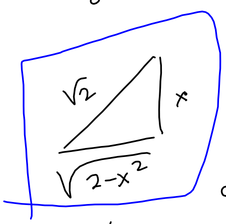


$S_1 = r\theta = \sqrt{2}\cdot\frac{\pi}{4}$ since $\theta = \frac{\pi}{4}$
$= S_2$,

So, $S = \displaystyle\int_0^1 \sqrt{1+\frac{x^2}{2-x^2}}\,dx = \int_0^1 \sqrt{\frac{2}{2-x^2}}\,dx$

Scratch: $1+\dfrac{x^2}{2-x^2} = \dfrac{2-x^2+x^2}{2-x^2} = \dfrac{2}{2-x^2}$

$= \sqrt{2}\displaystyle\int_0^1 \frac{dx}{\sqrt{2-x^2}} = A$ do trig substitution



$\dfrac{x}{\sqrt{2}} = \sin\theta \Rightarrow$

$x = \sqrt{2}\sin\theta \Rightarrow$

$dx = \sqrt{2}\cos\theta\,d\theta$

$\dfrac{\sqrt{2-x^2}}{\sqrt{2}} = \cos\theta$

$\Rightarrow \sqrt{2-x^2} = \sqrt{2}\cos\theta$ one way.

$\sqrt{2-x^2} = \sqrt{2-(\sqrt{2}\sin\theta)^2}$

$= \sqrt{2-2\sin^2\theta} = \sqrt{2(1-\sin^2\theta)}$

$= \sqrt{2}\sqrt{1-\sin^2\theta} = \sqrt{2}\sqrt{\cos^2\theta} = \sqrt{2}\,|\cos\theta|$

$A = \sqrt{2}\displaystyle\int_{x=0}^{x=1} \frac{\sqrt{2}\cos\theta\,d\theta}{\sqrt{2}\cos\theta} = \sqrt{2}\int_0^{\frac{\pi}{4}} d\theta$

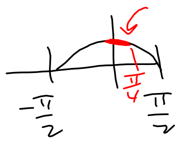
$= \sqrt{2}\cdot\frac{\pi}{4} = \boxed{\dfrac{\sqrt{2}\pi}{4} = \text{arc length.}}$

$x=0=\sqrt{2}\sin\theta \Rightarrow \theta = 0$

$x=1=\sqrt{2}\sin\theta$

$\sin\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}$

So.... $|\cos\theta| = \cos\theta$, bc $\cos\theta \ge 0$ on $[-\frac{\pi}{2}, \frac{\pi}{2}]$



2. Find the length of the curve $y = 2\ln\left(\sin\left(\frac{x}{2}\right)\right)$, for ~~$\frac{\pi}{3} \le x \le \pi$~~

$\frac{\pi}{3} \le x \le \frac{2\pi}{3}$

$y = 2\ln(\sin(\frac{x}{2}))$ $u = \frac{x}{2}$

$y' = 2\left(\frac{\frac{1}{2}\cos(\frac{x}{2})}{\sin(\frac{x}{2})}\right)$ $\frac{du}{dx} = \frac{1}{2}$

$= \tan(\frac{x}{2}) \Longrightarrow (y')^2 = \tan^2(\frac{x}{2})$

$S = \int_a^b \sqrt{1+(f'(x))^2}\,dx = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \sqrt{1+\tan^2(\frac{x}{2})}\,dx = \int_{\frac{\pi}{3}=x}^{\frac{2\pi}{3}=x} |\sec(\frac{x}{2})|\,dx$

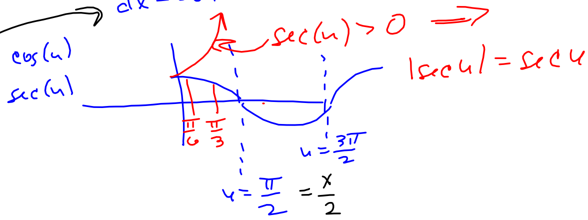
$\sqrt{1+\tan^2(\frac{x}{2})}$
$= \sqrt{\sec^2(\frac{x}{2})}$
$= |\sec(\frac{x}{2})|$

$u = \frac{x}{2}$ $x = \frac{\pi}{3} \Longrightarrow \frac{x}{2} = u = \frac{\pi}{6}$

$du = \frac{1}{2}dx$ $x = \frac{2\pi}{3} \Longrightarrow \frac{x}{2} = \frac{\pi}{3} = u$

$dx = 2du$

Don't forget me!

