1–10 Solve the differential equation.

1. $\frac{dy}{dx} = xy^2$

$$\int \frac{dy}{y^2} = \int x\,dx$$

$-2+1=-1$

$$\frac{y^{-1}}{-1} = \frac{1}{2}x^2 + C$$

$$-\frac{1}{y} = \frac{1}{2}x^2 + C$$

$$\boxed{\frac{-1}{\frac{1}{2}x^2 + C} = y}$$ for some C.

$$\frac{-y^{-1}}{-1} + C = \frac{1}{2}x^2$$

$$-\frac{1}{y} = \frac{1}{2}x^2 - C$$

$$-\frac{1}{y} = \frac{1}{2}x^2 + \hat{C}$$

4. $(y^2 + xy^2)y' = 1$

$$(1+x)y^2 y' = 1$$

$$y^2 y' = \frac{1}{x+1}$$

$$y^2 \frac{dy}{dx} = \frac{1}{x+1}$$

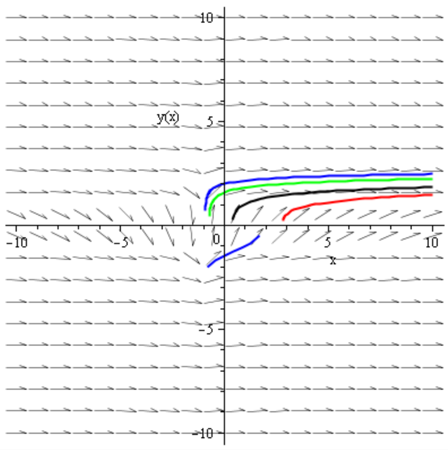
$$\int y^2\,dy = \int \frac{dx}{x+1}$$

$$\frac{1}{3}y^3 = \ln|x+1| + \hat{C}$$

$$y^3 = 3\ln|x+1| + 3\hat{C}$$

$$y = \sqrt[3]{3\ln|x+1| + C}$$

$with(DEtools) : with(plots) :$
$num4 := DEplot\left(\left[diff(y(x), x)\cdot\left(y(x)^2 + x\cdot y(x)^2\right) = 1\right], [y], x = -10..10, y = -10..10, [y(1) = -1], color = black, linecolor = blue, thickness = 2\right) : \% :$
$mysolutions := plot\left(\left[\sqrt[3]{3\cdot\ln(x+1) - 4}, \sqrt[3]{3\cdot\ln(x+1) + 8}, \sqrt[3]{3\cdot\ln(x+1) - 1}, \sqrt[3]{3\cdot\ln(x+1) + 4}\right], x = -10..10, y = -10..10, thickness = 2, color = [red, blue, black, green]\right) : \% :$
$display([num4, mysolutions])$



8. $\frac{dy}{d\theta} = \frac{e^y \sin^2\theta}{y \sec\theta}$

$$y e^{-y}\, dy = \frac{\sin^2\theta}{\sec\theta}\, d\theta = \sin^2\theta \cos\theta \, d\theta$$

$u = y \quad dv = e^{-y} dy$

$du = dy \quad v = -e^{-y}$

$uv - \int v\, du$

$-ye^{-y} + \int e^{-y} dy$

$$\boxed{-ye^{-y} - e^{-y} = \frac{1}{3}\sin^3\theta + C}$$

$u = \sin\theta, \quad du = \cos\theta\, d\theta$

$$\int \sin^2\theta \cos\theta \, d\theta = \int u^2\, du = \frac{1}{3}u^3 + C = \frac{1}{3}\sin^3\theta + C$$

11–18 Find the solution of the differential equation that satisfies the given initial condition.

14. $y' = \dfrac{xy \sin x}{y + 1}, \quad y(0) = 1$

$$\frac{y+1}{y} y' = x \sin x$$

$$\left(1 + \frac{1}{y}\right) y' = x \sin x$$

$$\int \left(1 + \frac{1}{y}\right) dy = \int x \sin x \, dx$$

$$\boxed{y + \ln|y| = \sin(x) - x\cos(x) + C}$$

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15. $x \ln x = y(1 + \sqrt{3 + y^2})y', \quad y(1) = 1$

