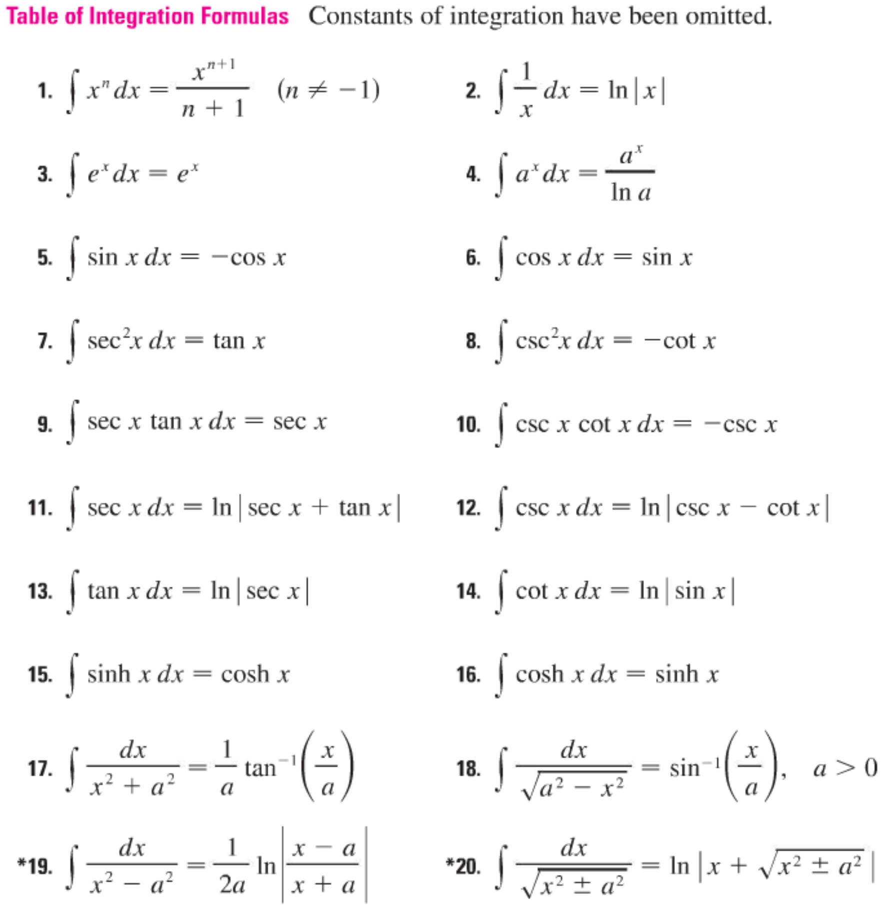


Table of Integration Formulas Constants of integration have been omitted.

1. $\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1)$

2. $\int \frac{1}{x}\,dx = \ln|x|$

3. $\int e^x\,dx = e^x$

4. $\int a^x\,dx = \frac{a^x}{\ln a}$

5. $\int \sin x\,dx = -\cos x$

6. $\int \cos x\,dx = \sin x$

7. $\int \sec^2 x\,dx = \tan x$

8. $\int \csc^2 x\,dx = -\cot x$

9. $\int \sec x \tan x\,dx = \sec x$

10. $\int \csc x \cot x\,dx = -\csc x$

11. $\int \sec x\,dx = \ln|\sec x + \tan x|$

12. $\int \csc x\,dx = \ln|\csc x - \cot x|$

13. $\int \tan x\,dx = \ln|\sec x|$

14. $\int \cot x\,dx = \ln|\sin x|$

15. $\int \sinh x\,dx = \cosh x$

16. $\int \cosh x\,dx = \sinh x$

17. $\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$

18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right), \quad a > 0$

***19.** $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|$

***20.** $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right|$

1. Simplify the Integrand if Possible

2. Look for an Obvious Substitution u - substitution

3. Classify the Integrand According to Its Form

- (a) *Trigonometric functions.*
- (b) *Rational functions.*
- (c) *Integration by parts.*
- (d) *Radicals.*

4. Try Again

§7.5 #s 1,4,7,10,13,16,19,20 | 22,23,25,31,33,40,47,67

I Due Tues | Due Wed II

1–82 Evaluate the integral.