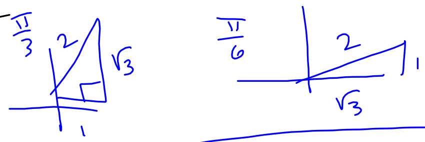


$= 2\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec u\,du$

$x = \pi$
$3\pi > x > \pi$ would make the $\sec(\frac{x}{2}) < 0$
$\nRightarrow$ $|\sec(\frac{x}{2})| = -\sec(\frac{x}{2})$

$= 2\ln|\sec u + \tan u|\Big]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$

$= 2\left[\ln|\sec\frac{\pi}{3} + \tan\frac{\pi}{3}| - \ln|\sec\frac{\pi}{6} + \tan\frac{\pi}{6}|\right]$



$$\boxed{= 2\left[\ln|2+\sqrt{3}| - \ln\left|\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right|\right]}$$

3. Find the area of the surface obtained by rotating the curve

$$9x = y^2 + 18, \text{ for } 2 \le x \le 6$$

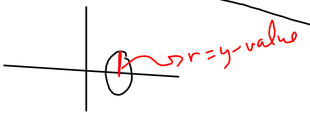
about the x-axis.

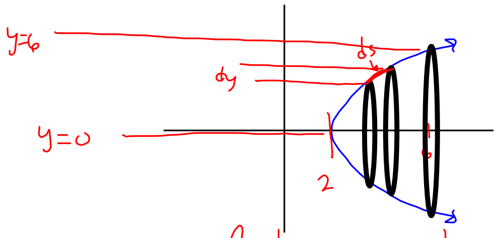
$x = g(y)$ sitch

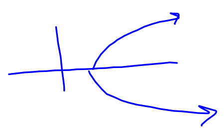
$ds = \sqrt{1 + g'(y)^2}\, dy$

$2\pi \int y\, ds$

$x = \frac{1}{9}y^2 + 2$







what if you're not good at graphing $x = g(y)$?

(Solve $x = g(y)$ for y!)

arc length increment

$$2\pi \int_{x=2}^{y=6} y \boxed{\sqrt{1+g'(y)^2}\, dy}$$

$g(y) = \frac{1}{9}y^2 + 2$

$g'(y) = \frac{2}{9}y$

$(g'(y))^2 = \frac{4}{81}y^2$

$$= 2\pi \int_0^6 y\sqrt{1 + \frac{4}{81}y^2}\, dy = A$$

$2 \lfloor 36$, $2 \lfloor 18$, $3 \lfloor 9$, 3 $\qquad$ $3 \lfloor 81$, $3 \lfloor 27$, $3 \lfloor 9$, 3

$u = \frac{4}{81}y^2 + 1$

$du = \frac{8}{81}y\, dy \Rightarrow dy = \frac{81}{8y}\, du$

$x=2$: $2 = \frac{1}{9}y^2 + 2$

$0 = \frac{1}{9}y^2 \Rightarrow y = 0$

$y = 0 \Rightarrow u = \frac{4}{81}y^2 + 1 = \frac{4}{81}(0)^2 + 1 = 1$

$x=6$: $6 = \frac{1}{9}y^2 + 2$

$y = 6 \Rightarrow u = \frac{4}{81}\cdot 36 + 1 = \frac{4}{9}\cdot 4 + 1 = \frac{16}{9} + 1 = \frac{25}{9}$

