

1. $\int u \, dv = uv - \int v \, du$

2. $\int u^n \, du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$

3. $\int \frac{du}{u} = \ln |u| + C$

4. $\int e^u \, du = e^u + C$

5. $\int a^u \, du = \frac{a^u}{\ln a} + C$

6. $\int \sin u \, du = -\cos u + C$

7. $\int \cos u \, du = \sin u + C$

8. $\int \sec^2 u \, du = \tan u + C$

9. $\int \csc^2 u \, du = -\cot u + C$

10. $\int \sec u \tan u \, du = \sec u + C$

11. $\int \csc u \cot u \, du = -\csc u + C$

12. $\int \tan u \, du = \ln |\sec u| + C$

13. $\int \cot u \, du = \ln |\sin u| + C$

14. $\int \sec u \, du = \ln |\sec u + \tan u| + C$

15. $\int \csc u \, du = \ln |\csc u - \cot u| + C$

16. $\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \frac{u}{a} + C, \quad a > 0$

17. $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$

18. $\int \frac{du}{u \sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{u}{a} + C$

19. $\int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left| \frac{u+a}{u-a} \right| + C$

20. $\int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \left| \frac{u-a}{u+a} \right| + C$

$$21. \int \sqrt{a^2 + u^2} du = \frac{u}{2} \sqrt{a^2 + u^2} + \frac{a^2}{2} \ln(u + \sqrt{a^2 + u^2}) + C$$

$$22. \int u^2 \sqrt{a^2 + u^2} du = \frac{u}{8} (a^2 + 2u^2) \sqrt{a^2 + u^2} - \frac{a^4}{8} \ln(u + \sqrt{a^2 + u^2}) + C$$

$$23. \int \frac{\sqrt{a^2 + u^2}}{u} du = \sqrt{a^2 + u^2} - a \ln \left| \frac{a + \sqrt{a^2 + u^2}}{u} \right| + C$$

$$24. \int \frac{\sqrt{a^2 + u^2}}{u^2} du = -\frac{\sqrt{a^2 + u^2}}{u} + \ln(u + \sqrt{a^2 + u^2}) + C$$

$$25. \int \frac{du}{\sqrt{a^2 + u^2}} = \ln(u + \sqrt{a^2 + u^2}) + C$$

$$26. \int \frac{u^2 du}{\sqrt{a^2 + u^2}} = \frac{u}{2} \sqrt{a^2 + u^2} - \frac{a^2}{2} \ln(u + \sqrt{a^2 + u^2}) + C$$

$$27. \int \frac{du}{u \sqrt{a^2 + u^2}} = -\frac{1}{a} \ln \left| \frac{\sqrt{a^2 + u^2} + a}{u} \right| + C$$

$$28. \int \frac{du}{u^2 \sqrt{a^2 + u^2}} = -\frac{\sqrt{a^2 + u^2}}{a^2 u} + C$$

$$29. \int \frac{du}{(a^2 + u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2 + u^2}} + C$$

30. $\int \sqrt{a^2 - u^2} du = \frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a} + C$

31. $\int u^2 \sqrt{a^2 - u^2} du = \frac{u}{8} (2u^2 - a^2) \sqrt{a^2 - u^2} + \frac{a^4}{8} \sin^{-1} \frac{u}{a} + C$

32. $\int \frac{\sqrt{a^2 - u^2}}{u} du = \sqrt{a^2 - u^2} - a \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$

33. $\int \frac{\sqrt{a^2 - u^2}}{u^2} du = -\frac{1}{u} \sqrt{a^2 - u^2} - \sin^{-1} \frac{u}{a} + C$

34. $\int \frac{u^2 du}{\sqrt{a^2 - u^2}} = -\frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a} + C$

35. $\int \frac{du}{u \sqrt{a^2 - u^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$

36. $\int \frac{du}{u^2 \sqrt{a^2 - u^2}} = -\frac{1}{a^2 u} \sqrt{a^2 - u^2} + C$

37. $\int (a^2 - u^2)^{3/2} du = -\frac{u}{8} (2u^2 - 5a^2) \sqrt{a^2 - u^2} + \frac{3a^4}{8} \sin^{-1} \frac{u}{a} + C$

38. $\int \frac{du}{(a^2 - u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2 - u^2}} + C$

39. $\int \sqrt{u^2 - a^2} du = \frac{u}{2} \sqrt{u^2 - a^2} - \frac{a^2}{2} \ln |u + \sqrt{u^2 - a^2}| + C$

40. $\int u^2 \sqrt{u^2 - a^2} du = \frac{u}{8} (2u^2 - a^2) \sqrt{u^2 - a^2} - \frac{a^4}{8} \ln |u + \sqrt{u^2 - a^2}| + C$

