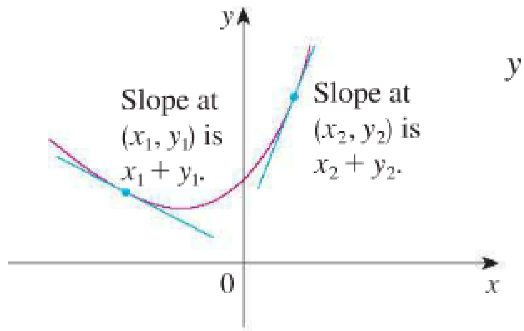
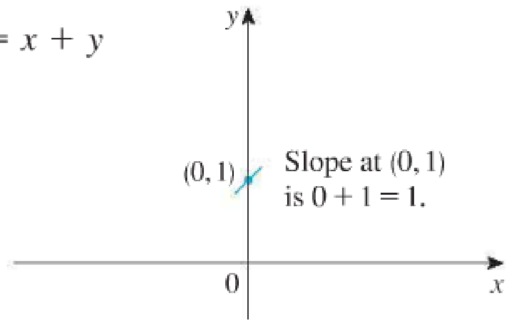
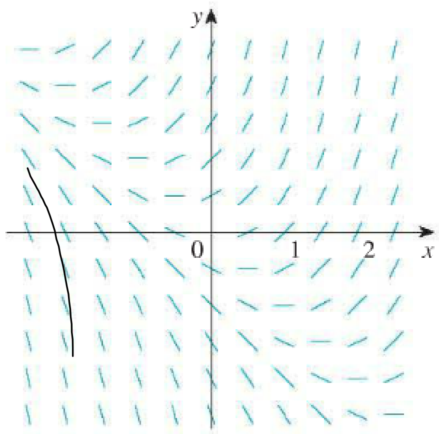
Section 9.2 - Direction Fields and Euler's Method