$\frac{1}{9}y^2 = 4$

$y^2 = 36$

$y = \pm 6 \rightsquigarrow y = 6$

$$2\pi \int_1^{25/9} y\sqrt{u}\, \frac{81}{8y}\, du$$

$$\frac{81\pi}{4}\int_1^{25/9} u^{\frac{1}{2}}\, du$$

$A = ?$

$$\frac{81\pi}{4}\int_1^{25/9} u^{\frac{1}{2}}\, du = \frac{81\pi}{4}\cdot\frac{2}{3}u^{\frac{3}{2}}\Big]_1^{25/9} = \frac{81\pi}{4}\cdot\frac{2}{3}\left[\left(\frac{25}{9}\right)^{\frac{3}{2}} - 1^{\frac{3}{2}}\right] = \frac{27\pi}{2}\left[\left(\frac{5}{3}\right)^3 - 1\right]$$

$$= \frac{27\pi}{2}\left[\frac{125}{27} - \frac{27}{27}\right] = \frac{27\pi}{2}\left[\frac{125-27}{27}\right] = \frac{27\pi}{2}\cdot\frac{98}{27} = \boxed{49\pi}$$

3. Find the area of the surface obtained by rotating the curve

$$9x = y^2 + 18, \quad \text{for } 2 \le x \le 6$$

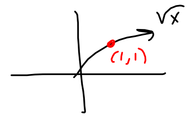
about the x-axis.

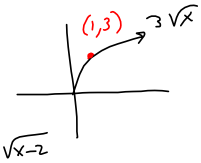
$x=g(y)$ is harder to graph

$y=f(x)$ is easier to graph, but may not be practical to find.

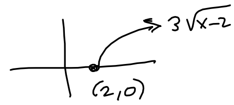
$y^2 = 9x-18$

$y = \pm\sqrt{9x-18} = \pm\sqrt{9(x-2)} = \pm 3\sqrt{x-2}$





We'll use the top half & spin it.



$y = 3\sqrt{x-2} = 3(x-2)^{\frac12}$

$\Rightarrow y' = 3\left(\frac12\right)(x-2)^{-\frac12} = \frac32 (x-2)^{-\frac12}$

$\left((x-2)^{-\frac12}\right)^2 = (x-2)^{-1}$

$\Rightarrow (y')^2 = \frac94 (x-2)^{-1} = \frac{9}{4(x-2)}$

$\Rightarrow \sqrt{1+(y')^2} = \sqrt{1+\frac{9}{4(x-2)}} = \sqrt{\frac{4x-8+9}{4x-8}}$

$= \sqrt{\frac{4x+1}{4x-8}}$ so $2\pi\int_2^6 y\,ds = 2\pi\int_2^6 f(x)\sqrt{1+(f'(x))^2}\,dx$

$= 2\pi\int_2^6 3\sqrt{x-2}\sqrt{\frac{4x+1}{4(x-2)}}\,dx =$

$= 2\pi\int_2^6 \frac32\sqrt{4x+1}\,dx$ $\quad \boxed{(4x+1)^{\frac12}}$

$= \frac34\pi\int_{x=2}^{x=6}\boxed{\sqrt{4x+1}}\cdot 4\,dx = \frac{3\pi}{4}\left[\frac23(4x+1)^{\frac32}\right]_2^6$

Scratch:

$\sqrt{x-2}\sqrt{\frac{4x-5}{4(x-2)}}$

$= \sqrt{\frac{\cancel{(x-2)}(4x-5)}{4\cancel{(x-2)}}}$

$u = 4x-5$

$du = 4dx$

$dx = \frac{du}{4}$, etc.

$\frac{3\pi}{6}\left[(24+1)^{\frac32} - (9)^{\frac32}\right]$

$= \frac{\pi}{2}\left[(25)^{\frac32} - 9^{\frac32}\right]$

$= \frac{\pi}{2}\left[5^3 - 3^3\right] = \frac{\pi}{2}\left[125-27\right] = \frac{\pi}{2}\left[98\right] = \boxed{49\pi}$!