41. $\int \frac{\sqrt{u^2 - a^2}}{u} du = \sqrt{u^2 - a^2} - a \cos^{-1} \frac{a}{|u|} + C$

42. $\int \frac{\sqrt{u^2 - a^2}}{u^2} du = -\frac{\sqrt{u^2 - a^2}}{u} + \ln |u + \sqrt{u^2 - a^2}| + C$

43. $\int \frac{du}{\sqrt{u^2 - a^2}} = \ln |u + \sqrt{u^2 - a^2}| + C$

44. $\int \frac{u^2 du}{\sqrt{u^2 - a^2}} = \frac{u}{2} \sqrt{u^2 - a^2} + \frac{a^2}{2} \ln |u + \sqrt{u^2 - a^2}| + C$

45. $\int \frac{du}{u^2 \sqrt{u^2 - a^2}} = \frac{\sqrt{u^2 - a^2}}{a^2 u} + C$

46. $\int \frac{du}{(u^2 - a^2)^{3/2}} = -\frac{u}{a^2 \sqrt{u^2 - a^2}} + C$

47. $\int \frac{u \, du}{a + bu} = \frac{1}{b^2} (a + bu - a \ln |a + bu|) + C$

48. $\int \frac{u^2 \, du}{a + bu} = \frac{1}{2b^3} [(a + bu)^2 - 4a(a + bu) + 2a^2 \ln |a + bu|] + C$

49. $\int \frac{du}{u(a + bu)} = \frac{1}{a} \ln \left| \frac{u}{a + bu} \right| + C$

50. $\int \frac{du}{u^2(a + bu)} = -\frac{1}{au} + \frac{b}{a^2} \ln \left| \frac{a + bu}{u} \right| + C$

51. $\int \frac{u \, du}{(a + bu)^2} = \frac{a}{b^2(a + bu)} + \frac{1}{b^2} \ln |a + bu| + C$

52. $\int \frac{du}{u(a + bu)^2} = \frac{1}{a(a + bu)} - \frac{1}{a^2} \ln \left| \frac{a + bu}{u} \right| + C$

53. $\int \frac{u^2 \, du}{(a + bu)^2} = \frac{1}{b^3} \left(a + bu - \frac{a^2}{a + bu} - 2a \ln |a + bu| \right) + C$

54. $\int u \sqrt{a + bu} \, du = \frac{2}{15b^2} (3bu - 2a)(a + bu)^{3/2} + C$

55. $\int \frac{u \, du}{\sqrt{a + bu}} = \frac{2}{3b^2} (bu - 2a) \sqrt{a + bu} + C$

56. $\int \frac{u^2 \, du}{\sqrt{a + bu}} = \frac{2}{15b^3} (8a^2 + 3b^2u^2 - 4abu) \sqrt{a + bu} + C$

$$\begin{aligned}
 57. \int \frac{du}{u\sqrt{a+bu}} &= \frac{1}{\sqrt{a}} \ln \left| \frac{\sqrt{a+bu} - \sqrt{a}}{\sqrt{a+bu} + \sqrt{a}} \right| + C, \quad \text{if } a > 0 \\
 &= \frac{2}{\sqrt{-a}} \tan^{-1} \sqrt{\frac{a+bu}{-a}} + C, \quad \text{if } a < 0
 \end{aligned}$$

$$58. \int \frac{\sqrt{a+bu}}{u} du = 2\sqrt{a+bu} + a \int \frac{du}{u\sqrt{a+bu}}$$

$$59. \int \frac{\sqrt{a+bu}}{u^2} du = -\frac{\sqrt{a+bu}}{u} + \frac{b}{2} \int \frac{du}{u\sqrt{a+bu}}$$

$$60. \int u^n \sqrt{a+bu} du = \frac{2}{b(2n+3)} \left[u^n(a+bu)^{3/2} - na \int u^{n-1} \sqrt{a+bu} du \right]$$

$$61. \int \frac{u^n du}{\sqrt{a+bu}} = \frac{2u^n \sqrt{a+bu}}{b(2n+1)} - \frac{2na}{b(2n+1)} \int \frac{u^{n-1} du}{\sqrt{a+bu}}$$

$$62. \int \frac{du}{u^n \sqrt{a+bu}} = -\frac{\sqrt{a+bu}}{a(n-1)u^{n-1}} - \frac{b(2n-3)}{2a(n-1)} \int \frac{du}{u^{n-1} \sqrt{a+bu}}$$