1. $\int \cos x\,(1 + \sin^2 x)\,dx$

$$= \int \cos x\,dx + \int \underbrace{\sin^2 x}_{u^2}\ \underbrace{\cos x\,dx}_{du}$$

$$= \boxed{\sin x + \frac{\sin^3 x}{3} + C}$$

4. $\int \frac{\sin^3 x}{\cos x}\,dx = -\int \frac{-\sin x\,dx}{\cos x} - \int \underset{u}{\sin x}\ \underbrace{\cos x\ dx}_{du} = -\ln|\cos x| - \frac{\sin^2 x}{2} + C$

$= \ln|\sec x| - \frac{1}{2}\sin^2 x + C$

$\sin^2 x \tan x$

$\frac{\sin^2 x\ \sin x}{\cos x} = \frac{(1-\cos^2 x)(\sin x)}{\cos x} = \frac{\sin x}{\cos x} - \frac{\sin x\ \cos^2 x}{\cos x}$

$= \frac{\sin x}{\cos x} - \sin x \cos x$

7. $\int_{-1}^{1} \frac{e^{\arctan y}}{1+y^2}\,dy$

11 **Table of Derivatives of Inverse Trigonometric Functions**

$$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\csc^{-1}x) = -\frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2}$$

$u = \arctan(y)$

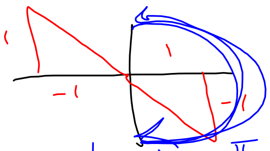
$du = \frac{dy}{y^2+1}$

$$= \int_{-1=y}^{1=y} e^{\arctan(y)} \frac{1}{y^2+1}\,dy = \int_{-1=y}^{1=y} e^u\,du = e^u\Big]_{y=-1}^{y=1} = e^{\arctan(y)}\Big]_{-1}^{1} = e^{\frac{\pi}{4}} - e^{-\frac{\pi}{4}}$$

$$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} e^u\,du = e^u\Big]_{-\frac{\pi}{4}}^{\frac{\pi}{4}} =$$

$y = -1 \longrightarrow$

$u = \arctan(y) = \arctan(-1) = -\frac{\pi}{4}$



$u = \arctan(1) = \frac{\pi}{4}$

Method 1

10. $\int_0^4 \frac{x-1}{x^2-4x-5}\,dx = \int_0^1 \frac{x-1}{(x-5)(x+1)}\,dx$

$u = x^2-4x-5$

$\boxed{du = 2x-4}$

Can't make this outta $x-1$

oh well.

x^2-4x-5

$= x^2-4x+2^2-2^2-5$

$= (x-2)^2-9$

Let $u = x-2$

Then $du = dx$ and

$x = u+2.$

This gives

$$\int \frac{u+2-1}{u^2-9}\,du = \int \frac{u+1}{u^2-9}\,du$$

one way

Partial Fracs Way

Don't need since x^2-4x-5 factors

$$= \frac{1}{2}\int \frac{2u}{u^2-9}\,du + \int \frac{1}{u^2-9}\,du$$

$v = u^2-9$

$dv = 2u\,du$

*19. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|$

$$= \frac{1}{2}\int \frac{dv}{v} + \frac{1}{(2)(3)}\ln\left|\frac{u-3}{u+3}\right| + C$$

$v = u^2-9$

$= (x-2)^2-9$

$= x^2-4x-5$

$$= \frac{1}{2}\ln|v| + \frac{1}{6}\ln\left|\frac{x-2-3}{x-2+3}\right| + C$$

$$= \frac{1}{2}\ln|x^2-4x-5| + \frac{1}{6}\ln\left|\frac{x-5}{x+1}\right| + C$$

$$\therefore \int_0^4 \frac{x-1}{x^2-4x-5}\,dx$$

$$= \frac{1}{2}\ln|-5| - \frac{1}{2}\ln|-5|$$

$$+ \frac{1}{6}\ln\left|\frac{4-5}{4+1}\right| - \frac{1}{6}\ln\left|\frac{-5}{1}\right|$$