4. Find the area of the surface obtained by rotating the curve

$y = \frac{x^2}{4} - \frac{\ln x}{2}$, $1 \le x \le 7$ → $1 \le x \le 2$

about the x-axis. → y-axis

will work this other version, too

$r = y = f(x)$

$y' = \frac{1}{2}x - \frac{1}{2}\cdot\frac{1}{x} = \frac{1}{2}x - \frac{1}{2x}$

$\Rightarrow (y')^2 = \left(\frac{1}{2}x\right)^2 - 2\left(\frac{1}{2}x\right)\left(\frac{1}{2x}\right) + \left(-\frac{1}{2x}\right)^2 = \frac{1}{4}x^2 - \frac{1}{2} + \frac{1}{4x^2}$

$\Rightarrow 1 + f'(x)^2 = 1 + \frac{1}{4}x^2 - \frac{1}{2} + \frac{1}{4x^2} = \frac{1}{4}x^2 + \frac{1}{2} + \frac{1}{4x^2} = \left(\frac{1}{2}x + \frac{1}{2x}\right)^2$

$\Rightarrow \text{Area} = 2\pi \int_{1=x}^{7=x} y\,ds = 2\pi \int_1^7 f(x)\,ds = 2\pi \int_1^7 \left(\frac{x^2}{4} - \frac{\ln x}{2}\right)\sqrt{\left(\frac{1}{2}x + \frac{1}{2x}\right)^2}\,dx$

$\left|\frac{1}{2}x + \frac{1}{2x}\right| = \frac{1}{2}x + \frac{1}{2x}$ $1 \le x \le 7$

$= 2\pi \int_1^7 \left(\frac{x^2}{4} - \frac{\ln x}{2}\right)\left(\frac{1}{2}x + \frac{1}{2x}\right)dx$

Almost $\int u\,du$, but the signs wrong

$= 2\pi \int_1^7 \left(\frac{x^3}{8} + \frac{x}{8} - \frac{1}{4}x\ln x - \frac{\ln x}{4x}\right)dx$

$= 2\pi \int_1^7 \left(\frac{x^3}{8} + \frac{x}{8} - \frac{1}{4}\frac{\ln x}{x}\right)dx - 2\pi \int_1^7 \frac{1}{4}x\ln x\,dx = A - B$

$A = 2\pi\left[\frac{1}{8}\cdot\frac{1}{4}x^4 + \frac{1}{8}\cdot\frac{1}{2}x^2 - \frac{1}{4}\cdot\frac{(\ln(x))^2}{2}\right]_1^7 = 2\pi\left[\frac{1}{32}(7)^4 + \frac{1}{16}(7)^2 - \frac{1}{8}(\ln(7))^2 - \left(\frac{1}{32}(1)^4 + \frac{1}{16}(1)^2 - \frac{1}{8}\underbrace{(\ln(1))^2}_{=0}\right)\right]$

$\frac{1}{4}\int_1^7 (\ln x)\left(\frac{1}{x}dx\right)$

$= \frac{1}{4}\int u\,du = \frac{1}{4}\cdot\frac{u^2}{2} + C$

where $u = \ln x$
$du = \frac{1}{x}dx$

$= 2\pi\left[\frac{7^4}{32} + \frac{7^2}{16} - \frac{(\ln(7))^2}{8} - \frac{1}{32} - \frac{1}{16}\right]$

$= 2\pi\left[\frac{7^4 + 7^2\cdot 2 - 4\ln(7)^2 - 1 - 2}{32}\right]$

$= 2\pi\left[\frac{7^4 + 98 - 4(\ln(7))^2 - 3}{32}\right]$

$= 2\pi\left[\frac{2496 - 4(\ln(7))^2}{32}\right]$

Clean as you can get it, if you want an exact representation.