$y' = x + y$





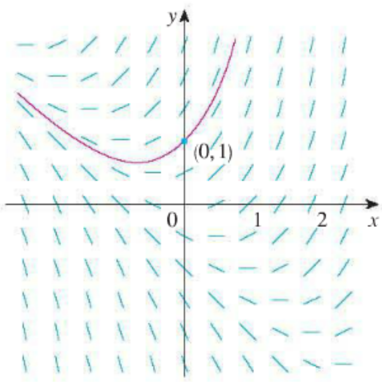


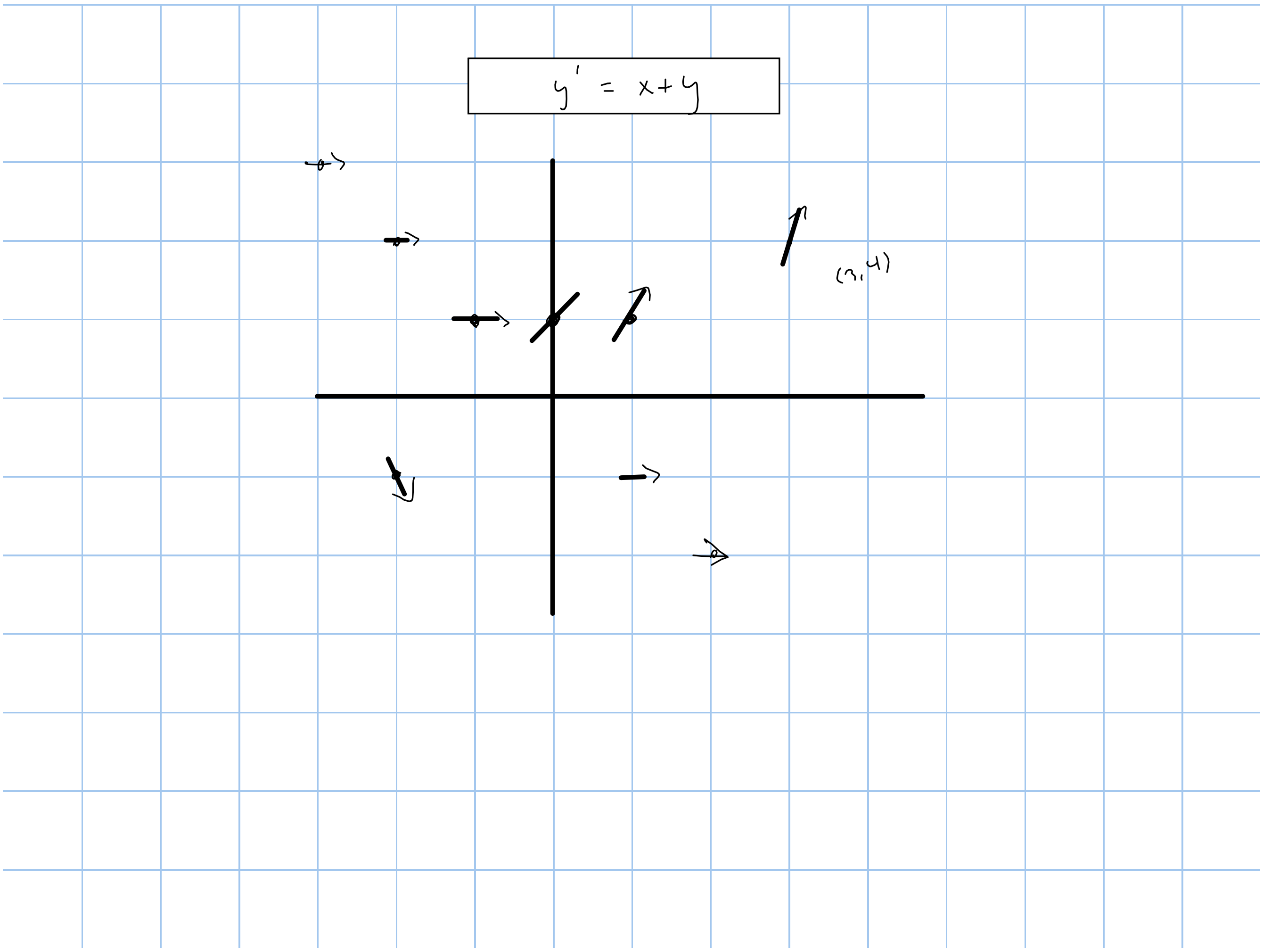
FIGURE 4
The solution curve through (0, 1)

$y = -x - 2$

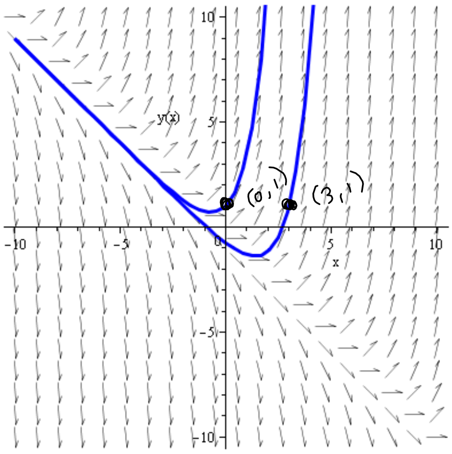
$y' = x + y$

$= x + (-x - 2)$

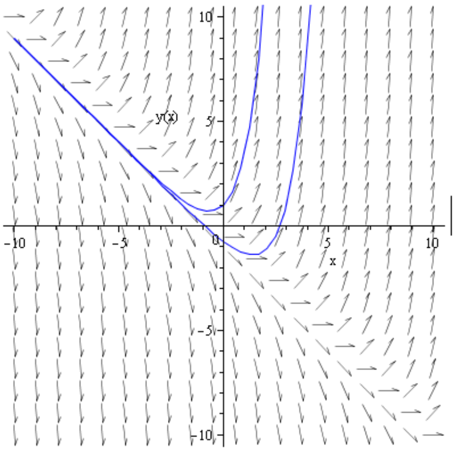
$= -2$


y' = x+y
(3,4)

$DEplot([diff(y(x), x) = x + y(x)], [y], x = -10..10, y = -10..10, [y(0) = 1, y(3) = 1], color = black, linecolor = blue)$