63. $\int \sin^2 u \, du = \frac{1}{2}u - \frac{1}{4} \sin 2u + C$

64. $\int \cos^2 u \, du = \frac{1}{2}u + \frac{1}{4} \sin 2u + C$

65. $\int \tan^2 u \, du = \tan u - u + C$

66. $\int \cot^2 u \, du = -\cot u - u + C$

67. $\int \sin^3 u \, du = -\frac{1}{3}(2 + \sin^2 u) \cos u + C$

68. $\int \cos^3 u \, du = \frac{1}{3}(2 + \cos^2 u) \sin u + C$

69. $\int \tan^3 u \, du = \frac{1}{2} \tan^2 u + \ln |\cos u| + C$

70. $\int \cot^3 u \, du = -\frac{1}{2} \cot^2 u - \ln |\sin u| + C$

71. $\int \sec^3 u \, du = \frac{1}{2} \sec u \tan u + \frac{1}{2} \ln |\sec u + \tan u| + C$

72. $\int \csc^3 u \, du = -\frac{1}{2} \csc u \cot u + \frac{1}{2} \ln |\csc u - \cot u| + C$

73. $\int \sin^n u \, du = -\frac{1}{n} \sin^{n-1} u \cos u + \frac{n-1}{n} \int \sin^{n-2} u \, du$

74. $\int \cos^n u \, du = \frac{1}{n} \cos^{n-1} u \sin u + \frac{n-1}{n} \int \cos^{n-2} u \, du$

76.
$$\int \cot^n u \, du = \frac{-1}{n-1} \cot^{n-1} u - \int \cot^{n-2} u \, du$$

77.
$$\int \sec^n u \, du = \frac{1}{n-1} \tan u \sec^{n-2} u + \frac{n-2}{n-1} \int \sec^{n-2} u \, du$$

78.
$$\int \csc^n u \, du = \frac{-1}{n-1} \cot u \csc^{n-2} u + \frac{n-2}{n-1} \int \csc^{n-2} u \, du$$

79.
$$\int \sin au \sin bu \, du = \frac{\sin(a-b)u}{2(a-b)} - \frac{\sin(a+b)u}{2(a+b)} + C$$

80.
$$\int \cos au \cos bu \, du = \frac{\sin(a-b)u}{2(a-b)} + \frac{\sin(a+b)u}{2(a+b)} + C$$

81.
$$\int \sin au \cos bu \, du = -\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)} + C$$

82.
$$\int u \sin u \, du = \sin u - u \cos u + C$$

83.
$$\int u \cos u \, du = \cos u + u \sin u + C$$

84.
$$\int u^n \sin u \, du = -u^n \cos u + n \int u^{n-1} \cos u \, du$$

85.
$$\int u^n \cos u \, du = u^n \sin u - n \int u^{n-1} \sin u \, du$$

86.
$$\int \sin^n u \cos^m u \, du = \frac{\sin^{n-1} u \cos^{m+1} u}{n+m} + \frac{n-1}{n+m} \int \sin^{n-2} u \cos^m u \, du$$

87.
$$\int \sin^{-1}u \, du = u \sin^{-1}u + \sqrt{1 - u^2} + C$$

88.
$$\int \cos^{-1}u \, du = u \cos^{-1}u - \sqrt{1 - u^2} + C$$

89.
$$\int \tan^{-1}u \, du = u \tan^{-1}u - \frac{1}{2} \ln(1 + u^2) + C$$

90.
$$\int u \sin^{-1}u \, du = \frac{2u^2 - 1}{4} \sin^{-1}u + \frac{u\sqrt{1 - u^2}}{4} + C$$

91.
$$\int u \cos^{-1}u \, du = \frac{2u^2 - 1}{4} \cos^{-1}u - \frac{u\sqrt{1 - u^2}}{4} + C$$

92.
$$\int u \tan^{-1}u \, du = \frac{u^2 + 1}{2} \tan^{-1}u - \frac{u}{2} + C$$

93.
$$\int u^n \sin^{-1}u \, du = \frac{1}{n+1} \left[u^{n+1} \sin^{-1}u - \int \frac{u^{n+1} du}{\sqrt{1 - u^2}} \right], \quad n \neq -1$$

94.
$$\int u^n \cos^{-1}u \, du = \frac{1}{n+1} \left[u^{n+1} \cos^{-1}u + \int \frac{u^{n+1} du}{\sqrt{1 - u^2}} \right], \quad n \neq -1$$

95.
$$\int u^n \tan^{-1}u \, du = \frac{1}{n+1} \left[u^{n+1} \tan^{-1}u - \int \frac{u^{n+1} du}{1 + u^2} \right], \quad n \neq -1$$

$$96. \int ue^{au} du = \frac{1}{a^2} (au - 1)e^{au} + C$$

$$97. \int u^n e^{au} du = \frac{1}{a} u^n e^{au} - \frac{n}{a} \int u^{n-1} e^{au} du$$

$$98. \int e^{au} \sin bu du = \frac{e^{au}}{a^2 + b^2} (a \sin bu - b \cos bu) + C$$

$$99. \int e^{au} \cos bu du = \frac{e^{au}}{a^2 + b^2} (a \cos bu + b \sin bu) + C$$