$\approx$ 487.1144918

$B = 2\pi \int_1^7 \frac{1}{4}x\ln x\,dx = \frac{\pi}{2}\int_1^7 (\ln x)(x\,dx) = uv - \int v\,du$

$u = \ln x$ $dv = x\,dx$
$du = \frac{1}{x}dx$ $v = \frac{1}{2}x^2$

$= \frac{\pi}{2}\left[\frac{1}{2}x^2\ln x\Big|_1^7 - \int_1^7 \frac{1}{2}x\,dx\right]$

$= \frac{\pi}{2}\left[\frac{1}{2}(7)^2\ln(7) - \frac{1}{2}(1)^2\underbrace{\ln(1)}_{0} - \frac{1}{4}x^2\Big|_1^7\right]$

$= \frac{\pi}{2}\left[\frac{49}{2}\ln(7) - \left(\frac{1}{4}(7^2 - 1^2)\right)\right]$

$= \frac{\pi}{2}\left[\frac{49}{2}\ln(7) - \frac{1}{4}(48)\right]$

$= \frac{\pi}{2}\left[\frac{49}{2}\ln(7) - 12\right]$

$= \frac{49\pi}{4}\ln(7) - 6\pi$

$\approx$ 56.03784269 $\approx B$

$\Rightarrow$ Area $= A - B \approx$ 487.1144918 – 56.03784269 = $\boxed{431.0766491 \approx \text{Area}}$

$\boxed{431.0766492 \text{ By Maple}}$

Mar 20-2:26 PM

Bonus – A gate in an irigation canal is constructed in the form of a trapezoid 3 ft wide at the bottom, 5 ft wide at the top, and 2 ft high. It is placed vertically in the canal, with the water extending to its top. Find the hydrostatic force on one side of the gate.

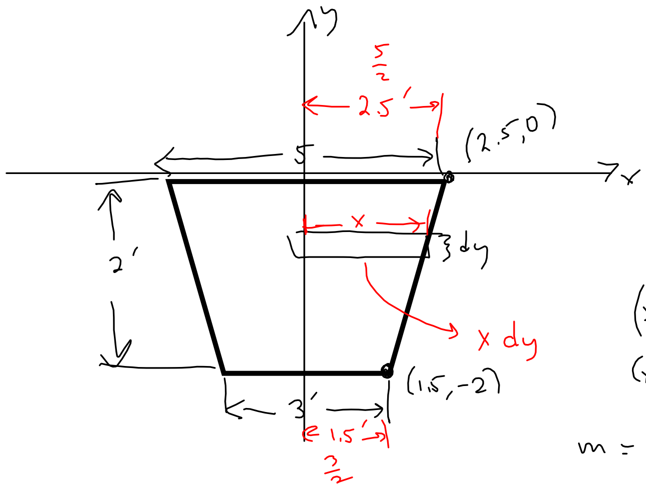


Figure out the force on $\frac{1}{2}$ the gate

$(x_1,y_1) = (2.5, 0)$

$(x_2,y_2) = (1.5, -2)$

$m = \frac{y_2-y_1}{x_2-x_1} = \frac{-2-0}{1.5-2.5} = \frac{-2}{-1} = 2$

$y = 2(x-2.5)+0$

$y = 2x-5 \Longrightarrow$

$2x-5 = y$

$2x = y+5$

$x = \frac{y+5}{2}$

Force on one rectangle

is $\left(\frac{\text{Force}}{\text{unit area}}\right)(\text{Area})$

is $\left(\frac{\text{weight}}{(\text{ft})^3} \cdot \text{depth}\right)(\text{Area})$

is $\left(\frac{62.5\text{ lb}}{\text{ft}^3}\right)(-y\text{ ft})\left(\frac{y+5}{2}\,dy\right)$

$\therefore$ Force $-62.5\int_{-2}^{0} y\left(\frac{y+5}{2}\right)dy = -\frac{62.5}{2}\int_{-2}^{0}(y^2+5y)\,dy$

$= -\frac{62.5}{2}\left[\frac{1}{3}y^3 + \frac{5}{2}y^2\right]_{-2}^{0} = -\frac{62.5}{2}\left[-\left(\frac{1}{3}(-2)^3 + \frac{5}{2}(-2)^2\right)\right]$

$= -\frac{62.5}{2}\left[-\left(-\frac{8}{3}\right)+10\right] = -\frac{62.5}{2}\left[\frac{8}{3} - \frac{10}{1}\cdot\frac{3}{3}\right] = -\frac{62.5}{2}\left[\frac{-22}{3}\right]$