$$= \frac{1}{2}\ln 5 - \frac{1}{2}\ln 5 + \frac{1}{6}\ln\left|\frac{-1}{5}\right| - \frac{1}{6}\ln|-5|$$

$$= \frac{1}{6}\ln\frac{1}{5} - \frac{1}{6}\ln 5 = \frac{1}{6}\ln\left(\frac{1}{5}\right) + \frac{1}{6}\ln\left(\frac{1}{5}\right)$$

$$= \frac{1}{3}\ln\left(\frac{1}{5}\right) = \boxed{-\frac{1}{3}\ln 5}$$

Method 2

10. $\int_0^4 \frac{x-1}{x^2-4x-5}\,dx$

$$\frac{x-1}{(x-5)(x+1)} = \frac{A}{x-5} + \frac{B}{x+1}$$

$$x-1 = A(x+1)+B(x-5)$$

$x=-1$:

$-1-1=-6B$

$\frac{1}{3} = \frac{-2}{-6} = \boxed{B = \frac{1}{3}}$

$x=5$:

$5-1=6A$

$\boxed{\frac{2}{3} = A}$

$$\int_0^4 \frac{x-1}{x^2-4x-5}dx$$

$$= \frac{2}{3}\int_0^4 \frac{dx}{x-5} + \frac{1}{3}\int_0^4 \frac{dx}{x+1}$$

$$= \frac{2}{3}\ln|x-5|\Big]_0^4 + \frac{1}{3}\ln|x+1|\Big|_0^4$$

$$= 0 - \frac{2}{3}\ln 5 + \frac{1}{3}\ln 5 - 0$$

$$= -\frac{1}{3}\ln 5$$

$$= \frac{1}{3}\ln\left(\frac{1}{5}\right) = -\frac{1}{3}\ln 5$$

13. $\int \sin^5 t \cos^4 t \, dt \;=\; \int (\cos^4 t - 2\cos^2 t + 1) \cos^4 t \cdot \sin t \, dt$

$$\sin^5 t = \sin^4 t \cdot \sin t = (\sin t)^4 \sin t = ((\sin t)^2)^2 \sin t$$

$$\sin t (1 - \cos^2 t)^2 = (1 - 2\cos^2 t + (\cos^2 t)^2) \sin t$$

$$= (1 - 2\cos^2 t + \cos^4 t) \sin t$$

$$= (\cos^4 t - 2\cos^2 t + 1) \sin t$$

$$= -\int \underset{u^8}{\cos^8 t} (-\sin t \, dt) + 2 \int \cos^6 t (-\sin t \, dt) + -\int \cos^4 t (-\sin t \, dt)$$

$$= \frac{1}{9}\cos^9 t + \frac{2}{7}\cos^7 t - \frac{1}{5}\cos^5 t + C$$

16. $\int_0^{\sqrt{2}/2} \frac{x^2}{\sqrt{1-x^2}}\,dx = \int_{x=0}^{x=\frac{\sqrt{2}}{2}} \frac{\sin^2\theta}{\cos\theta}\cos\theta\,d\theta = \int_{x=0}^{x=\frac{\sqrt{2}}{2}} \frac{1}{2}(1-\cos(2\theta))\,d\theta$

$= \frac{1}{2}\int_{x=0}^{x=\frac{\sqrt{2}}{2}} d\theta - \frac{1}{2}\cdot\frac{1}{2}\int_{x=0}^{x=\frac{\sqrt{2}}{2}} \cos(2\theta)\,(2d\theta)$

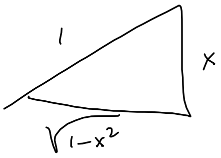
$= \frac{1}{2}\theta\Big]_{x=0}^{x=\frac{\sqrt{2}}{2}} - \frac{1}{4}\underset{\substack{\downarrow \\ 2\sin\theta\cos\theta}}{\sin(2\theta)}\Big]_{x=0}^{x=\frac{\sqrt{2}}{2}} = \frac{1}{2}\sin^{-1}(x)\Big]_0^{\frac{\sqrt{2}}{2}} - \frac{1}{4}x\cdot\sqrt{1-x^2}\Big]_0^{\frac{\sqrt{2}}{2}}\cdot 2$