Hyperbolic Forms

$$103. \int \sinh u du = \cosh u + C$$

$$104. \int \cosh u du = \sinh u + C$$

$$105. \int \tanh u du = \ln \cosh u + C$$

$$106. \int \coth u du = \ln |\sinh u| + C$$

$$107. \int \operatorname{sech} u du = \tan^{-1} |\sinh u| + C$$

$$100. \int \ln u \, du = u \ln u - u + C$$

$$101. \int u^n \ln u \, du = \frac{u^{n+1}}{(n+1)^2} [(n+1) \ln u - 1] + C$$

$$102. \int \frac{1}{u \ln u} \, du = \ln |\ln u| + C$$

$$108. \int \operatorname{csch} u \, du = \ln |\tanh \frac{1}{2} u| + C$$

$$109. \int \operatorname{sech}^2 u \, du = \tanh u + C$$

$$110. \int \operatorname{csch}^2 u \, du = -\coth u + C$$

$$111. \int \operatorname{sech} u \tanh u \, du = -\operatorname{sech} u + C$$

$$112. \int \operatorname{csch} u \coth u \, du = -\operatorname{csch} u + C$$

$$113. \int \sqrt{2au - u^2} du = \frac{u - a}{2} \sqrt{2au - u^2} + \frac{a^2}{2} \cos^{-1}\left(\frac{a - u}{a}\right) + C$$

$$114. \int u \sqrt{2au - u^2} du = \frac{2u^2 - au - 3a^2}{6} \sqrt{2au - u^2} + \frac{a^3}{2} \cos^{-1}\left(\frac{a - u}{a}\right) + C$$

$$115. \int \frac{\sqrt{2au - u^2}}{u} du = \sqrt{2au - u^2} + a \cos^{-1}\left(\frac{a - u}{a}\right) + C$$

$$116. \int \frac{\sqrt{2au - u^2}}{u^2} du = -\frac{2\sqrt{2au - u^2}}{u} - \cos^{-1}\left(\frac{a - u}{a}\right) + C$$

$$117. \int \frac{du}{\sqrt{2au - u^2}} = \cos^{-1}\left(\frac{a - u}{a}\right) + C$$

$$118. \int \frac{u \, du}{\sqrt{2au - u^2}} = -\sqrt{2au - u^2} + a \cos^{-1}\left(\frac{a - u}{a}\right) + C$$

$$119. \int \frac{u^2 \, du}{\sqrt{2au - u^2}} = -\frac{(u + 3a)}{2} \sqrt{2au - u^2} + \frac{3a^2}{2} \cos^{-1}\left(\frac{a - u}{a}\right) + C$$

$$120. \int \frac{du}{u \sqrt{2au - u^2}} = -\frac{\sqrt{2au - u^2}}{au} + C$$

$\int 7.6$ Integrating with tables & 'puters.

$\int 7.6 \#s 1, 5, 8, 10, 11, 15, 19, 22, 32, 38$

1-4 Use the indicated entry in the Table of Integrals on the Reference Pages to evaluate the integral.

$$1. \int_0^{\pi/2} \cos 5x \cos 2x \, dx; \text{ entry 80}$$

$$a = 5, b = 2 \quad a-b = 3$$

$$a+b = 7$$

$$\int \cos(ax) \cos(bx) \, dx =$$

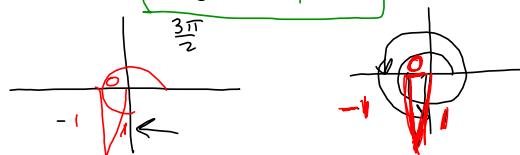
$$\frac{\sin((a-b)x)}{2(a-b)} + \frac{\sin((a+b)x)}{2(a+b)} + C$$

$$\int_0^{\frac{\pi}{2}} \cos(5x) \cos(2x) \, dx$$

$$= \left[\frac{\sin(3x)}{2(-3)} + \frac{\sin(7x)}{2(+7)} \right]_0^{\frac{\pi}{2}} = \frac{\sin(\frac{3\pi}{2})}{6} + \frac{\sin(\frac{7\pi}{2})}{14} - [0 + 0]$$

$$\frac{7\pi}{2} = \frac{5\pi}{2} + \frac{\pi}{2} = 3\pi + \frac{\pi}{2}$$

$$= \boxed{\frac{-1}{6} + \frac{-1}{14}}$$



5-32 Use the Table of Integrals on Reference Pages 6–10 to evaluate the integral.