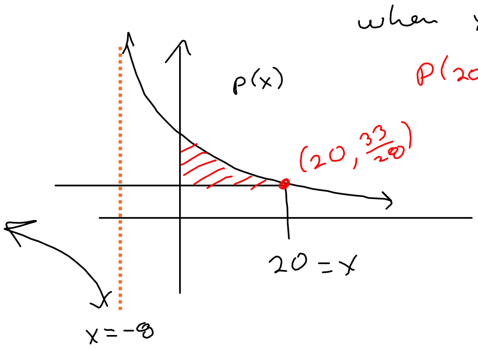
$= \frac{(62.5)(11)}{3} = 229.1\overline{6} \Longrightarrow$ Total force is $458.\overline{3}$ or about

$\boxed{458 \text{ lbs}}$

5. A demand curve is given by $p = \dfrac{33}{x+8}$. Find the consumer surplus when the selling price is \$20.

x = amount that can be sold.

p = price customers are willing to pay when x is the amount of product available.



$P(20) = \frac{33}{28}$

$CS = \int_0^{20} p(x)\,dx$ − Area of rectangle under P(20)

$= \int_0^{20} \frac{33}{x+8}\,dx - (20)\left(\frac{33}{28}\right)$

$= 33 \ln|x+8| \Big]_0^{20} - \frac{165}{7}$

$= 33\left[\ln 28 - \ln(7)\right] - \frac{165}{7}$

$= 33 \ln\left(\frac{28}{7}\right) - \frac{165}{7} = 33\ln(4) - \frac{165}{7}$

$\approx$ 22.17628535 $\approx \boxed{\$22.18}$

1. For what values of r does the function $y = e^{rt}$ satisfy the differential equation

$$y'' + y' - 20y = 0?$$

Operator Notation:

$Dy = y'$, $D^2 y = \frac{d^2}{dx}[y] = y''$

$(D^2 + D - 20)y = 0$

$(D+5)(D-4)y = 0$

$D = -5, 4 = r$-values!

$e^{-5t} = y \Rightarrow y' = -5e^{-5t}, y'' = 25e^{-5t}$

$y'' + y' - 20y = 25e^{-5t} + (-5e^{-5t}) - 20e^{-5t} = 0$ ✓

NOTE THAT Ce^{-5t} is also a sol'n $\forall C \in \mathbb{R}$

General Sol'n of the equation

is $C_1 e^{-5t} + C_2 e^{4t}$

and we'd need initial or boundary values (2 of 'em)