$= \frac{1}{2}\cdot\frac{\pi}{4} - \frac{1}{2}\cdot 0 - \frac{1}{4}\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}\cdot 2$

$= \frac{\pi}{8} - \frac{1}{8}\cdot 2 = \boxed{\frac{\pi}{8} - \frac{1}{4}}$

$\sqrt{1-x^2}$

thinking trig substitution



$x = \sin\theta$

$dx = \cos\theta\,d\theta$

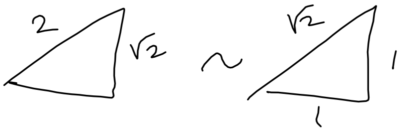
$x = \sin\theta$

$\sin^{-1}(x) = \sin^{-1}(\sin\theta)$

$= \theta$

$\sqrt{1-x^2} = \sqrt{1-\sin^2\theta} = \sqrt{\cos^2\theta} = |\cos\theta| = \cos\theta$

$\sqrt{1-(\frac{\sqrt{2}}{2})^2} = \sqrt{1-\frac{2}{4}} = \sqrt{1-\frac{1}{2}} = \frac{1}{\sqrt{2}}$



19. $\int e^{x+e^x} dx$

$$= \int \underbrace{e^x \cdot \overset{e^u}{\boxed{e^{e^x}}}\, dx}_{du} = \int e^u du = e^u + C = e^{e^x} + C$$

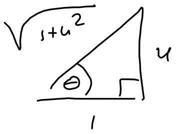
20. $\int e^2\,dx = e^2 \int dx = e^2 x + C$

e^2 is constant.

22. $\int \frac{\ln x}{x\sqrt{1+(\ln x)^2}}\,dx = \int \frac{u}{\sqrt{1+u^2}} \cdot du = \int \frac{\tan\theta}{\sqrt{1+\tan^2\theta}} \cdot \sec^2\theta \, d\theta$

$u = \ln x$

$du = \frac{1}{x}dx$



$u = \tan\theta \Longrightarrow -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$du = \sec^2\theta \, d\theta$

$= \int \frac{\tan\theta}{\sqrt{\sec^2\theta}} \sec^2\theta \, d\theta = \int \frac{\tan\theta}{|\sec\theta|} \sec^2\theta \, d\theta$

$= \int \frac{\tan\theta}{\sec\theta} \sec^2\theta \, d\theta = \int \sec\theta \tan\theta \, d\theta$

$= \sec\theta + C = \sqrt{1+u^2} + C = \sqrt{1+\ln(x)} + C$

23. $\int_0^1 (1+\sqrt{x})^8\,dx$

$\boxed{u = (1+\sqrt{x})}$ $dv = dx$

$du = \frac{1}{2\sqrt{x}}\,dx$ $v = x$

$dx = 2\sqrt{x}\,du$

$\boxed{u = \sqrt{x} + 1} \Rightarrow$

$u - 1 = \sqrt{x}$

$(u-1)^2 = x$

$$\int_{x=0}^{x=1} u^8 \cdot 2\sqrt{x}\,du$$

$$= \int_{x=0}^{x=1} u^8 \cdot 2(u-1)\,du = \int_{x=0}^{x=1} (2u^9 - 2u^8)\,du = \left[\frac{u^{10}}{5} - \frac{2}{9}u^9\right]_{x=0}^{x=1} = \left[\frac{1}{5}u^{10} - \frac{2}{9}u^9\right]_1^2$$

$$= \frac{1}{5}\cdot 2^{10} - \frac{2}{9}\cdot 2^9 - \left(\frac{1}{5}(1)^{10} - \frac{2}{9}(1)^9\right)$$

$$= \frac{2^{10}}{5} - \frac{1}{5} - \frac{2^{10}}{9} + \frac{2}{9} = \frac{9\cdot 2^{10} - 9 - 5\cdot 2^{10} + 10}{5\cdot 9} = \frac{4\cdot 2^{10} + 1}{45}$$