$$5. \int_0^{\pi/8} \arctan 2x \, dx$$

$$u = 2x, \, du = 2dx$$

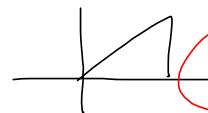
$$\# 89 \quad \int \tan^{-1}(u) \, du = u \tan^{-1}(u) - \frac{1}{2} \ln(1+u^2) + C$$

$$\tan^{-1}(u) = \arctan(u)$$

$$= \frac{1}{2} \int_{0=x}^{\frac{\pi}{8}=x} \arctan(2x) \cdot 2 \, dx = \frac{1}{2} \left[2x \arctan(2x) \right]_0^{\frac{\pi}{8}} - \frac{1}{2} \left[\ln(1+(2x)^2) \right]_0^{\frac{\pi}{8}}$$

$$= \frac{1}{2} \left[2\left(\frac{\pi}{8}\right) \arctan\left(\frac{2\pi}{8}\right) - 2(0) \arctan(2 \cdot 0) \right] - \frac{1}{2} \cdot \frac{1}{2} \left[\ln\left(1+\left(\frac{\pi}{4}\right)^2\right) - \ln(1) \right] = \frac{1}{2} \cdot 2 \frac{\pi}{8} \arctan \frac{\pi}{4} - \frac{1}{4} \ln\left(\frac{\pi^2}{16} + 1\right)$$

$$= \frac{\pi}{8} \arctan\left(\frac{\pi}{4}\right) - \frac{1}{4} \ln\left(\frac{\pi^2}{16} + 1\right)$$



Maple:

$$\frac{1}{8} \pi \arctan\left(\frac{1}{4} \pi\right) + \ln(2) - \frac{1}{4} \ln(16 + \pi^2)$$

$$\ln\left(\frac{\pi^2}{16} + 1\right) = \ln\left(\frac{\pi^2 + 16}{16}\right) = \ln(\pi^2 + 16) - \ln(16)$$

$$\therefore -\frac{1}{4} \ln\left(\frac{\pi^2}{16} + 1\right) = -\frac{1}{4} (\ln(\pi^2 + 16) - \ln(16))$$

$$= -\frac{1}{4} \ln(\pi^2 + 16) + \frac{1}{4} \ln(16) =$$

$$\ln\left(16^{\frac{1}{4}}\right) = \ln(2) !$$

$$\begin{aligned}
 8. \int \frac{\ln(1 + \sqrt{x})}{\sqrt{x}} dx &= \int \frac{\ln(u)}{\sqrt{x}} \cdot 2\sqrt{x} du = 2 \int \ln(u) \cdot \frac{1}{u} du \\
 u = 1 + \sqrt{x} &\quad = 1 + x^{\frac{1}{2}} & = 2 \int \ln(u) \cdot \frac{1}{u} du &= 2 \int u du = 2 \cdot \frac{1}{2} u^2 + C \\
 du = \frac{1}{2} x^{-\frac{1}{2}} dx &\quad = \frac{1}{2\sqrt{x}} dx & u = \ln(v) &\quad du = \frac{1}{v} dv \\
 \text{dx} = 2\sqrt{x} du &\quad \text{circled}
 \end{aligned}$$

$$\begin{aligned}
 &= 2 \int \ln(v) \cdot \frac{1}{v} dv & &= \left(\ln(v) \right)^2 + C \\
 & & &= \left(\ln(\sqrt{x} + 1) \right)^2 + C
 \end{aligned}$$

$$10. \int \frac{\sqrt{2y^2 - 3}}{y^2} dy \quad \#42 \quad \int \frac{\sqrt{u^2 - a^2}}{u^2} du = -\frac{\sqrt{u^2 - a^2}}{u} + \ln|u + \sqrt{u^2 - a^2}| + C$$

$$\begin{aligned} \sqrt{2(y^2 - \frac{3}{2})} &= \sqrt{2} \sqrt{y^2 - \frac{3}{2}} = \sqrt{2} \sqrt{y^2 - (\frac{\sqrt{2}}{2})^2} \\ &= \int \frac{\sqrt{2} \sqrt{y^2 - (\frac{\sqrt{2}}{2})^2}}{y^2} dy = \sqrt{2} \left[-\frac{\sqrt{y^2 - \frac{3}{2}}}{y} + \ln|y + \sqrt{y^2 - \frac{3}{2}}| \right] + C \\ &= \sqrt{2} \end{aligned}$$