to determine C_1 & C_2

2. For what nonzero values of k does the function $y = \sin(kt)$ satisfy $y'' + 4y = 0$?

$$(D^2+4)y=0 \Rightarrow D=\pm 2i \rightsquigarrow c_1e^{2it}+c_2e^{-2it}$$

$$= k\sin($$

$y = \sin(kt)$

$y' = k\cos(kt)$

$y'' = -k^2\sin(kt)$ $\quad y''+4y = -k^2\sin(kt)+4\sin(kt)=0$

$$(-k^2+4)\sin(kt)=0$$

$$-k^2+4=0$$

$$-k^2=-4$$

$$\boxed{k=\pm 2}$$

Mar 23-10:12 AM

3. Solve the differential equation $5yy' = 3x$.

$$5y\frac{dy}{dx} = 3x$$

$$5y\,dy = 3x\,dx$$

$$5\int y\,dy = 3\int x\,dx$$

$$\frac{5}{2}y^2 = \frac{3}{2}x^2 + \hat{C}$$

$$y^2 = \frac{2}{5}\cdot\frac{3}{2}x^2 + \frac{2}{5}\hat{C}$$

$$y^2 = \frac{3}{5}x^2 + C$$

$$y = \pm\sqrt{\frac{3}{5}x^2 + C}$$

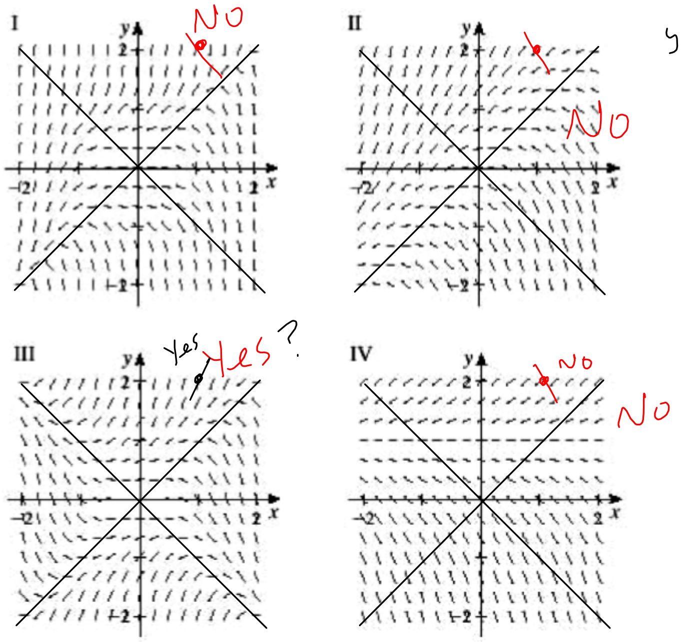
$$y' = \frac{1}{2}\left(\frac{3}{5}x^2 + C\right)^{-\frac{1}{2}}\left(\frac{6}{5}x\right)$$

$$3yy' = 5\left(\sqrt{\frac{3}{5}x^2+C}\right)\left(\left(\frac{x}{\sqrt{\frac{3}{5}x^2+C}}\right)\left(\frac{6}{5}\right)\left(\frac{1}{2}\right)\right)$$

$$= 3x \checkmark$$ (after some mess...)

Mar 19-10:04 AM

4. Select a direction field for the differential equation $y' = y^2 - x^2$ from a set of direction fields labeled I-IV.



$$\left.\begin{array}{l} y = x \\ y = -x \end{array}\right\} y' = 0$$

(1,2) just to check

$2^2 - 1^2 = 3$

But $y'\Big|_{\substack{x=1 \\ y=}}$

5. The functions $y = Ce^{2x^2}$ (for any constant C) are solutions of the differential equation $y' = 4xy$. Find the solution that satisfies the initial condition $y(1) = 1$.

Not asked, but a nice separable equation we can solve.

$y' = 4xy$

$\frac{y'}{y} = 4x$

$\frac{\frac{dy}{dx}}{y} = \frac{1}{y} \cdot \frac{dy}{dx} = 4x$

$\int \frac{dy}{y} = 4\int x\,dx$

$\ln|y| = 4 \cdot \frac{1}{2}x^2 + C = 2x^2 + \hat{C}$

(Assuming $y > 0$)

$\ln y = 2x^2 + \hat{C}$

$e^{\ln y} = e^{2x^2 + \hat{C}}$

$y = e^{2x^2} e^{\hat{C}}$

$y = e^{2x^2} \cdot C = Ce^{2x^2}$

$y(1) = 1$

$y(1) = Ce^{2(1)^2} = 1$

$Ce^2 = 1$

$\boxed{C = \frac{1}{e^2}}$

The actual work that was asked.

6. Solve the differential equation $y' = \dfrac{7x^6 y}{\ln y}$

$$\frac{\ln y}{y}\, y' = 7x^6$$

$$\int \ln y \left(\frac{1}{y}\right) dy = 7\int x^6\, dx$$

$$u \quad (\quad du \quad)$$

$u = \ln y$

$du = \frac{1}{y}dy$ ✓

$$\Longrightarrow \frac{(\ln y)^2}{2} = x^6 + \hat{C}$$

$$(\ln y)^2 = 2x^6 + 2\hat{C} = 2x^6 + C$$

$$\ln(y) = \pm\sqrt{2x^6 + C}$$

Sol'ns not func.!

$$y = e^{\pm\sqrt{2x^6+C}}$$

$e^{\sqrt{2x^6+C}}$

or

$e^{-\sqrt{2x^6+C}}$

S9.4 #7 $\boxed{P(1) = 2500}$ $P(0) = P_0 = 1000$

$M = 10{,}000$ = Carrying capacity.