$$x \ln x = y(1 + \sqrt{3+y^2}) \frac{dy}{dx} \quad ; \; y(1) = 1$$

$$\int x \ln x \, dx = \int y + \sqrt{3+y^2} \, y \, dy$$

$u = \ln x \qquad dv = x$

$du = \frac{1}{x} dx \qquad v = \frac{x^2}{2}$

$u = 3 + y^2$

$du = 2y \, dy$

$$uv - \int v \, du = \frac{1}{2}x^2 \ln x - \frac{1}{2}\int x^2 \cdot \frac{1}{x} dx = \frac{1}{2}y^2 + \frac{1}{2}\int (3+y^2)^{\frac{1}{2}} (2y\,dy)$$

$$\Rightarrow \frac{1}{2}x^2 \ln x - \frac{1}{2}\int x \, dx = \frac{1}{2}(3+y^2)^{\frac{3}{2}} \cdot \frac{2}{3} + \frac{1}{2}y^2$$

$$\Rightarrow \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C = \frac{1}{3}\sqrt{(3+y^2)^3} + \frac{1}{2}y^2$$

Use $y(1) = 1$ to find C:

$$\frac{1}{2}\ln(1) - \frac{1}{4} + C = \frac{1}{3}\sqrt{(3+1)^3} + \frac{1}{2}$$

$$-\frac{1}{4} + C = \frac{1}{3} \cdot 8 + \frac{1}{2}$$

$\sqrt{4^3}$
$= (\sqrt{4})^3$
$= 2^3 = 8$

$$-\frac{1}{4} + C = \frac{8}{3} \cdot \frac{2}{2} + \frac{1}{2} \cdot \frac{3}{3} = \frac{16+3}{6} = \frac{19}{6}$$

$$C = \frac{19}{6} \cdot \frac{2}{2} + \frac{1}{4} \cdot \frac{3}{3} = \frac{38+3}{12} = \boxed{\frac{41}{12} = C}$$

36. Find a function f such that $f(3) = 2$ and

$$(t^2+1)f'(t) + [f(t)]^2 + 1 = 0 \qquad t \neq 1$$

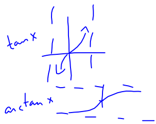
[*Hint:* Use the addition formula for $\tan(x+y)$ on Reference Page 2.]

$$(t^2+1)f'(t) = -f(t)^2 - 1$$

$$\frac{f'(t)}{-f(t)^2-1} = \frac{1}{t^2+1}$$

$$-\frac{\frac{df}{dt}}{(f^2+1)} = \frac{1}{t^2+1}$$

$$-\int \frac{df}{f^2+1} = \int \frac{dt}{t^2+1}$$



$$-\arctan(f) = \arctan(t) + C$$

$\frac{\pi}{4}$ for $f(3)=2$ prob.

$$\tan(\quad) = \tan(\quad)$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$-f = \tan(\arctan(t) + C)$$

$$-f = \frac{t + \tan(c)}{1 - t\tan(c)}$$

$$f = \frac{t + \tan(c)}{t\tan(c) - 1}$$

$\tan(-f) = -\tan(f)$ tangent is odd func.

$$f(3)=2 \Rightarrow \frac{3+\tan(c)}{3\tan(c)-1} = 2$$

$$\tan c + 3 = 6\tan c - 2$$

$$-5\tan c = -5$$

$$\tan c = 1$$

$$\boxed{c = \frac{\pi}{4}}$$

RESTART:

$$-\arctan(f) + C = \arctan(t)$$

$$C = \arctan(t) + \arctan(f)$$

$$\tan(c) = \tan(\arctan(t) + \arctan(f)) = \frac{\tan(\arctan(t)) + \tan(\arctan(f))}{1 - \tan(\arctan(t))\tan(\arctan(f))}$$

$$\tan(c) = \frac{t+f}{1-tf}$$

$f(3) = 2 \Rightarrow$

$$\tan(c) = \frac{3+2}{1-2\cdot 3} = \frac{5}{1-6} = \frac{5}{-5} = -1$$

$$\boxed{c = -\frac{\pi}{4}}$$

$$-\arctan(y) - \frac{\pi}{4} = \arctan(t)$$

$$-1 = \frac{t+f}{1-tf}$$

$$tf - 1 = t + f$$

$$tf - f = t + 1$$

$$f(t-1) = t+1$$

$$f = \frac{t+1}{t-1}$$

$$\boxed{f(t) = \frac{t+1}{t-1}}$$