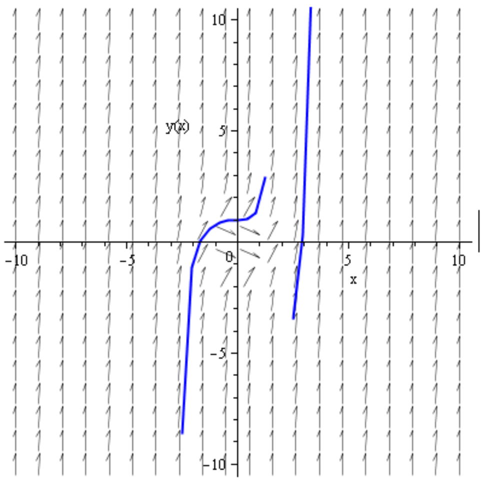


Dir. Field
for $y' = x+y$



with thinner
"thickness = 1"
argument
in DEplot.

$$y' = x^2 + y^2 - 1.$$



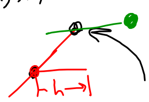
Euler's Method:

$y' = F(x,y)$ $\quad (x_0, y_0)$

Start with a point & it gives you

$y' = F(x_0, y_0)$ = slope at that point

(x_1, y_1)



(x_0, y_0)

$y_1 = y_0 + mh$

h = stepsize

$x_1 = x_0 + h$

$y_1 = y_0 + F(x_0, y_0) \cdot h$

$\vdots$

$y_{n+1} = y_n + F(x_n, y_n)\, h$

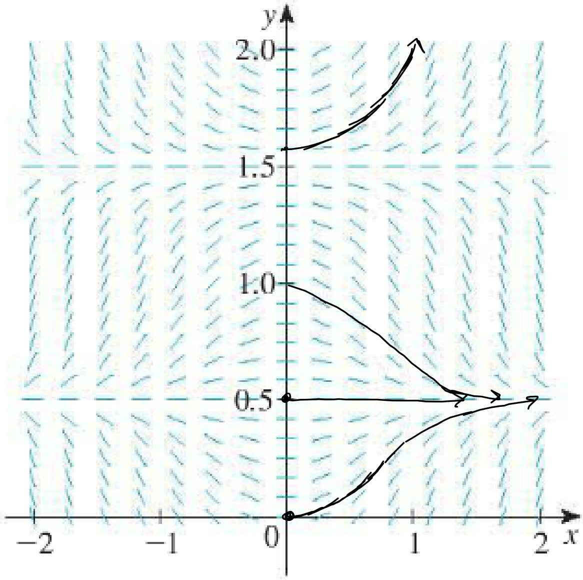
1. A direction field for the differential equation $y' = x \cos \pi y$ is shown.

(a) Sketch the graphs of the solutions that satisfy the given initial conditions.

(i) $y(0) = 0$ (ii) $y(0) = 0.5$

(iii) $y(0) = 1$ (iv) $y(0) = 1.6$

(b) Find all the equilibrium solutions.



(a)(i) $y(0)=0$ (iv) $y(0)=1.6$

(ii) $y(0)=.5$

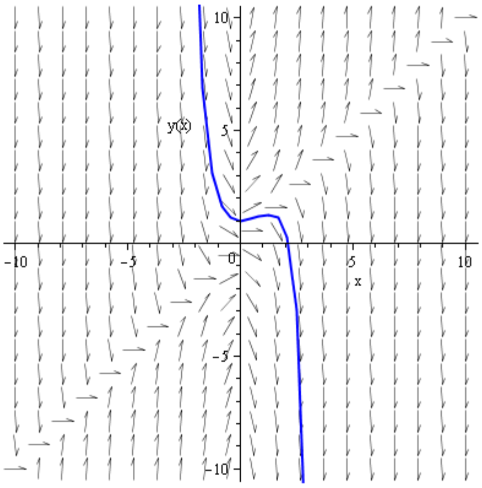
(iii) $y(0)=1$

(b) $y=1.5$ unstable

(c) $y=0.5$ stable

11–14 Sketch the direction field of the differential equation. Then use it to sketch a solution curve that passes through the given point.

12. $y' = xy - x^2, \quad (0, 1)$



21. Use Euler's method with step size 0.5 to compute the approximate y-values y_1, y_2, y_3, and y_4 of the solution of the initial-value problem $y' = y - 2x$, $y(1) = 0$.

See spreadsheet stuff