Logistic Model : $P(t) = \dfrac{M}{1+Ae^{-kt}}$

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$A = \dfrac{M - P_0}{P_0}$

$= \dfrac{10000}{1+9}$

$= \dfrac{10000 - 1000}{1000} = 9 = A$

$P(0) = \dfrac{10000}{1+9e^{-k(0)}}$ No joy.

$P(1) = \dfrac{10000}{1+9e^{-k(1)}} = \dfrac{10000}{1+9e^{-k}} \overset{\text{SET}}{=} 2500$

$\Rightarrow 10000 = 2500(9e^{-k}+1) = 22500e^{-k} + 2500$

$22500e^{-k} = 10000 - 2500$

$e^{-k} = \dfrac{7500}{22500} = \dfrac{\overset{3}{\cancel{75}}}{\underset{9}{\cancel{225}}} = \dfrac{1}{3}$

$\ln(\quad) = \ln(\quad)$

$-k = \ln\left(\frac{1}{3}\right) = -\ln(3)$

$k = \ln(3)$

Derivation?

$P(t) = \dfrac{10000}{1+9e^{-\ln(3)t}} = \boxed{\dfrac{10000}{1+9\left(\frac{1}{3}\right)^t} = P(t)}$

$e^{(-\ln(3))t} = e^{\left(\ln\left(\frac{1}{3}\right)\right)\cdot t} = \left(e^{\ln\left(\frac{1}{3}\right)}\right)^t$

$= \left(\frac{1}{3}\right)^t$

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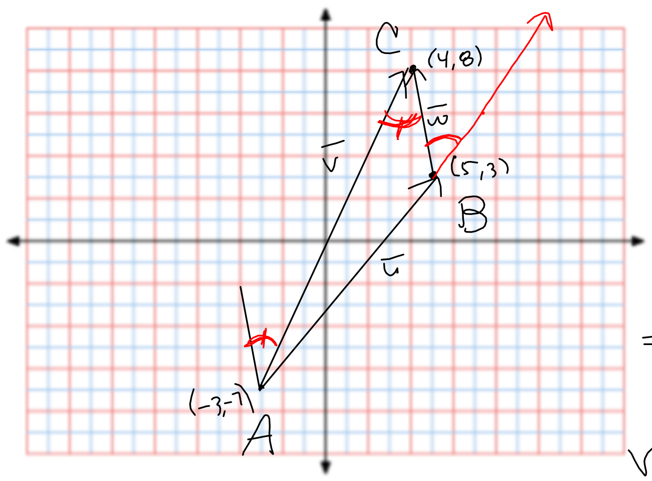
$(-3,-7)$, $(4,8)$, $(5,3)$

$$\cos\theta = \frac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|\|\bar{v}\|}$$



$\bar{w} = \langle 4-5, 8-3 \rangle = \langle -1, 5 \rangle$

$\bar{u} = \langle 5+3, 3+7 \rangle = \langle 8, 10 \rangle$

$\bar{v} = \langle 4+3, 15 \rangle = \langle 7, 15 \rangle$

$$\cos C = \frac{\bar{v}\cdot\bar{w}}{\|\bar{v}\|\|\bar{w}\|} = \frac{-7+75}{\sqrt{26}\sqrt{274}} = \frac{68}{2\sqrt{1781}} = \frac{34}{\sqrt{1781}}$$

$$\sqrt{26}\sqrt{274} = \sqrt{2\cdot 13\cdot 2\cdot 137} = 2\sqrt{1781}$$

$$\cos A = \frac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|\|\bar{v}\|} = \frac{206}{\sqrt{164}\sqrt{274}} \approx .9717846055$$

$$\Longrightarrow A \approx 13.64291478^\circ$$

$\bar{u}\cdot\bar{v} = \langle 8, 10 \rangle \cdot \langle 7, 15 \rangle$

$\sqrt{64+100}$ $\quad 49+225 = 274$

$$B = 180^\circ - \frac{\bar{u}\cdot\bar{w}}{\|\bar{u}\|\|\bar{w}\|}$$