#97

11. $\int_{-1}^0 t^2 e^{-t} dt$

$$\int u dv = uv - \int v du$$

$$\int t^n e^{at} dt = \frac{1}{a} t^n e^{at} - \frac{n}{a} \int t^{n-1} e^{at} dt$$

$$u = t^2 \quad dv = e^{-t} dt$$

$$du = 2t dt \quad v = -e^{-t}$$

$$uv - \int v du = -t^2 e^{-t} + 2 \int t e^{-t} dt = -t^2 e^{-t} - 2t e^{-t} + 2 \int e^{-t} dt$$

$$u = t \quad dv = e^{-t} dt$$

$$du = dt \quad v = -e^{-t}$$

$$= -t^2 e^{-t} - 2t e^{-t} - 2e^{-t} + C$$

$$\int_{-1}^0 t^2 e^{-t} dt = \left[-t^2 e^{-t} - 2t e^{-t} - 2e^{-t} \right]_{-1}^0$$

$$= 0 - 0 - 2e^0 - \left[-(-1)^2 e^{-(-1)} + 2(-1)e^{-(-1)} - 2e^{-(-1)} \right]$$

$$= -2 - \left[-e^1 + 2e^1 - 2e^1 \right] = -2 - \left[-e \right] = \cancel{-2}$$

$$e - 2$$

#92

15. $\int e^{2x} \arctan(e^x) dx$

$$\begin{array}{l} u = e^x \\ du = e^x dx \end{array}$$

$$e^{2x} = e^{x+x} = e^x e^x$$

$$(e^x)^2 = e^{2x}$$

$$\begin{aligned} &= \int e^x \arctan(e^x) \cdot e^x dx = \int u \arctan(u) du \\ &= \frac{u^2+1}{2} \arctan(u) - \frac{u}{2} + C \\ &= \frac{e^{2x}+1}{2} \arctan(e^x) - \frac{e^x}{2} + C \end{aligned}$$

19. $\int \sin^2 x \cos x \ln(\sin x) dx$

$$\begin{aligned} & \stackrel{\text{#101}}{=} \int u^2 \ln(u) du \\ &= \frac{u^{n+1}}{(n+1)} \left[(n+1) \ln(u) - 1 \right] + C \\ & \quad \begin{matrix} \uparrow \\ u = \sin x \\ du = \cos x dx \end{matrix} \\ &= \int u^2 \ln(u) du = \frac{u^3}{3} \left[3 \ln(u) - 1 \right] + C \\ &= \frac{\sin^3 x}{9} \left[3 \ln(\sin(x)) - 1 \right] + C \end{aligned}$$

$$22. \int_0^2 x^3 \sqrt{4x^2 - x^4} dx$$

$\sqrt{4x^2 - x^4}$

$$\sqrt{4x^2 - x^4} = \sqrt{x^2(4 - x^2)} = \sqrt{x^2} \sqrt{4 - x^2} = |x| \sqrt{4 - x^2}$$

$= x \sqrt{4 - x^2}$
 b/c $x \in [0, 2]$ on this integral.

$$= \int_0^2 x^4 \sqrt{4-x^2} dx = \int_{x=0}^{x=2} 2^4 \sin^4 \theta \cdot 2 \cos \theta \cdot 2 \cos \theta d\theta = 2^6 \int_{x=0}^{x=2} \sin^4 \theta \cos^2 \theta d\theta$$

$$\frac{x}{\sqrt{4-x^2}} = \sin \theta$$

$$x = 2 \sin \theta$$

$$dx = 2 \cos \theta d\theta$$

$$\begin{aligned} \sqrt{4-x^2} &= \sqrt{4-(2 \sin \theta)^2} \\ &= \sqrt{4-4 \sin^2 \theta} = \sqrt{4(1-\sin^2 \theta)} \\ &= 2 \sqrt{1-\sin^2 \theta} = 2 \sqrt{\cos^2 \theta} = 2 |\cos \theta| = 2 \cos \theta \end{aligned}$$

$$= 2^6 \int_{x=0}^{x=2} \sin^4 \theta \cos^2 \theta d\theta = 2^6 \int (\cos^2 \theta - 2 \cos^4 \theta + \cos^6 \theta) d\theta$$

$$(\sin^2 \theta)^2 = (1 - \cos^2 \theta)^2 = 1 - 2 \cos^2 \theta + \cos^4 \theta$$

$$(\sin^2 \theta)^2 (\cos^2 \theta) = \cos^2 \theta - 2 \cos^4 \theta + \cos^6 \theta$$

$$= \frac{1}{2}(1 + \cos(2\theta)) - 2((\cos^2 \theta)^2) + (\cos^2 \theta)^3$$

$$= \frac{1}{2} + \frac{1}{2} \cos \theta - 2 \left(\frac{1}{4}(1 + \cos(2\theta))^2 + \left(\frac{1}{2}(1 + \cos(2\theta)) \right)^3 \right)$$

$$= \frac{1}{2} + \frac{1}{2} \cos \theta - 2 \left[\frac{1}{4}(1 + 2\cos(2\theta) + \cos^2(2\theta)) \right]$$

