

$$\int_0^1 x \cdot \exp(x) \, dx = 1$$

The Midpoint Method:

`evalf(ApproximateInt(x · exp(x), 0 .. 1, 'partition' = 10, 'method' = midpoint, 'partitiontype' = normal, 'output' = 'value'))`
 0.9981523883 (1)

`evalf(ApproximateInt(x · exp(x), 0 .. 1, 'partition' = 5, 'method' = midpoint, 'partitiontype' = normal, 'output' = 'value'))`
 0.9926210129 (2)

`evalf(ApproximateInt(x · exp(x), 0 .. 1, 'partition' = 20, 'method' = midpoint, 'partitiontype' = normal, 'output' = 'value'))`
 0.9995379178 (3)

Left Endpoint:

`evalf(ApproximateInt(x · exp(x), 0 .. 1, 'partition' = 5, 'method' = left, 'partitiontype' = normal, 'output' = 'value'))`
 0.7429428906 (4)

`evalf(ApproximateInt(x · exp(x), 0 .. 1, 'partition' = 10, 'method' = left, 'partitiontype' = normal, 'output' = 'value'))`
 0.8677819517 (5)

`evalf(ApproximateInt(x · exp(x), 0 .. 1, 'partition' = 20, 'method' = left, 'partitiontype' = normal, 'output' = 'value'))`
 0.9329671699 (6)

Right Endpoint:

$$\text{evalf}(\text{ApproximateInt}(x \cdot \exp(x), 0..1, 'partition' = 5, 'method' = \text{right}, 'partitiontype' = \text{normal}, 'output' = 'value'))$$

$$1.286599256 \quad (7)$$

$$\text{evalf}(\text{ApproximateInt}(x \cdot \exp(x), 0..1, 'partition' = 10, 'method' = \text{right}, 'partitiontype' = \text{normal}, 'output' = 'value'))$$

$$1.139610134 \quad (8)$$

$$\text{evalf}(\text{ApproximateInt}(x \cdot \exp(x), 0..1, 'partition' = 20, 'method' = \text{right}, 'partitiontype' = \text{normal}, 'output' = 'value'))$$

$$1.068881261 \quad (9)$$

Trapezoid:

$$\text{evalf}(\text{ApproximateInt}(x \cdot \exp(x), 0..1, 'partition' = 5, 'method' = \text{trapezoid}, 'partitiontype' = \text{normal}, 'output' = 'value'))$$

$$1.014771073 \quad (10)$$

$$\text{evalf}(\text{ApproximateInt}(x \cdot \exp(x), 0..1, 'partition' = 10, 'method' = \text{trapezoid}, 'partitiontype' = \text{normal}, 'output' = 'value'))$$

$$1.003696043 \quad (11)$$

$$\text{evalf}(\text{ApproximateInt}(x \cdot \exp(x), 0..1, 'partition' = 20, 'method' = \text{trapezoid}, 'partitiontype' = \text{normal}, 'output' = 'value'))$$

$$1.000924216 \quad (12)$$

Simpson's:

$$\text{evalf}(\text{ApproximateInt}(x \cdot \exp(x), 0..1, 'partition' = 5, 'method' = \text{simpson}, 'partitiontype' = \text{normal}, 'output' = 'value'))$$

$$1.000004366 \quad (13)$$

$$\text{evalf}(\text{ApproximateInt}(x \cdot \exp(x), 0..1, 'partition' = 10, 'method' = \text{simpson}, 'partitiontype' = \text{normal}, 'output' = 'value'))$$

$$1.000004366 \quad (14)$$

$$\text{evalf}(\text{ApproximateInt}(x \cdot \exp(x), 0..1, 'partition' = 20, 'method' = \text{simpson}, 'partitiontype' = \text{normal}, 'output' = 'value'))$$

$$1.000000273 \quad (15)$$

$$|E_T| \leq \frac{K(b-a)^3}{12n^2} \quad E_M \leq \frac{K(b-a)^3}{24n^2} \quad \text{where } K \geq |f''(x)| \text{ on } [a,b]$$

$$|E_S| \leq \frac{K(b-a)^5}{180n^4} \quad \text{where } K \geq |f^{(4)}(x)| \text{ on } [a,b]$$

$$f(x) = xe^x$$

$$f'(x) = e^x + xe^x = (x+1)e^x$$

$$f''(x) = e^x + (x+1)e^x = \boxed{(x+2)e^x}$$

$$= e^x + xe^x + e^x = xe^x + 2e^x = (x+2)e^x$$

$$f'''(x) = e^x + (x+2)e^x = (x+3)e^x$$

$$f^{(4)}(x) = (x+4)e^x$$

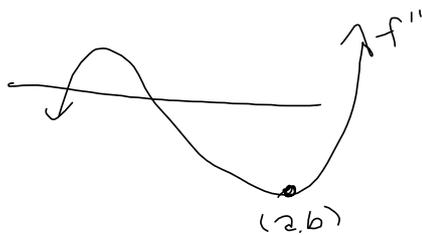
$$|f'(x)| \text{ on}$$

$$f''(x) = (x+2)e^x$$

$$f'''(x) = (x+3)e^x > 0 \text{ on } [0,1], \text{ so}$$

f'' increasing

$$f''(x) \leq f''(1) = \boxed{3e \equiv K}$$



If f'' is negative and its min is greater (in absolute value) than its positive max.

$$|f''(x)| \leq |b| = K$$