$$\int_0^1 (\text{sqrt}(x) + 1)^8\,dx = \frac{4097}{45}$$ ✓

$4\cdot 2^{10} = 2^2\cdot 2^{10} = 2^{12} = 4096$

$$4\cdot 2^{10} + 1 = 4097$$ ✓

25. $\int \frac{3x^2-2}{x^2-2x-8}\,dx$

Partial fractions route

$$\begin{array}{r} 3 + \frac{6x+22}{x^2-2x-8} \\ x^2-2x-8 \,\big)\, 3x^2 + 0x - 2 \\ -\left(3x^2-6x-24\right) \\ \hline 6x+22 \end{array}$$

$$\int\left(3+\frac{6x+22}{x^2-2x-8}\right)dx$$

$$= 3x + \int \frac{6x+22}{(x-4)(x+2)}\,dx$$

$$= 3x +$$

Completing square, looking for u^2-a^2 sitch -

$x^2-2x+1^2-1^2-8$

$= (x-1)^2-$

$= u^2-8$, where $u = x-1$

$\Rightarrow du = dx$

& $x = u+1$

$$\int \frac{3(u+1)^2-1}{u^2-9}\,du$$

$$= \int \frac{3(u^2+2u+1)-1}{\cancel{dx}}\cancel{dx} = \int \frac{3u^2+6u+3-1}{u^2-9}\,du =$$

Still lots of work, but

could do, say: $3\int\frac{u^2}{u^2-9}du + \boxed{6\int\frac{u}{u^2-9}du} + 2\int\frac{1}{u^2-9}du$

#44 #43

u-subst.

$\int\frac{x}{x^2-9}dx$

$u = x^2-9$

$du = 2udx$

$$\int \frac{6x+22}{(x-4)(x+2)}\,dx$$

$$\frac{6x+22}{(x-4)(x+2)} = \frac{A}{x-4} + \frac{B}{x+2}$$

$$6x+22 = A(x+2) + B(x-4)$$

$$= \frac{23}{3}\int \frac{1}{x-4}\,dx - \frac{5}{3}\int \frac{1}{x+2}\,dx$$

$x=-2$

$$6(-2)+22 = 10 = B(-2-4)$$

$$\boxed{= \frac{23}{3}\ln|x-4| - \frac{5}{3}\ln|x+2| + C}$$

$$-6B = 10$$

$$B = \frac{10}{-6} = \boxed{-\frac{5}{3} = B}$$

$\boxed{+3x}$ from previous.

$x=4$

$$46 = 6A$$

$$A = \frac{46}{6} = \frac{23}{3}$$

$$\int \frac{(3\cdot x^2 - 2)}{x^2 - 2\cdot x - 8}\,dx = 3x - \frac{5}{3}\ln(x+2) + \frac{23}{3}\ln(x-4) + C$$

$u = 1 - x^2$

$du = -2x\,dx$

31. $\int \sqrt{\frac{1+x}{1-x}}\,dx = \int \frac{dx}{\sqrt{1-x^2}} + \frac{1}{-2}\int \frac{-2x\,dx}{\sqrt{1-x^2}} = \arcsin(x) + -\frac{1}{2}\,\frac{\sqrt{1-x^2}}{\frac{1}{2}} + C$

See Example 5. Try $\frac{\sqrt{1+x}}{\sqrt{1-x}} \cdot \frac{\sqrt{1+x}}{\sqrt{1+x}} = \frac{1+x}{\sqrt{1-x^2}} = \frac{1}{\sqrt{1-x^2}} + \frac{x}{\sqrt{1-x^2}}$

$= \arcsin(x) - \sqrt{1-x^2} + C$

$$\frac{\sqrt{-\frac{1+x}{x-1}}\,(x-1)\left(\sqrt{1-x^2} - \arcsin(x)\right)}{\sqrt{-(x-1)(1+x)}}$$