$$+ \frac{1}{8}(1 + 3\cos(2\theta) + 3\cos^2(2\theta) + \cos^3(2\theta))$$

$$= \underline{\frac{1}{2} + \frac{1}{2} \cos \theta} - \underline{\frac{1}{2} \cos(2\theta)} - \underline{\frac{1}{2} \cos^2(2\theta)} + \underline{\frac{1}{8} + \frac{3}{8} \cos(2\theta)} + \underline{\frac{3}{8} \cos^2(2\theta)} + \underline{\frac{1}{8} \cos^3(2\theta)}$$

$$= \frac{1}{8} + \frac{1}{2} \cos \theta - \frac{5}{8} \cos(2\theta) - \frac{1}{8} \cos^2(2\theta) + \frac{1}{8} \cos^3(2\theta)$$

$$= \frac{1}{8} + \frac{1}{2} \cos \theta - \frac{5}{8} \cos(2\theta) - \frac{1}{8} \left[\frac{1}{2}(1 + \cos(4\theta)) \right] + \frac{1}{8} \cos(2\theta) - \frac{1}{8} \sin^2(2\theta) \cos(2\theta)$$

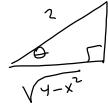
$$= \frac{1}{8} + \frac{1}{2} \cos \theta - \frac{1}{2} \cos(2\theta) - \frac{1}{16} - \frac{1}{8} \cos(4\theta) \quad \cos^3(2\theta) = \cancel{\cos^2(2\theta) \cos(2\theta)}$$

$$+ \frac{1}{8} \cos(2\theta) - \frac{1}{8} \sin^2(2\theta) \cos(2\theta)$$

$$= \boxed{\frac{1}{8} + \frac{1}{2} \cos(2\theta) - \frac{3}{8} \cos(2\theta) - \frac{1}{8} \cos(4\theta) - \frac{1}{8} \sin^2(2\theta) \cos(2\theta)}$$

is the integrand.

$$22. \int_0^2 x^3 \sqrt{4x^2 - x^4} dx$$


 $x^3 \sqrt{4x^2 - x^4} = x^3 \sqrt{x^2} \sqrt{4 - x^2} = x^3 \cdot x \sqrt{4 - x^2} = x^4 \sqrt{4 - x^2}$
 $\frac{x}{2} = \sin \theta \quad \text{This gives}$
 $x = 2 \sin \theta \quad dx = 2 \cos \theta d\theta$
 $\int_{x=0}^{x=2} 2 \sin \theta \cdot 2 \cos \theta \cdot 2 \cos \theta d\theta$

$$\begin{aligned}
 & \sqrt{4 - x^2} \\
 & = \sqrt{4 - 4 \sin^2 \theta} \\
 & = 2 \sqrt{1 - \sin^2 \theta} \\
 & = 2 \sqrt{\cos^2 \theta} \\
 & = 2 \cos \theta \quad (\cos \theta \geq 0 \text{ on } [0, \frac{\pi}{2}, \frac{\pi}{2}])
 \end{aligned}$$

$$\begin{aligned}
 & = 2^6 \int_{x=0}^{x=2} (\sin^2 \theta)^2 \cos^2 \theta d\theta \\
 & = 2^6 \int_{x=0}^{x=2} \left(\frac{1}{2}(1 - \cos(2\theta))\right)^2 \left(\frac{1}{2}(1 + \cos(2\theta))\right) d\theta \\
 & = 2^6 \cdot \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right) \int_{x=0}^{x=2} (1 - 2\cos(2\theta) + \cos^2(2\theta))(1 + \cos(2\theta)) d\theta \\
 & = \frac{2^6}{2^3} \int_{x=0}^{x=2} (1 + \cos(2\theta) - 2\cos(2\theta) - 2\cos^2(2\theta) + \cos^2(2\theta) + \cos^3(2\theta)) d\theta \\
 & = 2^3 \int_{x=0}^{x=2} d\theta - 2^3 \int_{x=0}^{x=2} \cos(2\theta) d\theta - 2^3 \int_{x=0}^{x=2} \cos^2(2\theta) d\theta + 2^3 \int_{x=0}^{x=2} \cos^3(2\theta) d\theta
 \end{aligned}$$

$$= 8\theta \Big|_{x=0}^{x=2} - \frac{3}{2} \int_{x=0}^{x=2} \cos(2\theta) \cdot 2d\theta - 8 \int_{x=0}^{x=2} \left(\frac{1}{2}(1 + \cos(4\theta))\right) d\theta + 8 \int_{x=0}^{x=2} (1 - \sin^2(2\theta)) \cos(2\theta) d\theta$$