$\sqrt{1-x^2}$

$\sqrt{\frac{1+x}{1-x}} \cdot \left(\sqrt{1-x^2}\right)(x-1)$

$= \sqrt{\frac{(1+x)(1-x)(1+x)}{1-x}}\;(x-1)$

$= \sqrt{(x+1)^2}\;(x-1) - (x-1)\sqrt{\frac{x+1}{1-x}}\;\arcsin(x) + C$

$= (x+1)(x-1)$

$= x^2 - 1$

33. $\int \sqrt{3 - 2x - x^2}\,dx$ $= \int \sqrt{4 - u^2}\,du$ $= \frac{u}{2}\sqrt{4-u^2} + \frac{4}{2}\sin^{-1}\left(\frac{u}{2}\right) + C$

#30

Formula $a = 2$

$-x^2 - 2x \quad + 3$

$-(x^2+2x+1^2) + 1^2 + 3$

$= -(x+1)^2 + 4$

$= 4 - (x+1)^2$

Let $u = x+1$

$du = dx$

$= \frac{x+1}{2}\sqrt{4-(x+1)^2} + 2\sin^{-1}\left(\frac{x+1}{2}\right) + C$

$$-\frac{1}{4}(-2-2x)\sqrt{3-2x-x^2} + 2\arcsin\left(\frac{1}{2}x + \frac{1}{2}\right)$$

40. $\int \frac{1}{\sqrt{4y^2 - 4y - 3}}\,dy = \frac{1}{2}\int \frac{dy}{\sqrt{(y-\frac{1}{2})^2 - 1}} = \frac{1}{2}\int \frac{dy}{\sqrt{u^2-1}} = \frac{1}{2}\ln\left| u + \sqrt{u^2-1} \right| + C$

Let $u = y - \frac{1}{2}$

$du = dy$

$= \frac{1}{2}\ln\left| y - \frac{1}{2} + \sqrt{(y-\frac{1}{2})^2 - 1} \right| + C$

$\sqrt{4y^2 - 4y - 3}$

$= \sqrt{4\left(y^2 - y + \left(\frac{1}{2}\right)^2\right) - 3 - 1}$

$= \sqrt{4\left(y - \frac{1}{2}\right)^2 - 4}$

$= \sqrt{4\left[\left(y - \frac{1}{2}\right)^2 - 1\right]}$

$= 2\sqrt{\left(y - \frac{1}{2}\right)^2 - 1}$

47. $\int x^3(x-1)^{-4}\,dx = \int \frac{(u+1)^3}{u^4}du = \int \frac{u^3+3u^2+3u+1}{u^4}du$

$u = x-1 \Longrightarrow$

$x = u+1$ & $du = dx$ $\quad = \int \frac{du}{u} + \int \frac{3du}{u^2} + \int \frac{3du}{u^3} + \int \frac{du}{u^4}$

$$= \ln|u| + \frac{3}{-1}u^{-1} + \frac{3}{-2}u^{-2} + -\frac{1}{3}u^{-3} + C$$

$$\boxed{= \ln|x-1| - 3(x-1)^{-1} - \frac{3}{2}(x-1)^{-2} - \frac{1}{3}(x-1)^{-3} + C}$$

$$= \ln|x-1| - \frac{3}{x-1} - \frac{3}{2(x-1)^2} - \frac{1}{3(x-1)^3} + C$$

67. $\int \frac{1}{\sqrt{x+1}+\sqrt{x}}\,dx = \int\left(\sqrt{x+1}-\sqrt{x}\right)dx = \int u^{\frac{1}{2}}du + \int x^{\frac{1}{2}}dx$

$u = x+1$

$du = dx$

$= \frac{2}{3}u^{\frac{3}{2}} + \frac{2}{3}x^{\frac{3}{2}} + C$

$$\left(\frac{1}{\sqrt{x+1}+\sqrt{x}}\right)\left(\frac{\sqrt{x+1}-\sqrt{x}}{\sqrt{x+1}-\sqrt{x}}\right) = \frac{\sqrt{x+1}-\sqrt{x}}{x+1-x} = \sqrt{x+1}-\sqrt{x}$$

$$= \boxed{\frac{2}{3}(x+1)^{\frac{3}{2}} + \frac{2}{3}x^{\frac{3}{2}} + C}$$