$$\begin{aligned}
 & = 8 \arcsin\left(\frac{x}{2}\right) \Big|_0^{x=2} - 4 \sin 2\theta \Big|_{x=0}^{x=2} - \frac{3}{2} \int_{x=0}^{x=2} d\theta - \frac{3}{2} \cdot \frac{1}{4} \int_{x=0}^{x=2} \cos(4\theta) \cdot 4d\theta + 8 \cdot \frac{1}{2} \int_{x=0}^{x=2} \cos(2\theta) \cdot 2d\theta \\
 & \quad - 8 \cdot \frac{1}{2} \int_{x=0}^{x=2} \sin^2(2\theta) \cos(2\theta) \cdot 2d\theta \\
 & = 8 \left(\arcsin(1) - \arcsin(0) \right) - 4 \left(2 \sin \theta \cos \theta \right) \Big|_{x=0}^{x=2} - 4 \left[\arcsin(1) - \arcsin(0) \right] \\
 & \quad - \sin(4\theta) \Big|_{x=0}^{x=2} + 4 \sin(2\theta) \Big|_{x=0}^{x=2} - 4 \cdot \frac{\sin^3(2\theta)}{3} \Big|_{x=0}^{x=2} \\
 & = 8 \left(\frac{\pi}{2} \right) - 8 \left[\frac{x \sqrt{4-x^2}}{2} \right]_0^{x=2} - 4 \left[\frac{\pi}{2} \right] - \frac{1}{4} \left(x \sqrt{4-x^2} \right) \Big|_0^{x=2} + 4 \left(2 \sin \theta \cos \theta \right) \Big|_{x=0}^{x=2} \\
 & \quad - \frac{4}{3} \left(2 \sin \theta \cos \theta \right)^3 \Big|_{x=0}^{x=2}
 \end{aligned}$$

$$= 4\pi - 2\pi - \frac{1}{4} \cdot 0 + 8 \left(\frac{x}{2} \right) \left(\frac{\sqrt{4-x^2}}{2} \right) \Big|_0^{x=2} - \frac{8}{3} \left(\left(\frac{x}{2} \right) \left(\frac{\sqrt{4-x^2}}{2} \right) \right)^3 \Big|_{x=0}^{x=2} = 2\pi$$

$$\begin{aligned}
 \sin(4\theta) & = 2 \sin(2\theta) \cos(2\theta) \\
 & = 2(2 \sin \theta \cos \theta)(\cos^2 \theta - \sin^2 \theta) \\
 & = 4 \left(\frac{x}{2} \cdot \frac{\sqrt{4-x^2}}{2} \right) \left(\frac{4-x^2}{4} - \frac{x^2}{4} \right) \\
 & = \frac{1}{4} \left(x \sqrt{4-x^2} \right) (4-2x^2)
 \end{aligned}$$

$$\begin{aligned}
 32. \int \frac{\sec^2 \theta \tan^2 \theta}{\sqrt{9 - \tan^2 \theta}} d\theta &= \int \frac{\tan^2 \theta}{\sqrt{9 - \tan^2 \theta}} \sec^2 \theta d\theta = \int \frac{u^2}{\sqrt{9 - u^2}} du \\
 u = \tan \theta & \quad \text{# 34} \\
 du = \sec^2 \theta d\theta & \\
 &= -\frac{u}{2} \sqrt{9 - u^2} + \frac{9}{2} \sin^{-1}\left(\frac{u}{3}\right) + C \\
 &= -\frac{1}{2} \tan \theta \sqrt{9 - \tan^2 \theta} + \frac{3}{2} \arcsin\left(\frac{\tan \theta}{3}\right) + C
 \end{aligned}$$

$$38. \int \csc^5 x dx = -\frac{1}{4} \cot x \csc^3 x + \frac{3}{4} \int \csc^3 x dx$$

#78

$$= -\frac{1}{4} \cot x \csc^3 x + \frac{3}{4} \left[-\frac{1}{2} \csc x \cot x + \frac{1}{2} \ln |\csc x - \cot x| \right] + C$$

↙ ↘

#72

46. Computer algebra systems sometimes need a helping hand from human beings. Try to evaluate

$$\int (1 + \ln x) \sqrt{1 + (x \ln x)^2} dx$$

with a computer algebra system. If it doesn't return an answer, make a substitution that changes the integral into one that the CAS *can* evaluate.

$$\begin{aligned} & \int (\ln(x) + 1) \cdot \sqrt{1 + (x \cdot \ln(x))^2} dx \\ & \int \sqrt{1+u^2} du \quad \text{Red arrow} \quad u = x \ln x \\ & \quad du = \left(\ln x + x \cdot \frac{1}{x} \right) dx \\ & \quad = (1 + \ln x) dx \\ & \quad \int (\ln(x) + 1) \sqrt{1 + x^2 \ln(x)^2} dx \\ & \quad \frac{1}{2} u \sqrt{1+u^2} + \frac{1}{2} \operatorname{arcsinh}(u) \\ & = \boxed{\left(\frac{1}{2} x \ln(x) \sqrt{1 + (x \ln(x))^2} + \frac{1}{2} \operatorname{arcsinh}(x \ln(x)) \right) + C} \end{aligned}$$

